Gravitationally produced dark matter and its entropy

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Motivation

- Existence of Dark Matter (DM) has been confirmed by many astrophysical and cosmological observations.
- Yet, direct detection remains elusive and all the evidence is consistent with DM interacting solely with gravity.
- Cosmological particle production : Feasible mechanism to produce DM particle candidates interacting only with gravity.
- The DM abundance only depends on cosmological history and mass, independent of couplings to any other degrees of freedom.

Summary of main results:

- Bosonic light (DM) saturates the dark matter abundance for $m \simeq 1.5 \times 10^{-5}\mathrm{eV}$ with an equation of state parameter $w \simeq 10^{-14}$ (CDM) and a free streaming length $\lambda_{fs} \simeq 70\,\mathrm{pc}$. Its distribution function is enhanced in the infrared describing a near Bose Condensed phase.
- Fermionic (DM) features a nearly thermal distribution function with an "emergent temperature" $T_H = H_0\sqrt{\Omega_R} \simeq 10^{-36}(\mathrm{eV})$ and saturates the (CDM) abundance for a mass $m \simeq 10^8\,\mathrm{GeV}$.
- After renormalizing the zero point contribution and averaging over fast interference terms, the energy momentum tensor features the kinetic fluid form to leading order in the adiabatic expansion during matter domination.
- Fast oscillations from "out" particle-antiparticle interference, introduce decoherence and the emergence of a coarse grained entropy. It features a kinetic form and is recognized as the entanglement entropy as a consequence of gravitational production resulting in entangled pairs.

Preliminaries and main assumptions

- Spatially flat FRW cosmology in conformal time $d\eta = dt/a$:

$$g_{\mu\nu}(\eta) = a^2(\eta)\,\mathrm{diag}(1,-1,-1,-1)\,.$$

- The DM particle only interacts with gravity and has no vev.
- The cosmological dynamics is considered as a background: during inflation - the inflaton field and during RD - by SM degrees of freedom (and/or beyond).
- All DM fields are in their Bunch-Davies vacuum state during inflation, which is closest to Minkowski at short distance.
- Light fields: mass $\ll H_{dS}$, $H_{dS} \lesssim 10^{13}\,\mathrm{GeV}$ (upper bound from Planck) is the Hubble scale during inflation (taken as de Sitter space-time).

Preliminaries and main assumptions (contd)

- Inflation $\xrightarrow[\textit{transition}]{\textit{instantaneous}}$ Post-inflation RD era.
- Occurs at around $a_{ds} \simeq 10^{-28} \ll a_{eq} \simeq 10^{-4}$, (from upper bound on H_{dS} from Planck),
- The cosmologically relevant modes are super-Hubble at the end of inflation, with comoving wavevectors k for which $\lambda \gg \mathrm{few\,mts}$.
- They are causally disconnected and insensitive to the reheating dynamics post-inflation, thus removing model dependence.

Cosmological Particle Production

- In state : Bunch-Davies vacuum state $|0_I\rangle$ during inflation.
- Asymptotic out states during (MD): Adiabatic WKB form of mode function solns: "in" $\longrightarrow$ "out" $\longrightarrow$ **particle production**.
- We will analyse the energy-momentum tensor at the time when matter begins to dominate the expansion.
- Bogoliubov transformation: a unitary transformation relating the in and out states, from which we extract the distribution function of the out particles.

Complex Scalar Fields : Model

- In conformal time and upon conformal field rescaling, the action is given by:

$$S = \int d^3x d\eta \left\{ \chi^{\dagger\,'} \chi' - \nabla\chi^\dagger \nabla\chi - M^2(\eta)\chi^\dagger\chi \right\},$$

$$M^2(\eta) = m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)}$$

- Time dependent *mass* $\rightarrow$ *particle production*.
- $\chi(\vec{x},\eta) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \left[a_{\vec{k}}\, g_k(\eta)\, e^{-i\vec{k}\cdot\vec{x}} + b^\dagger_{\vec{k}}\, g^*_k(\eta)\, e^{i\vec{k}\cdot\vec{x}} \right]$
- $g^{''}_k(\eta) + \left[k^2 + m^2\, a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \right] g_k(\eta) = 0$
- Will analyse mode functions corresponding to inflation $g^<_k(\eta)$ and RD era(including adiabatic regime) $g^>_k(\eta)$.

