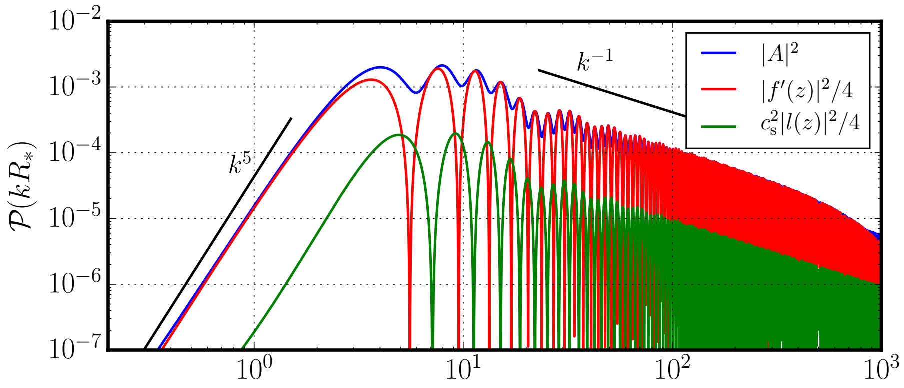
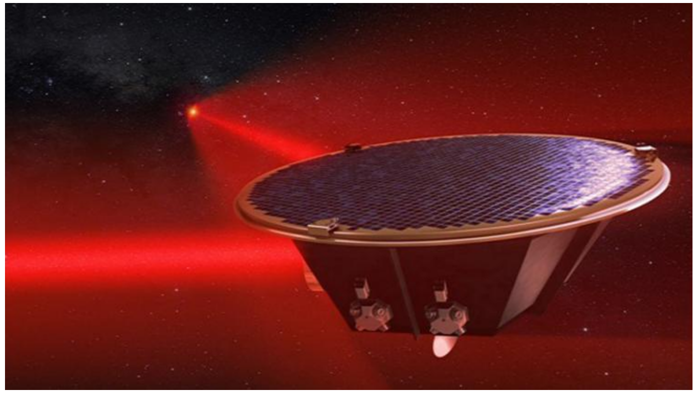




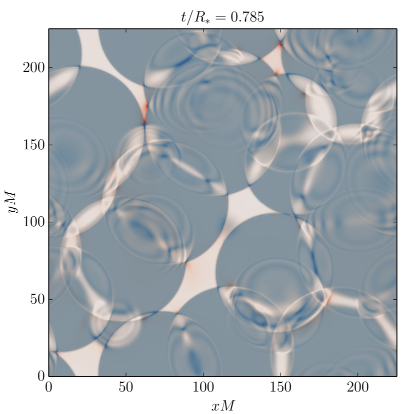
Gravitational waves from phase transitions in the early universe

Mark Hindmarsh

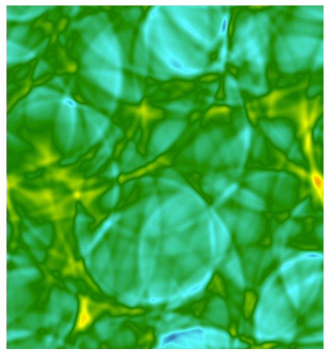
Helsinki Institute of Physics & Dept of Physics, University of Helsinki

and

Dept of Physics and Astronomy, University of Sussex



Kings College London
17. maaliskuuta 2021



Phase transitions in the early Universe

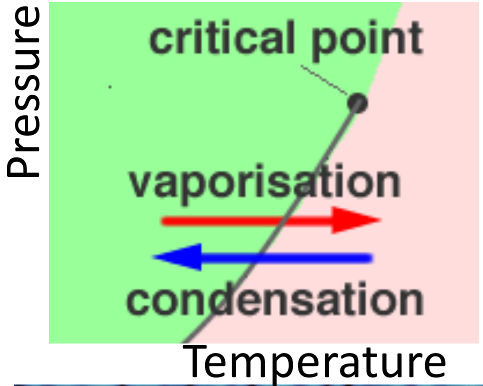
- At very high temperatures and pressures, the state of matter in the Universe changes
  - T_c ~ 100 MeV (1 ms) QCD
  - T_c ~ 100 GeV (10 ps) Electroweak
  - T_c >> 100 GeV new symmetries?
- Departures from equilibrium and homogeneity (shear stress)
  - First order phase transition: relativistic condensation or `fizz'

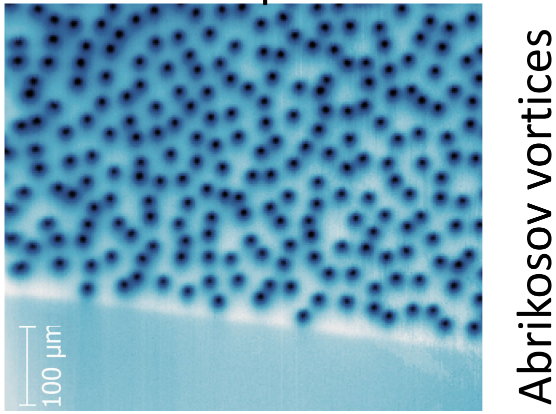
    Steinhardt (1982)
  - Formation of topological defects

