DIRAC NEUTRINOS IN WIMP AND AXION DARK MATTER SCENARIOS

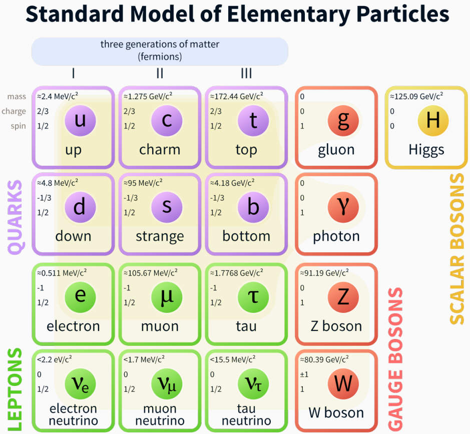
Julio Leite

TPPC online seminar
King's College London
2nd December 2020

Based on
arXiv: 2005.03600 (**JL**, A. Morales, J. W. F. Valle, C. A. Vaquera-Araujo)
arXiv: 2008.10650 (A. G. Dias, **JL** , J. W. F. Valle, C. A. Vaquera-Araujo)

THE STANDARD MODEL

- $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ (Accidental/Automatic anomalous B and L)



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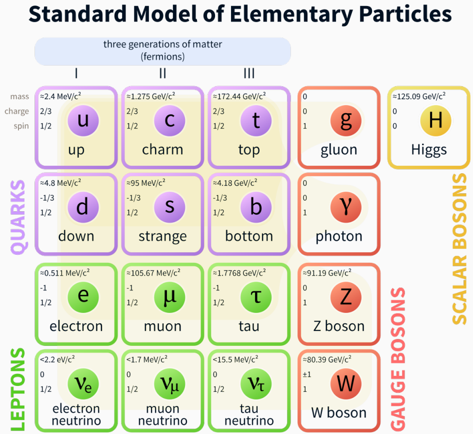
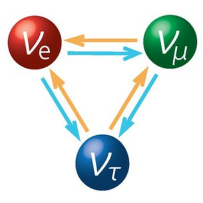


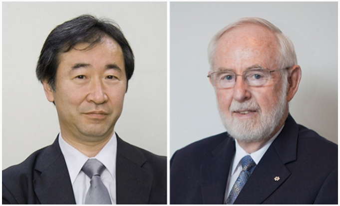
- 3 fermion generations
- Fermion mass hierarchies: $m_f = \lambda_f v_{EW} \Rightarrow \lambda_u/\lambda_t \sim \mathcal{O}(10^{-5})$.
- Lack of suitable **DM** candidates.
- Neutrinos remain massless.

NEUTRINO MASSES

Neutrino masses

- Neutrino oscillations (2015 Nobel Prize)





- $\Delta m^2_{21}, \Delta m^2_{31} \neq 0$ (but very small)

[1] S. Weinberg (1979)

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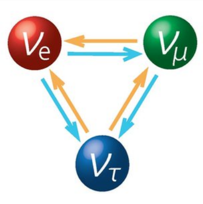
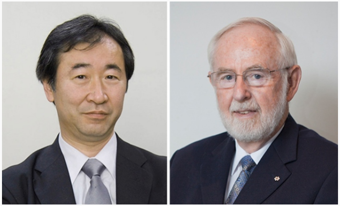
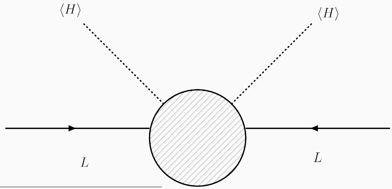




- $\Delta m^2_{21}, \Delta m^2_{31} \neq 0$ (but very small)
- Only LH neutrinos in the SM $\Rightarrow \frac{1}{\Lambda_{NP}} L_L L_L H H$[1]
- The D5WO breaks L (or $B - L$) by 2 units, and the resulting neutrinos are Majorana fermions.



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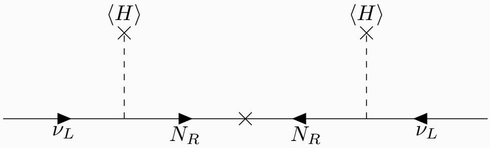
- Simplest tree-level realisation of the D5WO: (type-I) seesaw mechanism.
- SM + N_R (singlets).



$$m_\nu \sim m_D^2/M \propto \left(\frac{v_{EW}}{\Lambda_{NP}}\right) v_{EW} \lesssim \mathcal{O}(0.1)\text{ eV} \quad \text{with} \quad \Lambda_{NP} \sim \mathcal{O}(10^{14})\text{ GeV} \tag{1}$$

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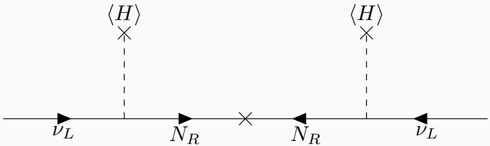

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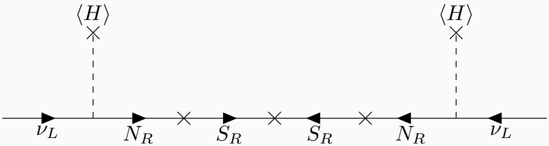
- Low-scale seesaw[2]: SM + N_R (singlets) + S_R (singlets). $((\nu_L)^c, N_R, S_R)$



$$\mathcal{M}^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix} \Rightarrow m_\nu \sim \mu \frac{m_D^2}{M^2} \propto \left(\frac{\mu v_{EW}}{\Lambda_{NP}^2}\right) v_{EW} \tag{2}$$

with $\Lambda_{NP} \sim \mathcal{O}(10)$ TeV and $\mu \sim \mathcal{O}(1)$ keV.