Adiabatic Regime

- Late during (RD) and throughout (MD) there exists a wide separation of time scales: $\frac{1}{m} \ll \frac{1}{H(t)}$ where $1/H(t)$: expansion time scale and $1/m$:microscopic time scale.
- Adiabaticity Condition: during (RD) epoch $a(\eta) = H_R\, \eta$; $H_R \approx 10^{-35}\, eV$ and $\omega_k(\eta) = \sqrt{k^2 + m^2\, a^2(\eta)}$ imply **slowly varying frequencies**

$$\frac{\omega_k'(\eta)}{\omega_k^2(\eta)} \ll 1 \implies \frac{a'(\eta)}{m\, a^2(\eta)} = \frac{H_R}{m\, a^2(\eta)} \ll 1 \Longrightarrow a(\eta) \gg \frac{10^{-17}}{\sqrt{m_{ev}}}$$

- In (RD) the mode functions are combinations of Weber parabolic cylinder functions with "out" boundary conditions $f_k(\eta)$ with the asymptotic behavior

$$f_k(\eta) \to \frac{e^{-i\int^{\eta} \omega_k(\eta')d\eta'}}{\sqrt{2\omega_k(\eta)}}$$

defining "particles" in the out state.

Complex Scalar Fields : Mode Functions

- The matching conditions from Inflation to RD ($\eta = \eta_R$):

$$\begin{aligned} g^{<}_k(\eta_R) &= g^{>}_k(\eta_R) \\ \frac{d}{d\eta} g^{<}_k(\eta)\Big|_{\eta_R} &= \frac{d}{d\eta} g^{>}_k(\eta)\Big|_{\eta_R} . \end{aligned} \tag{1}$$

- In state mode functions during inflation satisfying Bunch-Davies vacuum bc. with $m << H_{dS}$: Hankel functions.
- The mode function $g^{>}_k(\eta)$ is a linear combination of Weber parabolic cylinder functions with "out" particle boundary conditions with coefficients $A_k; B_k$:
-

$$g^{>}_k(\eta) = A_k\, f_k(\eta) + B_k\, f^{*}_k(\eta)$$

Complex Scalar Fields : Distribution Function

- A, B are obtained from matching conditions at the inflation-(RD) transition. Expand field in terms of the solution f_k with "out" boundary conditions, and creation/annihilation operators related to a_k, b_K by a Bogoliubov transformation determined by the coefficients A_k, B_k.
- Number of out (anti)particles is obtained from this Bogoliubov transformation: **distribution function**

$$N_k = |B_k|^2 = \overline{N}_k \, .$$

- $N_k \simeq \frac{1}{16\sqrt{2}} \left(\frac{H_{dS}}{m}\right)^2 \frac{D(z)}{z^3}$, $z = \frac{k}{\sqrt{2mH_R}}$ with $D(z)$ a smooth function with a rapid fall off for $z \gg 1$
- Infrared enhancement of $N_k \propto 1/k^3$ and $H_{dS}/m \gg 1$ implies $N_k \gg 1$ for $z \ll \sqrt{H_{dS}/m}$. Consequence of IR behavior of mode functions for light scalar fields during inflation. The distribution function during (RD,MD) "inherits" IR enhancement from inflation.
  Like a Bose Einstein condensate: large occupation in a narrow range of momenta, but without SSB.

Fermionic Fields : Model

- Spinors are defined via a spin connection, but simplify in spatially flat FRW with a conformal rescaling to a field ψ and Minkowski γ^μ matrices.
- Upon conformal rescaling of fields and in conformal time, the action is given by:

$$S = \int d^3x \, d\eta \, \overline{\psi} \Big[i \not\partial - M(\eta) \Big] \psi \; ; M(\eta) = m \, a(\eta)$$

- $\psi(\vec{x},\eta) = \frac{1}{\sqrt{V}} \sum_{\vec{k},s} \Big[b_{\vec{k},s} \, U_s(\vec{k},\eta) + d^\dagger_{-\vec{k},s} \, V_s(-\vec{k},\eta) \Big] \, e^{i\vec{k}\cdot\vec{x}}$
- Dirac equations :

$$\Big[i \, \gamma^0 \, \partial_\eta - \vec{\gamma} \cdot \vec{k} - M(\eta) \Big] U_s(\vec{k},\eta) = 0 \tag{2}$$

$$\Big[i \, \gamma^0 \, \partial_\eta - \vec{\gamma} \cdot \vec{k} - M(\eta) \Big] V_s(-\vec{k},\eta) = 0 \, . \tag{3}$$