    Kibble (1976)
  - Baryon asymmetry

    Kuzmin, Rubakov, Shaposhnikov (1985)
- Phase transitions can produce GWs

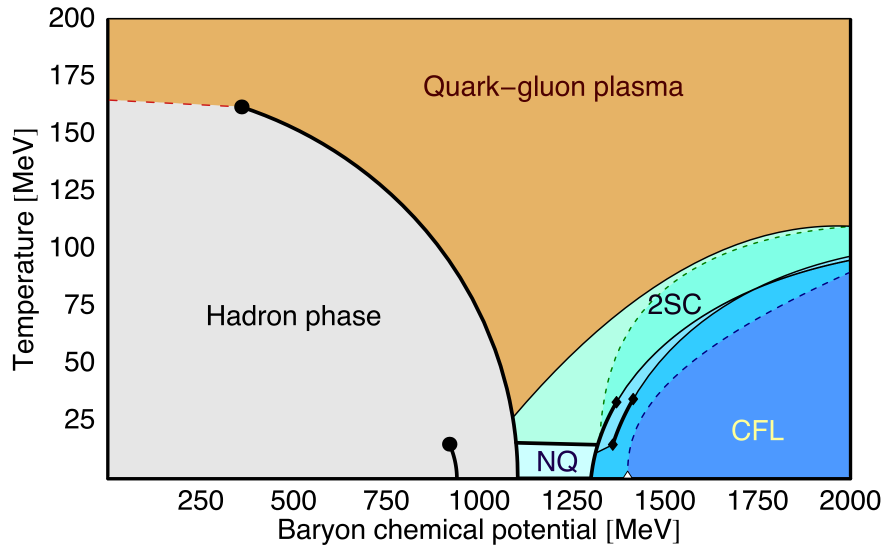
  Witten (1984)

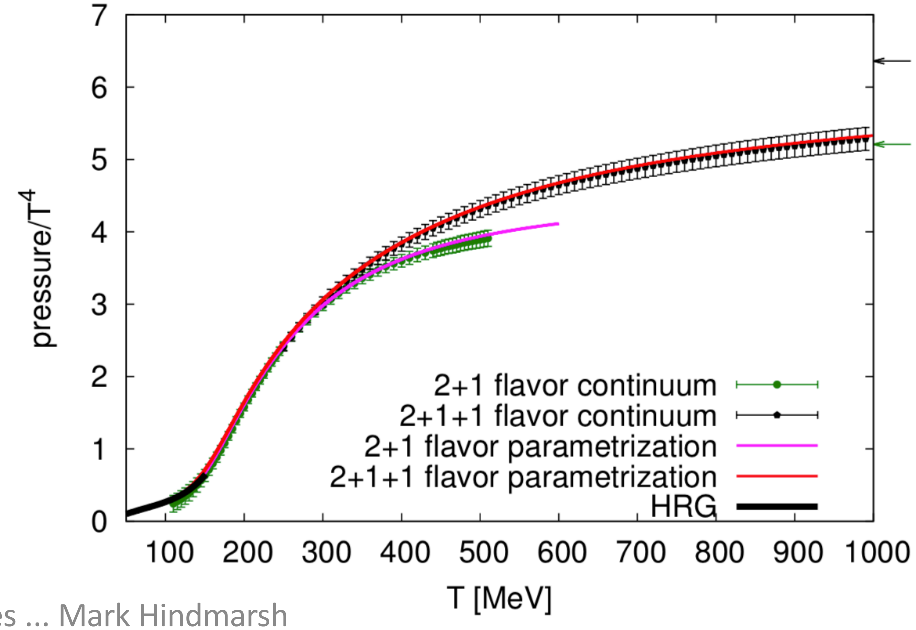




QCD phases

- QCD: rich phase diagram
- Universe: $n_B/n_\gamma \approx 6.1 \times 10^{-10}$
- Behaviour at low chemical potential well-established by lattice QCD Borsanyi et al (2016)
- Transition from QGP to hadronic phase is a smooth cross-over
- Departures from equilibrium very small: no GWs