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But it doesn't have to be this way...

- In general, $U(1)_L$ can be broken to Z_N.
- Z_N: cyclic group of N elements x, with $x^{N+1} = x$.
- Generated by $\omega = \exp(2\pi i/N)$.

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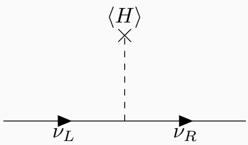

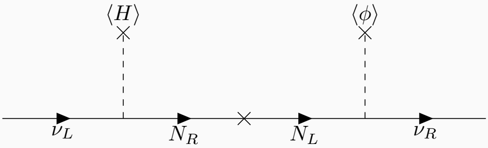

Diracness requires a symmetry.

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Neutrino mass generation

- Neutrinos are Majorana fermions only when
  - no Z_N is left from the breaking of L $(B - L)$
  - or L $(B - L)$ is broken to Z_{2N} with $L_L \sim \omega^N L_L = -L_L$ (as in the D5WO case)

- In addition to tree-level mechanisms, radiative mass generation mechanisms are of particular interest:
  - loop-suppressed masses to explain neutrino lightness.
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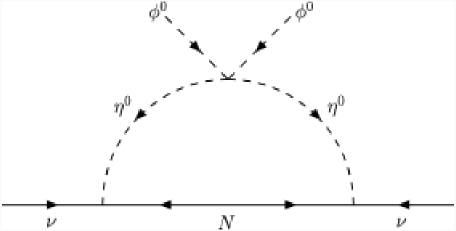
  - new physics at lower scale: richer pheno.
- Symmetries are also generally required to forbid tree-level terms.
- Could we use the symmetries to help us with yet another important issue as that of DM?

- Scotogenic model:[4]
  - SM field content + an inert doublet (η) and fermion singlets (N).
  - *ad hoc* Z_2 symmetry, under which only the BSM fields are odd.



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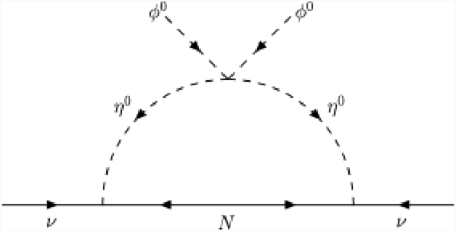

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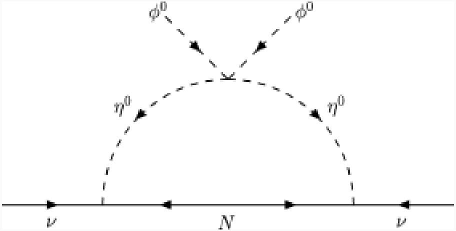

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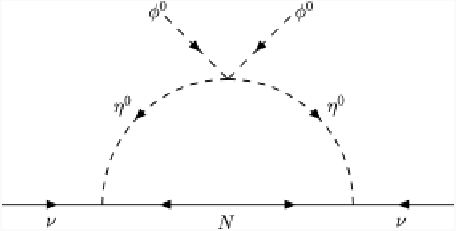

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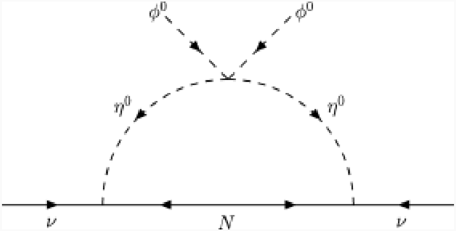

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- That's what we propose next in the context of 3-3-1 models...

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3-3-1 models

SM		3-3-1[5]
$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$\Rightarrow$	$SU(3)_c \otimes SU(3)_L \otimes U(1)_X$
$Q = T_3 + Y$	$\Rightarrow$	$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X$

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$$Q_{aL} = (u_{aL} \ \ d_{aL})^T \sim 2 \quad \Rightarrow \quad Q_{\alpha L} = (d_{\alpha L} \ \ -u_{\alpha L} \ \ D_{\alpha L})^T \sim 3^*$$

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- The charged RH fields come as singlets of $SU(3)_L$: $e_{aR}, d_{aR}, u_{aR}, D_{\alpha R}, U_{3R}$.