- DE is first order in time $\Longrightarrow$ matching condition from inflation to RD :

$$\psi^<(\vec{x},\eta_R) = \psi^>(\vec{x},\eta_R) \, .$$

Fermionic Fields : Mode Functions

- U, V spinors solved exactly:
  - Inflation (IN state) : Hankel Functions
  - RD (OUT state with adiabatic BC) : Parabolic Cylinder functions.
- Matching conditions imply :

$$U^{<}_s(\vec{k},\eta_R) = U^{>}_s(\vec{k},\eta_R)\,, \tag{4}$$

$$V^{<}_s(-\vec{k},\eta_R) = V^{>}_s(-\vec{k},\eta_R)\,. \tag{5}$$

- During (RD) :

$$U^{>}_s(\vec{k},\eta) = A_{k,s}\,\mathcal{U}_s(\vec{k},\eta) + B_{k,s}\,\mathcal{V}_s(-\vec{k},\eta) \tag{6}$$

$$V^{>}_s(-\vec{k},\eta) = C_{k,s}\;\mathcal{V}_s(-\vec{k},\eta) + D_{k,s}\,\mathcal{U}_s(\vec{k},\eta)\,. \tag{7}$$

- with $\mathcal{U}_s(\vec{k},\eta)$; $\mathcal{V}_s(-\vec{k},\eta)$ spinor solns during RD: combinations of parabolic cylinder functions with asymptotic "out" particle/antiparticle BC:

$$\mathcal{U}_s(\vec{k},\eta) \to\propto\; e^{-i\int^{\eta}\omega_k(\eta')\,d\eta'} \quad ; \quad \mathcal{V}_s(\vec{k},\eta) \to\propto\; e^{i\int^{\eta}\omega_k(\eta')\,d\eta'}\,.$$

Fermionic Fields : Distribution Function

- Use matching conditions from inflation to (RD), obtain the coefficients above. Expand the Fermi field in terms of the solutions with OUT boundary conditions with $\widetilde{b}\,,\widetilde{d}$ as creation/annihilation operators (OUT), :

$$\psi^{out}(\vec{x},\eta) = \frac{1}{\sqrt{V}} \sum_{\vec{k},s} \left[\widetilde{b}_{\vec{k},s}\, \mathcal{U}_s(\vec{k},\eta) + \widetilde{d}^{\,\dagger}_{-\vec{k},s}\, \mathcal{V}_s(-\vec{k},\eta) \right] e^{i\vec{k}\cdot\vec{x}}$$

  $\widetilde{b}\,,\widetilde{d}$ are related to the original $b\,,d$ via a Bogoliubov transformation determined by the coefficients A, B, C, D.

- Number of asymptotic (anti)particles in BD vacuum: **distribution function**

$$\langle 0_I | \widetilde{b}^{\dagger}_{\vec{k},s} \widetilde{b}_{\vec{k},s} | 0_I \rangle = \langle 0_I | \widetilde{d}^{\dagger}_{-\vec{k},s} \widetilde{d}_{-\vec{k},s} | 0_I \rangle = |B_{k,s}|^2 \equiv N_k$$

- We find a *nearly thermal* spectrum but w/o an event horizon!!

$$N_k = |B_{k,s}|^2 = \frac{1}{2}\left[1 - \left(1 - e^{-\frac{k^2}{2mT_H}} \right)^{1/2} \right] ; \; T_H = \frac{H_R}{2\pi} \simeq 10^{-36}\,\mathrm{eV}$$

Energy Momentum Tensor

- $T^{\mu\nu} = -dS_M/dg_{\mu\nu}$
  - Expand the fields in terms of the creation/annihilation operators and mode functions with "OUT" BC.
  - Take the expectation value of $T^{\mu\nu}$ in the IN state: Heisenberg picture.
- These expectation values feature: **i)** a vacuum term, **ii):** an interference term between asymptotic "out particle/antiparticle" modes, and **iii:)** the particle production contribution determined by the distribution function.
- The vacuum term features zero-point UV divergent contributions and are absorbed into a renormalization : Cosmological constant, Newton's constant plus corrections to Einstein-Hilbert action proportional to higher curvature terms.
- The interference terms oscillate rapidly on the short time scale $1/m$, thus averaging out on the longer time scale $1/H$, with $H \ll m$ consistent with the adiabatic approximation.
- On the long time scale and to leading adiabatic order, $\langle 0_I | T^{\mu\nu} | 0_I \rangle$ is of the kinetic fluid form and determined by distribution functions given by N_k.