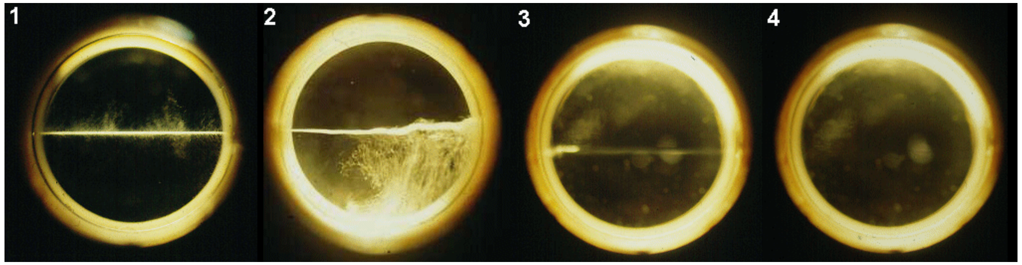


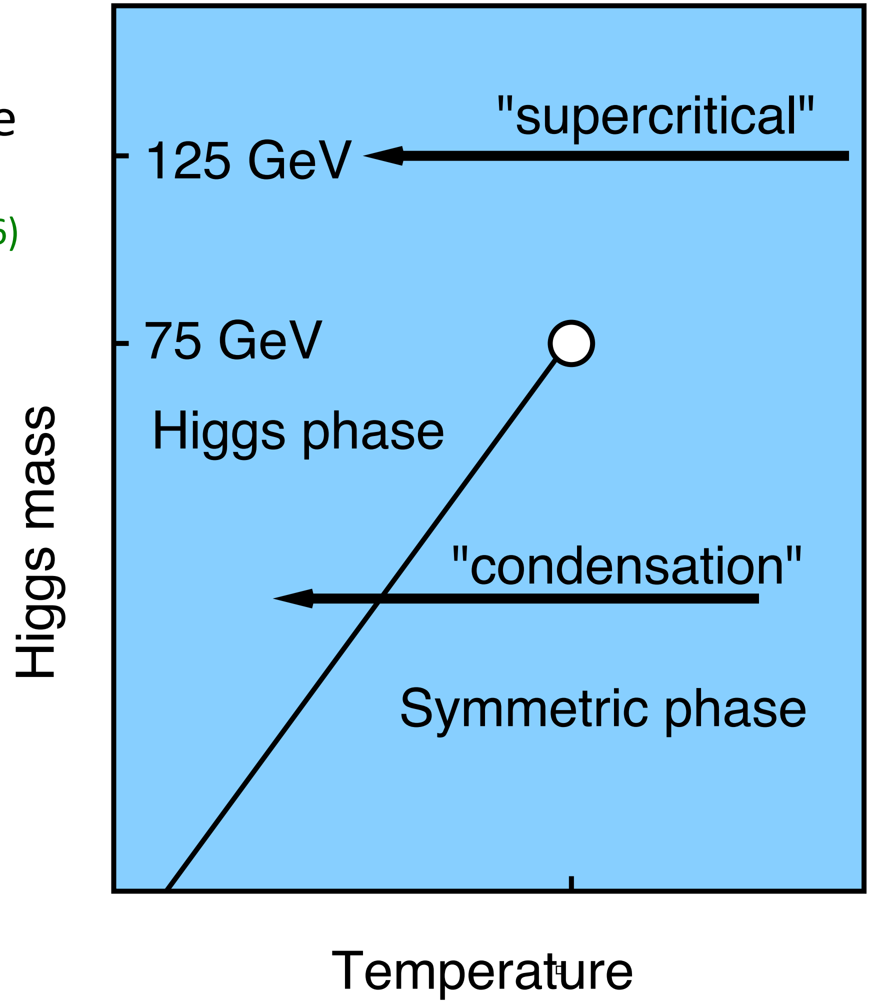


Borsanyi et al 2016

Electroweak transition

- SM is not weakly coupled at high T
- Non-perturbative techniques:
  - Dimensional reduction to 3D effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - SU(2)-Higgs on 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV is a cross-over - a supercritical fluid