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- Anomaly cancellation + asymptotic freedom: # generations = # colours = 3
- $B - L$ symmetry has a non-trivial realisation given by (3-3-1-1[6])

$$B - L = -\frac{4}{\sqrt{3}} T_8 + N \,, \tag{3}$$

and can be promptly promoted to local.

[5] M. Singer, J. W. F. Valle, J. Schechter (1980)

[6] P. V. Dong, T. D. Tham, H. T. Hung (PRD 2013)

3-3-1 Models

- The minimal scalar sector contains

$$\Phi_1, \Phi_3 \equiv \begin{pmatrix} \phi_1^0 \\ \phi_1^- \\ \tilde{\phi}_1^0 \end{pmatrix}, \begin{pmatrix} \phi_3^0 \\ \phi_3^- \\ \tilde{\phi}_3^0 \end{pmatrix}; \Phi_2 \equiv \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \\ \tilde{\phi}_2^+ \end{pmatrix}, \ \langle \phi_1^0 \rangle^2 + \langle \phi_2^0 \rangle^2 = v_{EW}^2 \text{ and } \langle \tilde{\phi}_3^0 \rangle \propto w \quad (4)$$

- Symmetry breaking happens in two stages:

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \xrightarrow{w} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{v_{EW}} SU(3)_c \otimes U(1)_Q \,. \quad (5)$$

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- In the gauge sector, we have extra bosons: $Z_2, V^\pm, V^0, V^{0\dagger}$, with $m \propto w$.
- At tree level, $M_{Z_2}^2/M_{V^0}^2 \approx 1.48$ and $M_{V^\pm}^2 - M_{V^0}^2 = M_{W^\pm}^2$.

$$\Delta\rho_0 \equiv \frac{M_W^2}{\cos^2\theta_W M_{Z_1}^2} - 1 \approx \frac{(v_{EW}/w)^2}{4\cos^4\theta_W} \quad \Rightarrow \ w \gtrsim 6.5 \text{ TeV} \quad (6)$$

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- The model is such that by forbidding $\Phi_1\Phi_2\Phi_3$ and $\overline{\Psi_L}\Phi_2^*(\Psi_L)^c$, a Peccei-Quinn symmetry[7] appears.
- $\overline{\Psi_L}\Phi_2^*(\Psi_L)^c$ generates a massless and two degenerate neutrinos $m_\nu \propto \langle \phi_2^0 \rangle$.

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SCOTOGENIC 3-3-1-1 MODEL

- The fermions and scalars are[8]

Field	3-3-1-1 rep	$B-L$	$M_P = (-1)^{3(B-L)+2s}$	$\mathrm{U}(1)_{\mathrm{PQ}}$
$Q_{\alpha L}$	$(\mathbf{3}, \mathbf{3}^*, 0, -\frac{1}{3})$	$(\frac{1}{3}, \frac{1}{3}, -\frac{5}{3})^T$	$(+++)^T$	1
Q_{3L}	$(\mathbf{3}, \mathbf{3}, \frac{1}{3}, 1)$	$(\frac{1}{3}, \frac{1}{3}, \frac{7}{3})^T$	$(+++)^T$	1
u_{aR}	$(\mathbf{3}, \mathbf{1}, \frac{2}{3}, \frac{1}{3})$	$\frac{1}{3}$	$+$	4
U_{3R}	$(\mathbf{3}, \mathbf{1}, \frac{2}{3}, \frac{7}{3})$	$\frac{7}{3}$	$+$	4
d_{aR}	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, \frac{1}{3})$	$\frac{1}{3}$	$+$	-2
$D_{\alpha R}$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -\frac{5}{3})$	$-\frac{5}{3}$	$+$	-2
ψ_{aL}	$(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, -\frac{1}{3})$	$(-1, -1, +1)^T$	$(+++)^T$	-3
e_{aR}	$(\mathbf{1}, \mathbf{1}, -1, -1)$	-1	$+$	-6
S_{aR}	$(\mathbf{1}, \mathbf{1}, 0, 0)$	0	$-$	0
$\mathbf{\Phi}_1$	$(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, \frac{2}{3})$	$(0, 0, 2)^T$	$(+++)^T$	-3
$\mathbf{\Phi}_2$	$(\mathbf{1}, \mathbf{3}, \frac{2}{3}, \frac{2}{3})$	$(0, 0, 2)^T$	$(+++)^T$	3
$\mathbf{\Phi}_3$	$(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, -\frac{4}{3})$	$(-2, -2, 0)^T$	$(+++)^T$	-3
$\mathbf{\Phi}_4$	$(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, -\frac{1}{3})$	$(-1, -1, 1)^T$	$(---)^T$	-3

- Neutrino masses – $\overline{\mathbf{\Psi}_L}\mathbf{\Phi}_2^*(\mathbf{\Psi}_L)^c \to v_2\overline{\nu_L}\nu_R$ – are forbidden by $U(1)_{PQ}$.
- However, $U(1)_{PQ}$ is softly broken by $\mathbf{\Phi}_1\mathbf{\Phi}_2\mathbf{\Phi}_3$ (no visible axion field).