Density Matrix : From Pure to mixed state

- the IN state $|0_I\rangle$ is a superposition of back-to-back "OUT" **entangled** particle-antiparticle pairs with amplitudes determined by Bogoliubov coeffs. In Schroedinger picture

$$|0_I\rangle = \Pi_{\vec{k}} \sum_{n_{\vec{k}}=0}^{\infty} \mathcal{C}_{n_{\vec{k}}}(k,\eta)\,|n_{\vec{k}};\overline{n}_{-\vec{k}}\rangle \quad (8)$$

$$\mathcal{C}_{n_{\vec{k}}}(k,\eta) \propto e^{2i\, n_{\vec{k}} \int_{\eta_i}^{\eta} \omega_k(\eta')\, d\eta'} \times \text{Bogoliubovs}$$

- Density matrix describes a pure state $\rho_S(\eta) = |0_I\rangle\langle 0_I|$. In the OUT basis, it features off-diagonal elements: particle-antiparticle interference with very fast oscillations (time scales 1/m), much shorter than the adiabatic time scales of the diagonal matrix elements $\simeq 1/H$.
- These rapid oscillations lead to decoherence by **dephasing** (averaging out of the off diagonal elements), leaving a diagonal density matrix in the out basis: **a mixed state**.

Entropy from Decoherence

- The Von-Neumann entropy associated with this mixed state, $S = Tr\rho \log \rho$ is **exactly the entanglement entropy obtained by tracing over one member of the particle-antiparticle pair in the density matrix** $\longrightarrow$ reduced density matrix $\rho_r = \mathrm{Tr}_{\overline{n}} \ \rho$.
- The Von-Neumann Entropy is completely determined by the Bogoliubov coefficients and the occupation number of the gravitationally produced **pairs** and is similar to the kinetic form (with a subtle difference).
- For Complex Fields,

$$S^{(d)} = \sum_{\vec{k}} \Big\{ (1+N_k) \ln(1+N_k) - N_k \ln N_k \Big\} .$$

- For Fermionic Fields :

$$S^{(d)} = -2 \sum_{\vec{k}} \Big\{ (1-N_k) \ln(1-N_k) + N_k \ln N_k \Big\} .$$

- **Same for either Dirac or Majorana**.

Results I: Complex Scalar Fields

- Comoving Number density of gravitationally produced **particle-antiparticle pairs**:

$$\mathcal{N}_{p\overline{p}} = \frac{1}{\pi^2} \int_0^\infty k^2 N_k \, dk$$

- Energy Density:

$$\overline{\rho}_{p\overline{p}}(\eta) = \frac{1}{\pi^2 \, a^4(\eta)} \int_0^\infty k^2 N_k \, \omega_k(\eta) \, dk$$

- Pressure:

$$\overline{P}_{p\overline{p}}(\eta) = \frac{1}{3\pi^2 \, a^4(\eta)} \int_0^\infty \frac{k^4}{\omega_k(\eta)} N_k \, dk$$

- Entropy density = Entanglement entropy :

$$S_{p\overline{p}} = \frac{1}{2\pi^2} \int_0^\infty k^2 \Big[(1+N_k) \ln[1+N_k] - N_k \ln N_k \Big] \, dk$$

Results : Fermionic Fields

- Comoving Number density:

$$\mathcal{N}_{p\overline{p}} = \frac{2}{\pi^2} \int_0^\infty k^2 \, N_k \, dk \,,$$

- Energy Density:

$$\overline{\rho}_{p\overline{p}}(\eta) = \frac{2}{\pi^2 a^4(\eta)} \int_0^\infty k^2 \, N_k \, \omega_k(\eta) \, dk \,,$$

- Pressure:

$$\overline{P}_{p\overline{p}}(\eta) = \frac{2}{3\pi^2 a^4(\eta)} \int_0^\infty k^2 \, N_k \, \frac{k^2}{\omega_k(\eta)} \, dk \,,$$

- Entropy density = Entanglement entropy for each spin/helicity of Dirac or Majorana:

$$\mathcal{S}_{p\overline{p}} = -\frac{2}{2\pi^2} \int_0^\infty k^2 \left\{ (1-N_k) \ln(1-N_k) + N_k \ln N_k \right\} dk \,,$$

Results

- For Complex scalar fields, specific entropy :

$$\frac{\mathcal{S}_{p\overline{p}}}{\mathcal{N}_{p\overline{p}}} \ll 1\,,$$

which is consistent with an Bose-Einstein condensate: a large occupation of a few states $\longrightarrow$ a low entropy state akin to a Bose Einstein condensate but w/o SSB.