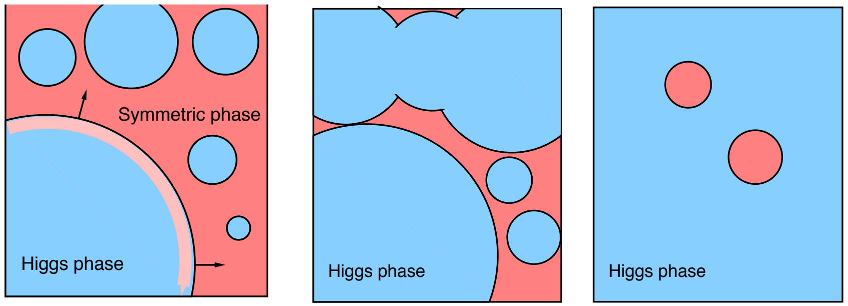
- Search for 1st order transition is a search for physics beyond SM

Little bangs in the Big Bang

- 1st order transition by nucleation of bubbles of low-T phase

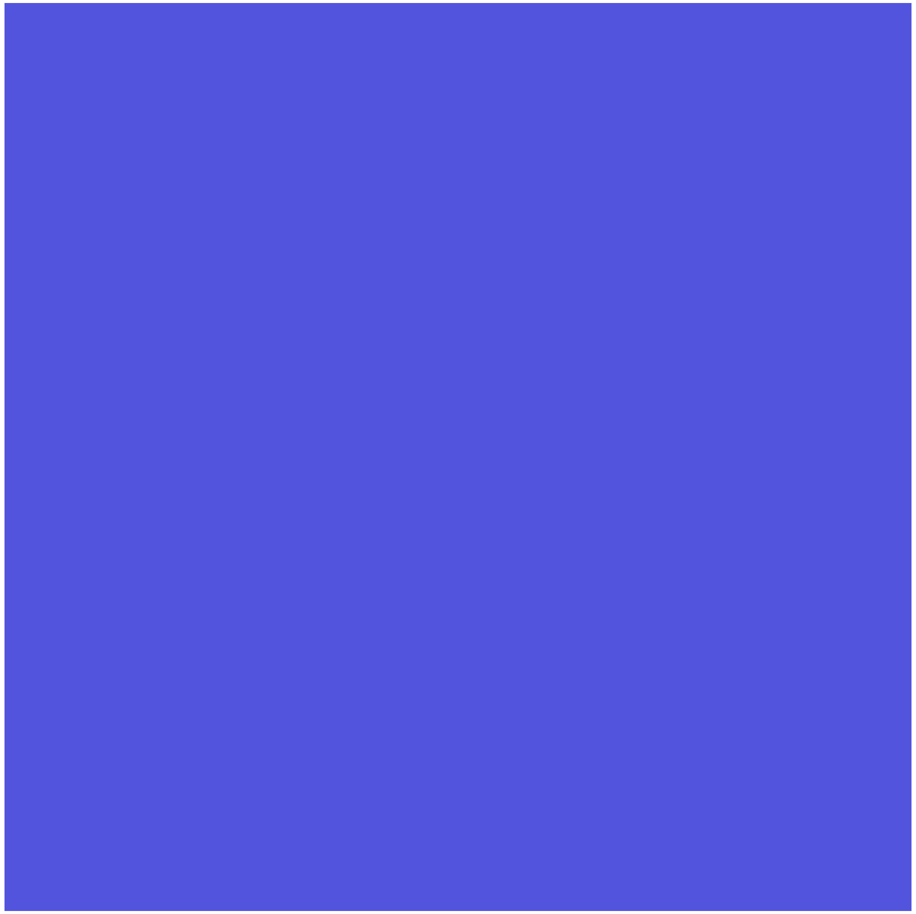
  Langer 1969, Coleman 1974, Linde 1983

- Nucleation rate/volume $p(t)$ rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Universal "fizz"
- Gravitational wave production