[8] **JL**, A. Morales, J. W. F. Valle and C. A. Vaquera-Araujo (2005.03600 – PRD 2020)

- The resulting scalar potential is given by

$$V = \sum_{i=1}^{4}\left[\mu_i^2\Phi_i^\dagger\Phi_i + \lambda_i(\Phi_i^\dagger\Phi_i)^2\right] + \sum_{i<j}\left[\lambda_{ij}(\Phi_i^\dagger\Phi_i)(\Phi_j^\dagger\Phi_j) + \tilde{\lambda}_{ij}(\Phi_i^\dagger\Phi_j)(\Phi_j^\dagger\Phi_i)\right]$$
$$+\left(-\frac{\mu_\phi}{\sqrt{2}}\Phi_1\Phi_2\Phi_3 + \frac{\lambda'}{2}\Phi_1^\dagger\Phi_4\Phi_3^\dagger\Phi_4 + \mathrm{h.c.}\right), \quad (7)$$

where the cubic term characterised by μ_ϕ breaks $\mathrm{U}(1)_{\mathrm{PQ}}$ softly.

- The scalars follow The scalar multiplets are decomposed as

$$\Phi_1 = \begin{pmatrix} \frac{v_1+s_1+ia_1}{\sqrt{2}} \\ \phi_1^- \\ \tilde{\phi}_1^0 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2+s_2+ia_2}{\sqrt{2}} \\ \tilde{\phi}_2^+ \end{pmatrix}, \quad \Phi_3 = \begin{pmatrix} \phi_3^0 \\ \phi_3^- \\ \frac{w+s_3+ia_3}{\sqrt{2}} \end{pmatrix}, \quad \Phi_4 = \begin{pmatrix} \phi_4^0 \\ \phi_4^- \\ \tilde{\phi}_4^0 \end{pmatrix},$$

with $w^2 \gg v_1^2 + v_2^2 \equiv v_{EW}^2$.

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with $w^2 \gg v_1^2 + v_2^2 \equiv v_{EW}^2$.

- The soft breaking of $U(1)_{PQ}$ leads to a massive pseudoscalar

$$A_1 = \frac{v_2 w a_1 - v_1 w a_2 + v_1 v_2 a_3}{\sqrt{v_1^2 v_2^2 + v_1^2 w^2 + v_2^2 w^2}}, \qquad \text{with} \qquad m_{A_1}^2 = \frac{\mu_\phi\left(v_1^2 v_2^2 + v_1^2 w^2 + v_2^2 w^2\right)}{2 v_1 v_2 w}.$$

- In the limit $\mu_\phi \to 0$, $U(1)_{PQ}$ is recovered and A_1 would become a visible axion.

FERMION MASSES

Quark masses

- Allowed Yukawa interactions for the quarks

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Yq}} \quad = \quad & y^{u}_{\alpha a}\,\overline{Q^{\alpha}_{L}}\,\Phi_2^{*}\,u^{a}_{R} + y^{u}_{3a}\,\overline{Q^{3}_{L}}\,\Phi_1\,u^{a}_{R} + y^{U}_{33}\,\overline{Q^{3}_{L}}\,\Phi_3\,U^{3}_{R} \\
& + y^{d}_{3a}\,\overline{Q^{3}_{L}}\,\Phi_2\,d^{a}_{R} + y^{d}_{\alpha a}\,\overline{Q^{\alpha}_{L}}\,\Phi_1^{*}\,d^{a}_{R} + y^{D}_{\alpha\beta}\,\overline{Q^{\alpha}_{L}}\,\Phi_3^{*}\,D^{\beta}_{R} + \mathrm{h.c.}
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- leading to, in the bases (u_a, U_3) and (d_a, D_α), respectively,

$$M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} -v_2 y^{u}_{11} & -v_2 y^{u}_{12} & -v_2 y^{u}_{13} & 0 \\ -v_2 y^{u}_{21} & -v_2 y^{u}_{22} & -v_2 y^{u}_{23} & 0 \\ v_1 y^{u}_{31} & v_1 y^{u}_{32} & v_1 y^{u}_{33} & 0 \\ 0 & 0 & 0 & w y^{U}_{33} \end{pmatrix} = \begin{pmatrix} m^{u}_{3\times 3} & 0_{3\times 1} \\ 0_{1\times 3} & \frac{w y^{U}_{33}}{\sqrt{2}} \end{pmatrix}, \tag{9}$$

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- $\mathrm{U(1)_{B-L}}$ conservation ensures that the standard and the new quarks do not mix.
- The CKM matrix is unitary and defined as $\mathsf{V_{CKM}} = (\mathcal{U}^{\mathsf{u}}_{\mathsf{L}})^\dagger \mathcal{U}^{\mathsf{d}}_{\mathsf{L}}$.

Lepton masses

- For the leptons: $-\mathcal{L}_1 = y^e\,\overline{\psi_L}\,\Phi_2 e_R + y^S_{ab}\,\overline{\psi^b_L}\,\Phi_4 S^a_R + \frac{M^S_{ab}}{2}\overline{(S^a_R)^c}S^b_R + \mathrm{h.c.}$
- Charged lepton masses $\Rightarrow M_e = y^e v_2/\sqrt{2}$.