- For the heavy fermionic DM, specific entropy :

$$\frac{\mathcal{S}_{p\overline{p}}}{\mathcal{N}_{p\overline{p}}} \simeq 1.8,$$

which is consistent with an emerging a (NR) near thermal spectrum.

Connection to Observables

- For Complex fields, $m \approx 10^{-5}$ eV yields correct DM abundance for (CDM) with $w \simeq 10^{-14}$ and $\lambda_{fs} \simeq 70pc$ (cutoff in the power spectrum of density perturbations).
- Its comoving entropy is much smaller than that of the (CMB) today

$$\frac{\mathcal{S}_{p\overline{p}}}{\mathcal{S}_{cmb}} \simeq 10^{-45} \,.$$

- For Fermionic fields, $m \approx 10^{8}$ GeV yields correct DM abundance for (CDM).
- The ratio of its comoving entropy to that of the (CMB) today is,

$$\frac{\mathcal{S}_{p\overline{p}}}{\mathcal{S}_{cmb}} \simeq 10^{-15} \,.$$

- For light bosonic (DM), the entropy is consistent with a Bose-Einstein condensate, few modes of large occupation and low entropy, whereas for fermions it is consistent with a (NR) thermal species and Pauli blocking.

Comments:

- The entanglement entropy is *NOT* to be associated with entropy (isocurvature) perturbations!!. It is a ubiquitous outcome of gravitational production of back-to-back **pairs** and decoherence by rapid dephasing. Q: what is its role if any in (DM) clustering? (TBD).
- Familiar (kinetic) form of $T^{\mu\nu}$ emerges after renormalization and neglecting fast oscillations (decoherence) in the adiabatic regime.
- The von Neumann-entanglement- entropy and the kinetic fluid form of the energy momentum are all a consequence of decoherence of the density matrix in the out basis. A (subtle) factor 2 and similarity between Dirac and Majorana for the entropy is a consequence of **entangled pairs**, not single particles.

Questions

- Are superhorizon modes insensitive to the post-inflation transition? (this was the assumption, needs confirmation!).
- The expectation value $\langle 0_I | T^{\mu\nu} | 0_I \rangle$, after renormalization, and to leading adiabatic order is of the fluid form, how to extract density perturbations in the QFT approach?
- Since the (DM) fields do not feature an VEV during inflation there is no *linear* isocurvature perturbation, is there a **non-linear** one?, what about renormalization of $T^{\mu\nu}$ correlations?
- If there are either self-interactions or interactions wih other d.o.f, does the entanglement entropy become the thermal entropy, with which it shares the same kinetic form? (in which case the distribution function should thermalize?)
- What is the role of entropy in clustering?? is it similar to the role of coarse grained entropy in "violent relaxation"??

Final conclusions

- Gravitational production is a suitable mechanism for (DM) production, the abundance only depends on the expansion history and mass. The seeds were planted at inflation, and harvested at (MD).
- Light bosonic fields inherit the infrared enhancement of the inflationary era. Their distribution function describes a nearly Bose Condensed state without (SSB), a mass $m \simeq 10^{-5}\,\mathrm{eV}$ yields (CDM), and saturates the (DM) abundance with a $\lambda_{fs} \simeq 70\,\mathrm{pc}$.
- Fermionic (DM) is produced with a nearly thermal (NR) distribution function with a $T_H \simeq 10^{-36}\,\mathrm{eV}$ (without an event horizon!), saturating the (CDM) abundance for $m \simeq 10^{8}\,\mathrm{GeV}$.
- Particle-antiparticle interference in the "out" state leads to decoherence of the density matrix by dephasing, the Von-Neuman entropy is recognized as the entanglement entropy by tracing one member of the pair. It features a kinetic fluid form. For bosons it is in agreement with a low entropy Bose-condensed phase. For fermions it displays Pauli blocking.
- Many questions remain....(to be continued in season 2!)