Steinhardt (1982); Hogan (1983,86);
Gyulassy et al (1984); Witten (1984)

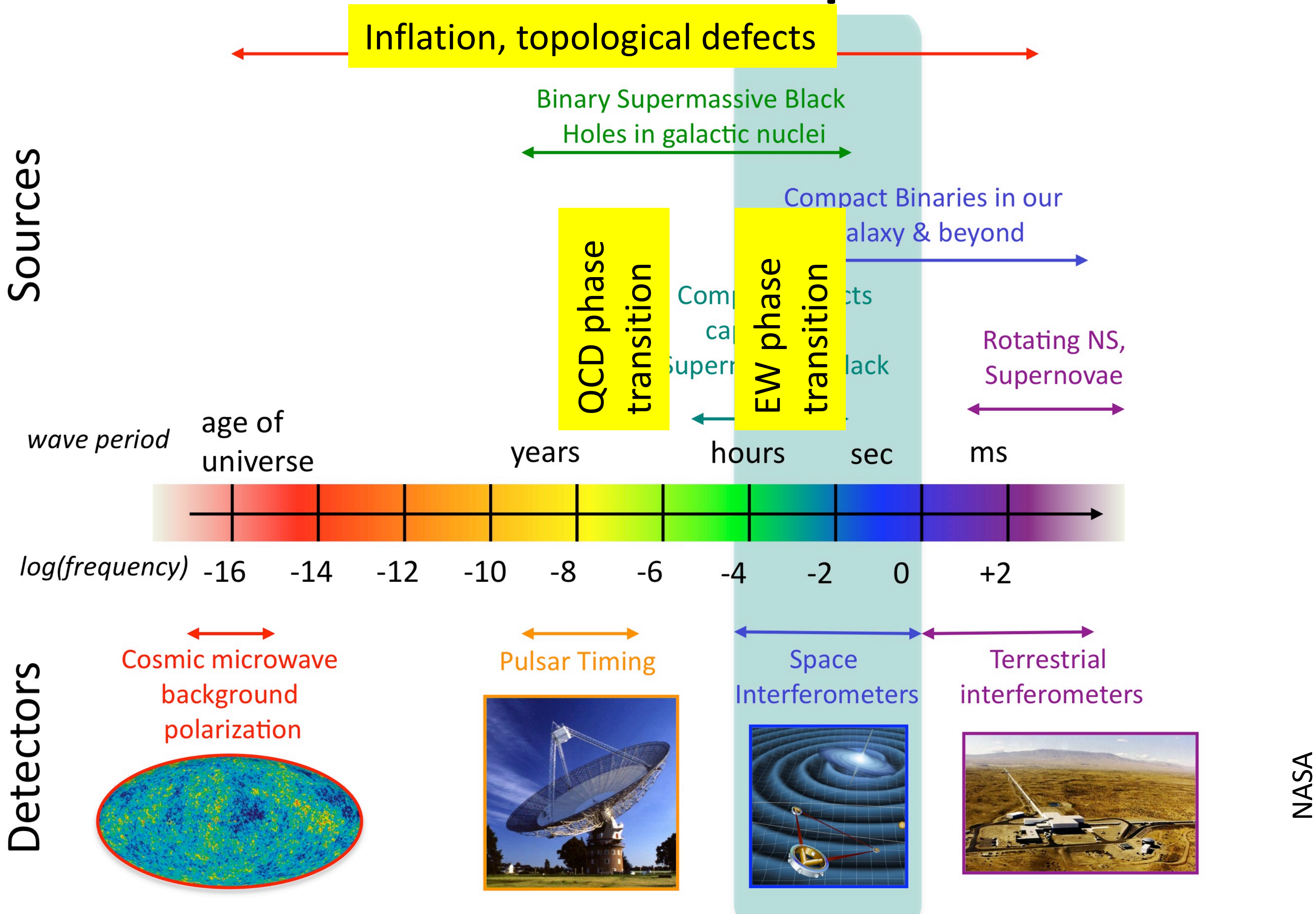
Fluid kinetic energy



David Weir

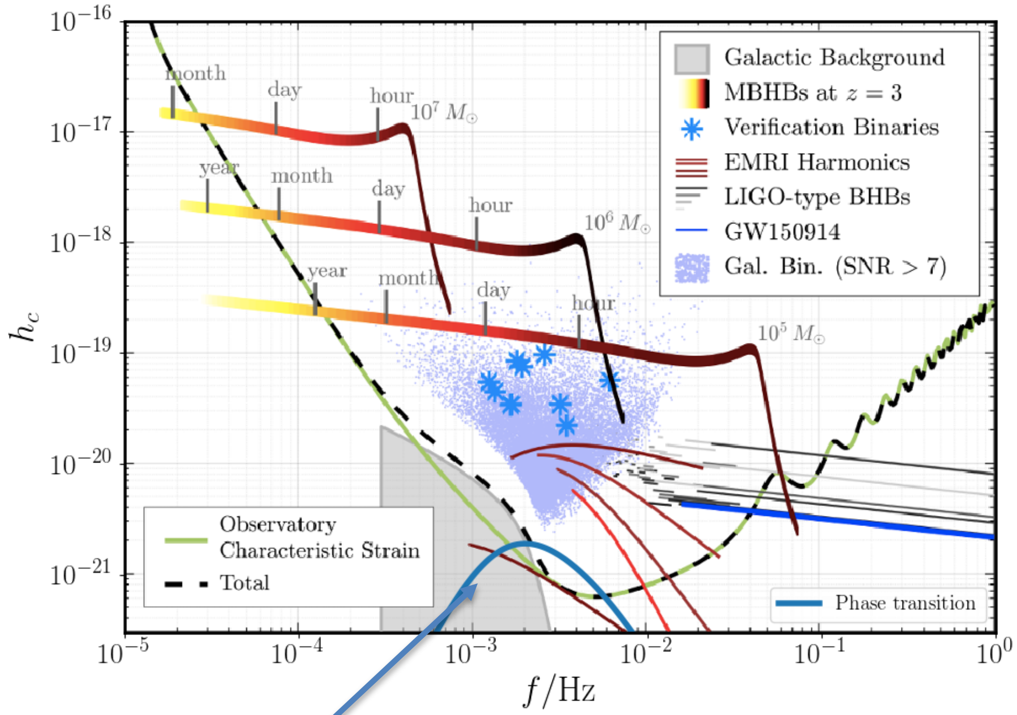
MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

Gravitational wave spectrum

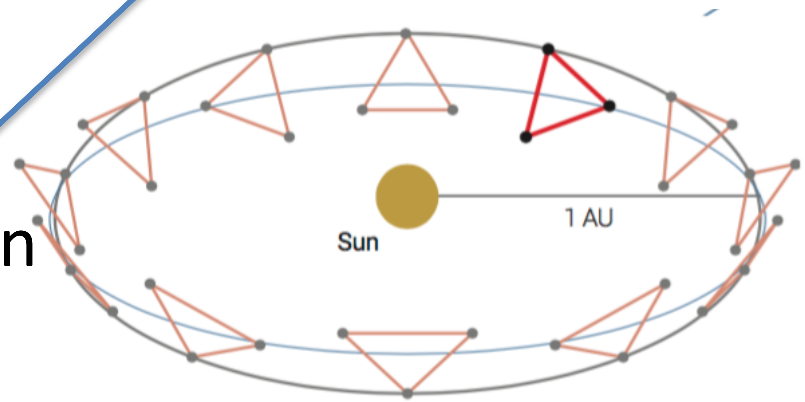


Laser Interferometer Space Antenna

- Launch by 2034
- 4-year mission (up to 10 years)
- 2.5M km arms
- Science objectives:
  - White dwarves
  - Black holes
  - Galaxy mergers
  - Extreme gravity
  - TeV-scale early Universe
- Other missions: Taiji, Tianqin