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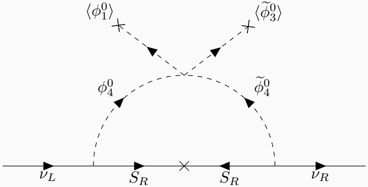
- As $\mathrm{U(1)_{B-L}}$ remains exactly conserved, Φ_4 does not acquire a vev and neutrino masses are only generated at loop level.



$$(m_\nu)_{ab} = \sum_{k=1}^{3} \frac{M_k y^S_{ka} y^S_{kb} \sin 2\theta}{32\pi^2} \left[\frac{m^2_{\eta_1}}{m^2_{\eta_1} - M^2_k} \ln \frac{m^2_{\eta_1}}{M^2_k} - \frac{m^2_{\eta_2}}{m^2_{\eta_2} - M^2_k} \ln \frac{m^2_{\eta_2}}{M^2_k} \right]. \tag{11}$$

with $(\eta_1, \eta_2) = R(\theta)(\widetilde{\phi}^0_4, \phi^{0*}_4)$ and $\tan 2\theta = \frac{v_2 w \lambda'}{v_2^2 \widetilde{\lambda}_{24} - w^2 \widetilde{\lambda}_{34}}$.

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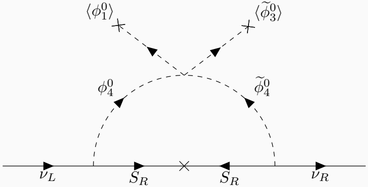

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- The internal fields are M_P-odd so that the lightest, either $\underline{\eta_1}$ or $\underline{S_1}$, is stable.

- To illustrate the **DM** viability, we consider a simplified scenario:
  - where all the non-SM fields, except $\eta_{1,2}$, are heavy and decouple
  - η_1 is the lightest and mostly a SM singlet;
  - $0 < |\lambda| < 1$;
  - $0 < |\theta| < 0.01$ (only natural since $\theta \propto v/w$ - avoids DD constraints);
  - $0 < m_{\eta_1} < 10^4$ GeV and $m_{\eta_1} < m_{\eta_2} < 1.1 m_{\eta_1}$ (allowing for co-annihilation);

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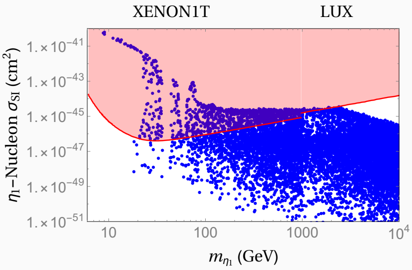

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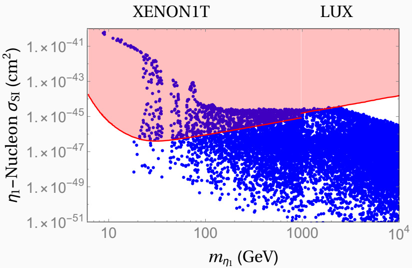

- In this very simplified scenario: only the Higgs and Z portals are available.
- The parameter space can become much richer if S_k, Z' and Z'' become lighter.

Axion and Dirac Neutrinos in 3-3-1 Theory

Field[9]	3-3-1-1 rep	$B-L$	$\mathrm{U(1)_{PQ}}$
ψ_{aL}	$\left(1,3,-\frac{1}{3},-\frac{1}{3}\right)$	$(-1,-1,+1)^T$	$\frac{1}{2}(-PQ_\sigma+PQ_{\Phi_1}+PQ_{\Phi_3})$
e_{aR}	$(1,1,-1,-1)$	-1	$\frac{1}{2}(PQ_\sigma+3PQ_{\Phi_1}+3PQ_{\Phi_3})$
$Q_{\alpha L}$	$\left(3,3^*,0,-\frac{1}{3}\right)$	$\left(\frac{1}{3},\frac{1}{3},-\frac{5}{3}\right)^T$	$PQ_{Q_{\alpha L}}$
Q_{3L}	$\left(3,3,\frac{1}{3},1\right)$	$\left(\frac{1}{3},\frac{1}{3},\frac{7}{3}\right)^T$	$PQ_{Q_{\alpha L}}-PQ_\sigma-PQ_{\Phi_3}$
u_{aR}	$\left(3,1,\frac{2}{3},\frac{1}{3}\right)$	$\frac{1}{3}$	$PQ_{Q_{\alpha L}}-(PQ_\sigma+PQ_{\Phi_1}+PQ_{\Phi_3})$
U_{3R}	$\left(3,1,\frac{2}{3},\frac{7}{3}\right)$	$\frac{7}{3}$	$PQ_{Q_{\alpha L}}-PQ_\sigma-2PQ_{\Phi_3}$
d_{aR}	$\left(3,1,-\frac{1}{3},\frac{1}{3}\right)$	$\frac{1}{3}$	$PQ_{Q_{\alpha L}}+PQ_{\Phi_1}$
$D_{\alpha R}$	$\left(3,1,-\frac{1}{3},-\frac{5}{3}\right)$	$-\frac{5}{3}$	$PQ_{Q_{\alpha L}}+PQ_{\Phi_3}$
S_{aL}	$(1,1,0,-1)$	-1	$\frac{1}{2}(PQ_\sigma-PQ_{\Phi_1}+PQ_{\Phi_3})$
S_{aR}	$(1,1,0,-1)$	-1	$\frac{1}{2}(-PQ_\sigma-PQ_{\Phi_1}+PQ_{\Phi_3})$
Φ_1	$\left(1,3,-\frac{1}{3},\frac{2}{3}\right)$	$(0,0,2)^T$	PQ_{Φ_1}
Φ_2	$\left(1,3,\frac{2}{3},\frac{2}{3}\right)$	$(0,0,2)^T$	$-(PQ_\sigma+PQ_{\Phi_1}+PQ_{\Phi_3})$
Φ_3	$\left(1,3,-\frac{1}{3},-\frac{4}{3}\right)$	$(-2,-2,0)^T$	PQ_{Φ_3}
σ	$(1,1,0,0)$	0	PQ_σ