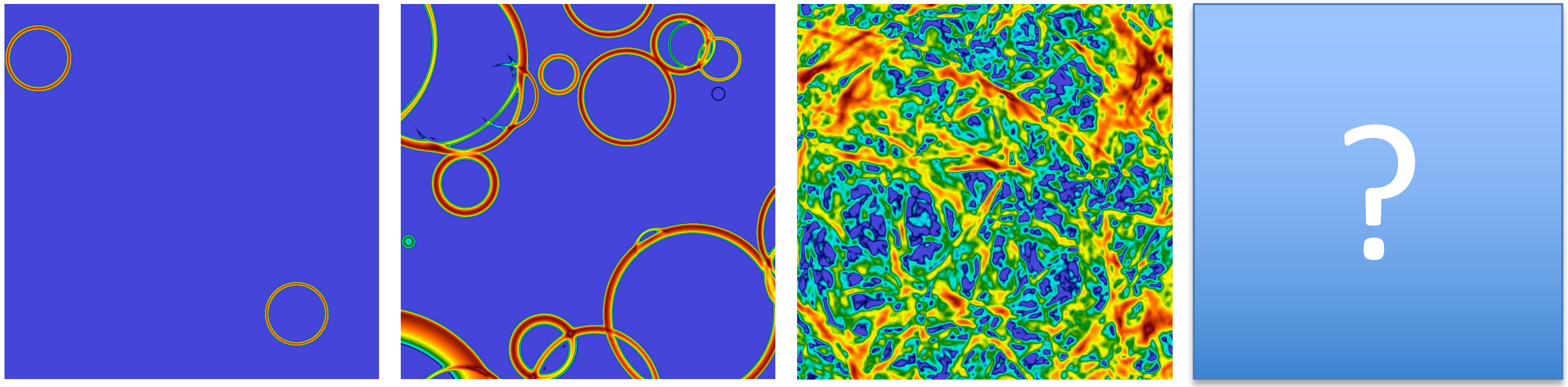


LISA sensitivity



LISA

Phases of a phase transition



1 2 3 4

1. Nucleation and expansion
2. Collision
3. Acoustic
4. Non-linear (shocks, turbulence)

$$\tau_{\rm nl} \sim L_f/\bar{U}_f$$

$L_{\rm f}$ – fluid flow length scale
$U_{\rm f}$ – RMS fluid velocity

Ellis Lewicki No 2018

'exponential' nucleation

$$p(t) = p_n e^{\beta(t-t_n)}$$

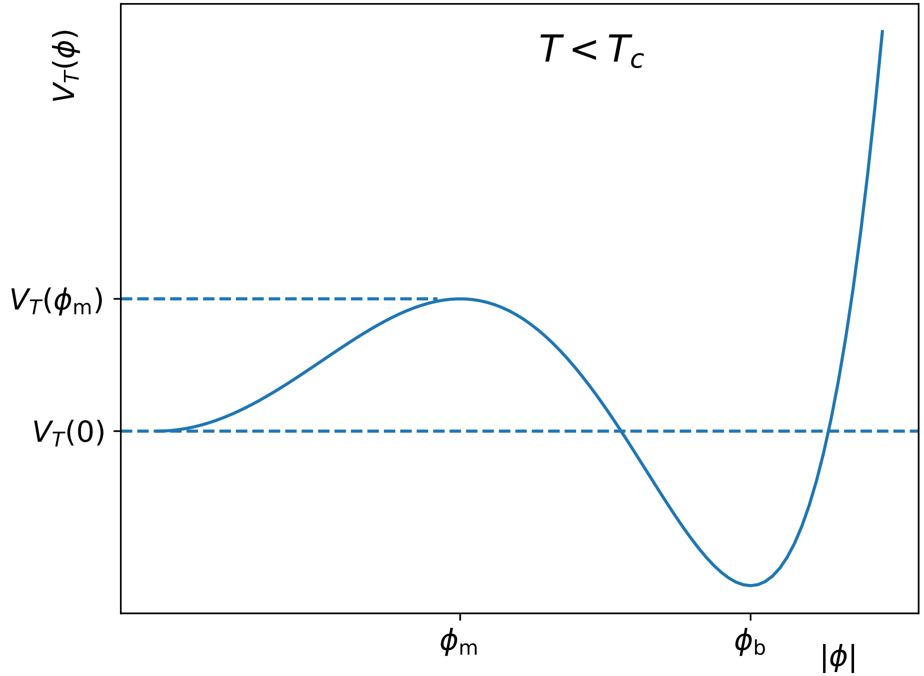
$$\tau_{\rm co} = \beta^{-1}$$

Guth, Weinberg 1983; Enqvist et al 1992; Turner, Weinberg, Widrow 1992;

p – nucleation rate/volume
β – transition rate parameter

Review: MH, Lüben, Lumma, Pauly 2021

Effective theory of phase tr…



- Ingredients:
  - Higgs field $-\ddot{\phi} + \nabla^2\phi - \ldots$
    - η coupling to fluid (models e…
  - Relativistic fluid

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi}W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_i P + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

    - E energy density, Z_i momentum density, V_i velocity, W γ-factor
  - Discretisation
  - Metric perturbation

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

Wilson & Matthews (2003)
Different approach: Giblin, Mertens (2013)

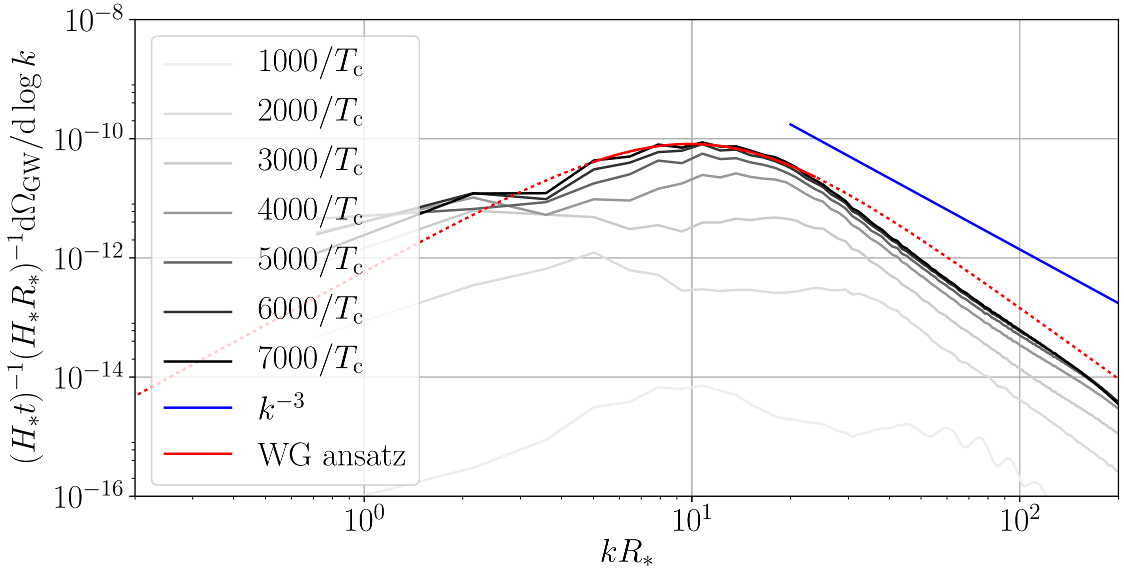
Garcia-Bellido, Figueroa, Sastre (2008)

Simulations of phase transitions

- 2015: 1M hrs CSC, Finland
- 2015/6: 17M CPU-hours
  Tier-0 (Hazel Hen, Stuttgart)
- 4200^3 lattice on 24k cores
- Output: GW power spectrum

$$\frac{d\Omega_{\rm gw}}{d\ln f} = \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm gw}}{d\ln f} = \frac{8\pi^2}{3H^2} f^3 S_h(f)$$

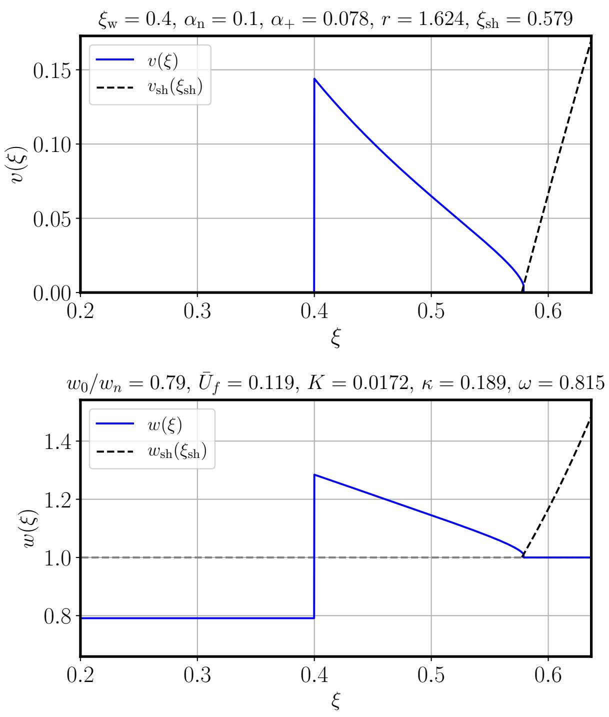
Transition strength: α = 0.0046
Wall speed: $v_{\rm w}$ = 0.44



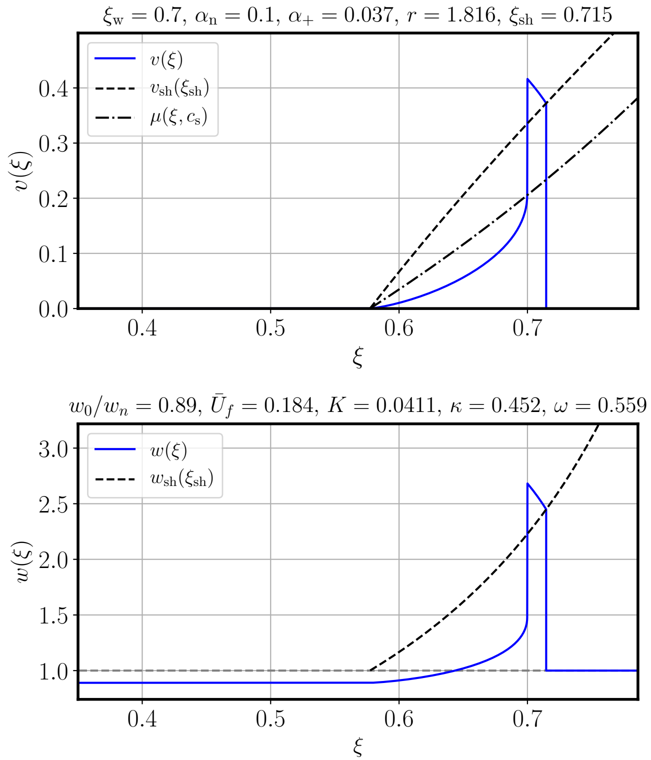
SUOMEN AKATEMIA
FINLANDS AKADEMI
ACADEMY OF FINLAND

Generic features:
- "Domed" peak at $kR_* \sim 10$
- Approx k^{-3} spectrum at high k

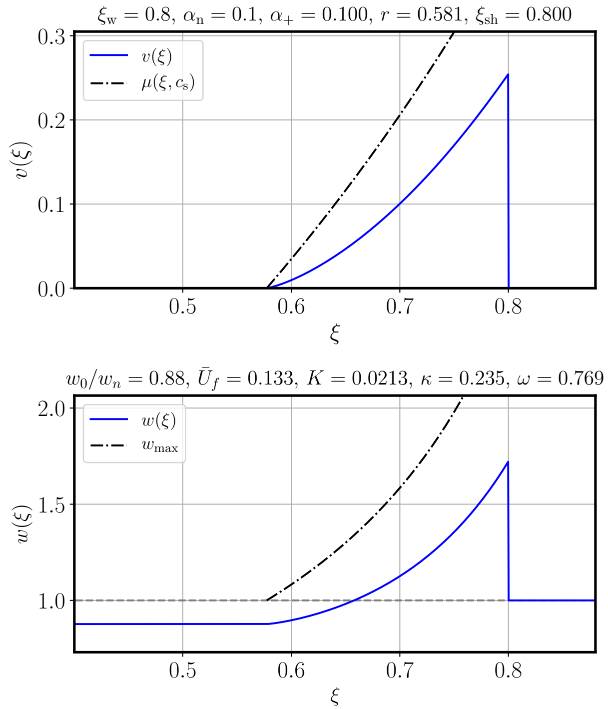
Hindmarsh, Huber, Rummukainen, Weir 2017



Deflagration



Supersonic deflagration ("hybrid")



Detonation

Landau & Lifshitz; Steinhardt (1984)
Kurki-Suonio, Laine (1991), Espinosa et al (2010)