- $[\mathrm{SU(3)_c}]^2\times\mathrm{U(1)_{PQ}}$ anomaly coefficient: $C_{ag}=PQ_\sigma$.

[9] A. G. Dias, **JL** , J. W. F. Valle, C. A. Vaquera-Araujo (2008.10650 – PLB 2020)

NEUTRINO MASSES

Neutrino masses

- Charged fermion masses are generated in the same way as in the previous model.

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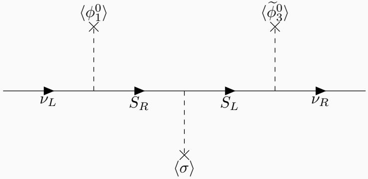
- On the other hand, neutrino masses arise from

$$- \mathcal{L}_{\mathrm{Y}_\nu} = y^{\nu_1}\, \overline{\psi_L}\, \Phi_1\, S_R + y^{\nu_2}\, \overline{\psi_L}\, \Phi_3\, (S_L)^c + y^S\, \overline{S_L}\, \sigma\, S_R + \mathrm{h.c.} \tag{12}$$

- In the basis $N = (\nu, S)$, we have $\Rightarrow M_D = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & y^{\nu_1} v_1 \\ (y^{\nu_2})^T w & y^S v_\sigma \end{pmatrix}$.
- Since $v_\sigma \gg w \gg v$, we have a Type-I Dirac seesaw mechanism:

$$m_\nu^D \simeq \frac{y^{\nu_1} (y^S)^{-1} (y^{\nu_2})^T}{\sqrt{2}} v_1 \left(\frac{w}{v_\sigma} \right).$$



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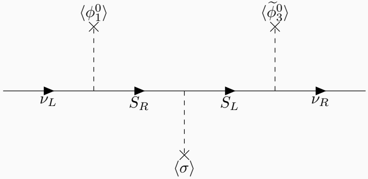

- The Dirac seesaw with $v_\sigma \simeq v_{PQ}$ suggests *New Physics* at a lower (3-3-1) scale w.
- For $v_1, w, v_\sigma \sim 10^2, 10^4, 10^{12}$ GeV $\Rightarrow m_\nu^D \sim 0.1$ eV for $y^S \sim 1$, $y^{\nu_1,\nu_2} \sim 10^{-2}$.

AXION PROPERTIES

- The axion profile is given by

$$a = \frac{1}{f_{PQ}} \left[v_1 PQ_{\Phi_1}\, a_1 + v_2 PQ_{\Phi_2}\, a_2 + w PQ_{\Phi_3}\, a_3 + v_\sigma PQ_\sigma\, a_\sigma \right] , \tag{13}$$

where $f_{PQ} = PQ_\sigma \sqrt{v_\sigma^2 + \frac{v_1^2 v_2^2 w^2}{v_1^2 v_2^2 + v_{EW}^2 w^2}}$, with

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-

$$\frac{PQ_{\Phi_1}}{PQ_\sigma} = -(\cos\delta \cos\beta)^2, \quad \frac{PQ_{\Phi_2}}{PQ_\sigma} = -(\cos\delta \sin\beta)^2, \quad \frac{PQ_{\Phi_3}}{PQ_\sigma} = -(\sin\delta)^2, \tag{14}$$

where

$$\tan\delta = \frac{v_1 v_2}{w v_{EW}} \quad \text{and} \quad \tan\beta = \frac{v_1}{v_2}. \tag{15}$$

- The charges satisfy $\sum_i PQ_{\Phi_i} = PQ_\sigma$ as a result of $\sigma\Phi_1\Phi_2\Phi_3$ in the potential.

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- The charges satisfy $\sum_i PQ_{\Phi_i} = PQ_\sigma$ as a result of $\sigma\Phi_1\Phi_2\Phi_3$ in the potential.
- Therefore, for $v_\sigma \gg w \gg v_1, v_2$, we have $a \simeq a_\sigma$.

- The axion couplings to charged leptons are $-ig_{ae}\, a\, \overline{e'}\gamma^5 e'$ with

$$g_{ae} = \frac{\text{diag}(m_e, m_\mu, m_\tau)}{f_a} c_{ae} \quad \text{and} \quad c_{ae} = \frac{C_{ae}}{C_{ag}} = -\cos^2\delta\, \sin^2\beta. \tag{16}$$

  Since $\delta \to 0$ $(w \gg v_{EW}) \Rightarrow g_{ae} \sim 3 \times (g_{ae})_{DFSZ}$.[10]

- $f_a = f_{PQ}/N_{DW}$, where $N_{DW} = 1$, whereas $N_{DW} = 3$ in the DFSZ model.

[10] Dine, M. and Fischler, W. and Srednicki, M. (PLB 1981); Zhitnitsky, A.R., (SJNP 1980).

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- $f_a = f_{PQ}/N_{DW}$, where $N_{DW} = 1$, whereas $N_{DW} = 3$ in the DFSZ model.
- For the quarks, we have flavour-violating interactions

$$ia\, \overline{q'_i}\left[(g^V_{aq})_{ij} - (g^A_{aq})_{ij}\gamma^5\right] q'_j, \tag{17}$$

with $q' = u', d'$ and

$$(g^V_{au})_{ij} = \frac{m^u_i - m^u_j}{2f_a}\cos^2\delta \times X^u_{ij}, \quad (g^A_{au})_{ij} = \frac{m^u_i + m^u_j}{2f_a}\cos^2\delta\left[\sin^2\beta \times \delta_{ij} + X^u_{ij}\right],$$
$$(g^V_{ad})_{ij} = \frac{m^d_i - m^d_j}{2f_a}\cos^2\delta \times X^d_{ij}, \quad (g^A_{ad})_{ij} = \frac{m^d_i + m^d_j}{2f_a}\cos^2\delta\left[\cos^2\beta \times \delta_{ij} + X^d_{ij}\right],$$

with

$$X^q_{ij} = \left[(\mathcal{U}^q_L)^\dagger \mathrm{diag}\,(0,0,-1)\,\mathcal{U}^q_L\right]_{ij}, \qquad q = u, d.$$

[10] Dine, M. and Fischler, W. and Srednicki, M. (PLB 1981); Zhitnitsky, A.R., (SJNP 1980).

- From the $[\mathrm{U}(1)_{\mathrm{Q}}]^2 \times \mathrm{U}(1)_{\mathrm{PQ}}$ anomaly coefficient: $C_{a\gamma} = -\frac{4}{3} PQ_\sigma$, we can calculate

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma}}{4} a \, F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{18}$$

where the axion-to-photon coupling is

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{C_{a\gamma}}{C_{ag}} - \frac{2}{3}\frac{4+z}{1+z}\right) \approx \frac{\alpha}{2\pi f_a}\left(-\frac{4}{3} - 1.95\right). \tag{19}$$

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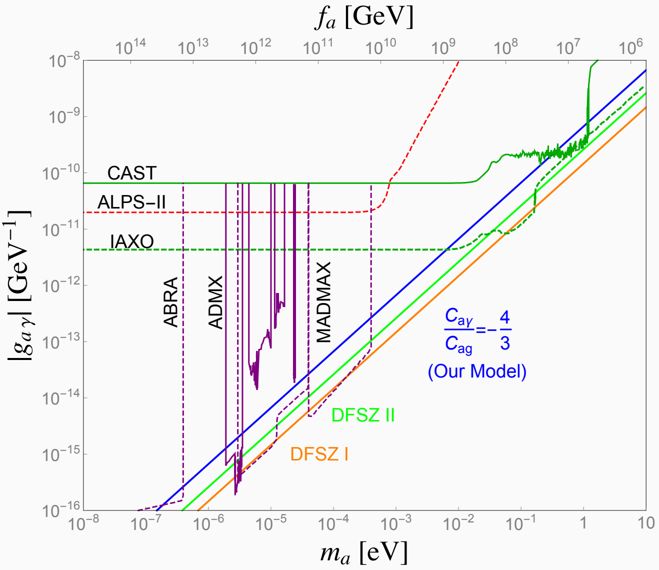
- Enhanced value of $|g_{a\gamma}| \sim 3.3\frac{\alpha}{2\pi f_a}$ when compared to popular axion models, such as
  - DFSZ-I: $C_{a\gamma}/C_{ag} = 8/3$, $|g_{a\gamma}| \sim 0.7\frac{\alpha}{2\pi f_a}$.
  - DFSZ-II: $C_{a\gamma}/C_{ag} = 2/3$, $|g_{a\gamma}| \sim 1.3\frac{\alpha}{2\pi f_a}$
  - KSVZ[11]: $C_{a\gamma}/C_{ag} = 0$, $|g_{a\gamma}| \sim 1.95\frac{\alpha}{2\pi f_a}$

[11] Kim, J. E. (PRL 1979); Shifman, M. A. and Vainshtein, A.I. and Zakharov, V. I., (NPB 1980)



f_a [GeV]
10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8} 10^{7} 10^{6}
$|g_{a\gamma}|$ [GeV^{-1}]
10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12} 10^{-13} 10^{-14} 10^{-15} 10^{-16}
CAST
ALPS–II
IAXO
ABRA
ADMX
MADMAX
$\frac{C_{a\gamma}}{C_{ag}} = -\frac{4}{3}$
(Our Model)
DFSZ II
DFSZ I
10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10
m_a [eV]

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- ν masses can be generated in a number of ways, all of which require BSM physics.
- The smallness of ν masses suggests a suppression mechanism:
  - (high-scale) seesaw
  - radiative generation
- ν's nature (Dirac or Majorana) is related to the $U(1)_L$ symmetry, accidental in the SM.
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- We have presented two 3-3-1 constructions where Dirac neutrino masses and dark matter are interconnected.
- Scotogenic model:
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  - **DM** can be either a scalar or a fermion.
- Axion model:
  - Dirac seesaw: $m_\nu^D \propto v_1(w/v_\sigma) \Rightarrow$ NP at a relatively low (3-3-1) scale: $w(\sim 10\text{TeV}) \ll v_\sigma(\sim 10^{12}\text{GeV})$.
  - The model is a hybrid DFSZ-KSVZ construction, but with dominant DFSZ-like features.
  - Flavour-violating axion couplings to quarks follow from the gauge structure.
  - Axion couplings to electrons and photons are enhanced.

Cheers!

BACK-UPS

Stueckelberg mechanism

- Covariant derivative $SU(3)_L \otimes U(1)_X \otimes U(1)_N$

$$D_\mu \Phi_i = \left[\partial_\mu + i g_L \frac{\lambda^{a\,*}}{2} W^a_\mu - i g_X X B_\mu - i g_N N C_\mu\right] \Phi_i = \left(\partial_\mu + i \frac{g_L}{2} \mathcal{P}_\mu\right) \Phi_i \tag{20}$$

the diagonal terms of $\mathcal{P}_\mu$, with $t_{X,N} = g_{X,N}/g_L$, are

$$\left(W^3 + \frac{W^8}{\sqrt{3}} - 2\left(t_X X B + t_N N C\right), -W^3 + \frac{W^8}{\sqrt{3}} - 2\left(t_X X B + t_N N C\right), -2\left(\frac{W^8}{\sqrt{3}} + t_X X B + t_N N C\right)\right)_\mu, \tag{21}$$

- After SSB, in the basis $\mathcal{B}^T_\mu = (W^3_\mu, W^8_\mu, B_\mu, C_\mu)$, we have

$$\mathcal{L} \supset \frac{1}{2} \mathcal{B}^T_\mu M_0^2 \mathcal{B}^\mu + \frac{1}{2}(m C^\mu - \partial^\mu \sigma)^2 + \mathcal{L}^{\mathrm{St}}_{\mathrm{gf}}. \tag{22}$$

M_0^2 comes from the Higgs mechanism; m is the Stueckelberg mass of the C^μ; and σ is the scalar Stueckelberg compensator

$$C^\mu \to C^\mu + \partial^\mu \Omega(x), \quad \sigma \to \sigma + m\Omega(x). \tag{23}$$

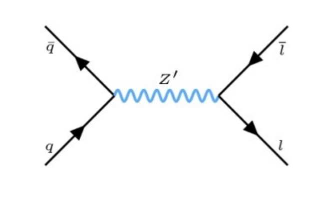
- Diagonalising by blocks, we find $m^2_Z \simeq m^2_{Z_{SM}} + \mathcal{O}((v_{EW}/w)^2)$ and

$$m^2_{Z', Z''} = \frac{1}{18}\left\{w^2 g_L^2\left(16 t_N^2 + t_X^2 + 3\right) + 9m^2 \mp \mathcal{G}\sqrt{\left[w^2 g_L^2\left(16 t_N^2 - t_X^2 - 3\right) + 9m^2\right]^2 + 64 w^4 g_L^4 t_N^2\left(t_X^2 + 3\right)}\right\},$$

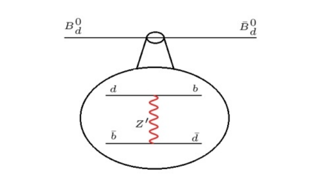
with $\mathcal{G} = \mathrm{sign}[w^2 g_L^2\left(16 t_N^2 - t_X^2 - 3\right) + 9m^2]$.

- In the limit $m \to 0$, one of these becomes massless.

- di-leptons searches at LHC[12]



- $B_d^0 - \bar{B}_d^0$ mass difference



[12] F. S. Queiroz, C. Siqueira and J. W. F. Valle (PLB 2016)