- Scalar potential energy converted to kinetic energy, heat energy
- Wall velocity v_w (pressure difference ΔV, scalar-fluid coupling η)
- Result: radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$
  - **Similarity solution $v(r/t)$, $w(r/t)$**
- Some cases … runaway ($v_w \rightarrow 1$) (weakly coupled near-vacuum transition)

Bodeker Moore 2010, 2017

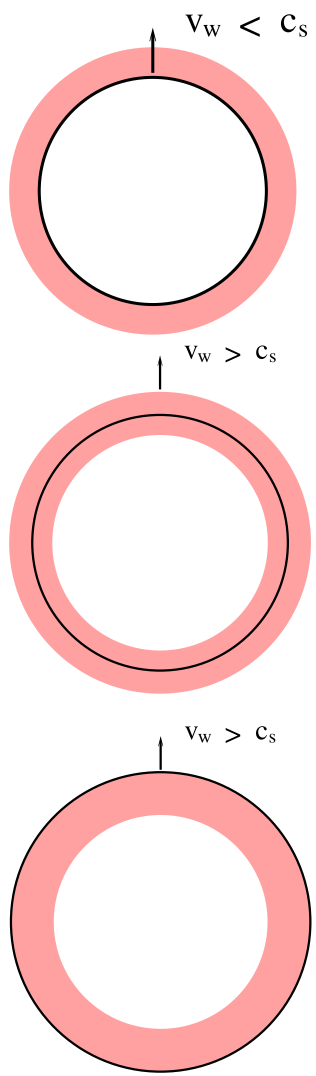
GWs from first order phase transitions

- Parameters of transition:
  - T_n = Temperature at nucleation
  - β = transition rate (= - d log p / dt)
  - v_w = Bubble wall speed
  - α = ("Potential energy")/("Heat energy")
  - c_s = sound speed (here taken $1/\sqrt{3}$)
- Derived parameters:
  - r_* = (mean bubble separation)/Hubble length
  - K = fluid kinetic energy fraction
- Aim: GW power spectrum

$$\frac{d\Omega_{\rm gw}}{d\ln f} = \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm gw}}{d\ln f} = \frac{8\pi^2}{3H^2} f^3 S_h(f)$$

Giese et al 2020

Steinhardt '84
Espinosa et al 2010



Estimating GW power

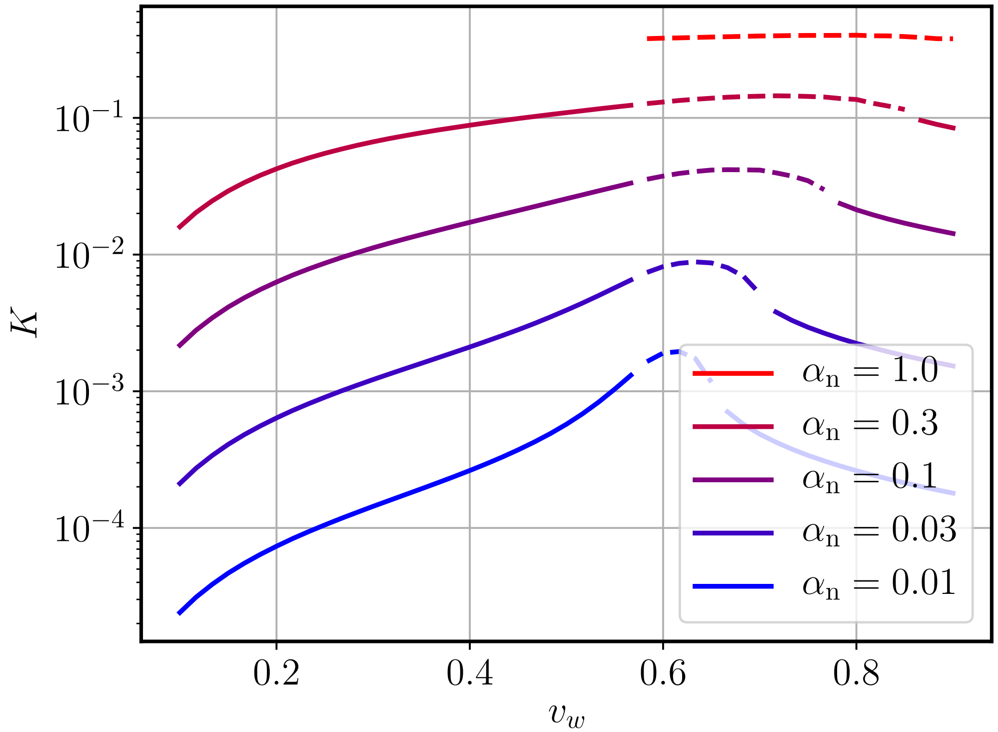
$$\Omega_{\rm gw} = \rho_{\rm gw}/\rho_{\rm c} \sim (H_{\rm n}\tau_{\rm v})(H_{\rm n}\tau_{\rm c})K^2$$

- GW energy fraction:
  - H_n Hubble rate at nucleation
  - τ_v duration of stresses
  - τ_c coherence time
- Coherence time:
  - $\tau_c \sim R_*$ (bubble separation)
- Stress lifetime (shocks):
  - $\tau_v = H_n^{-1}/(1 + U_f/r_*)$
  - U_f (RMS velocity) $\sim \sqrt{K}$
  - Or $K = (4/3)U_f^2$

$$\Omega_{\rm gw} \simeq \tilde{\Omega}_{\rm gw} \frac{r_*}{1+\sqrt{K}/r_*} K^2$$

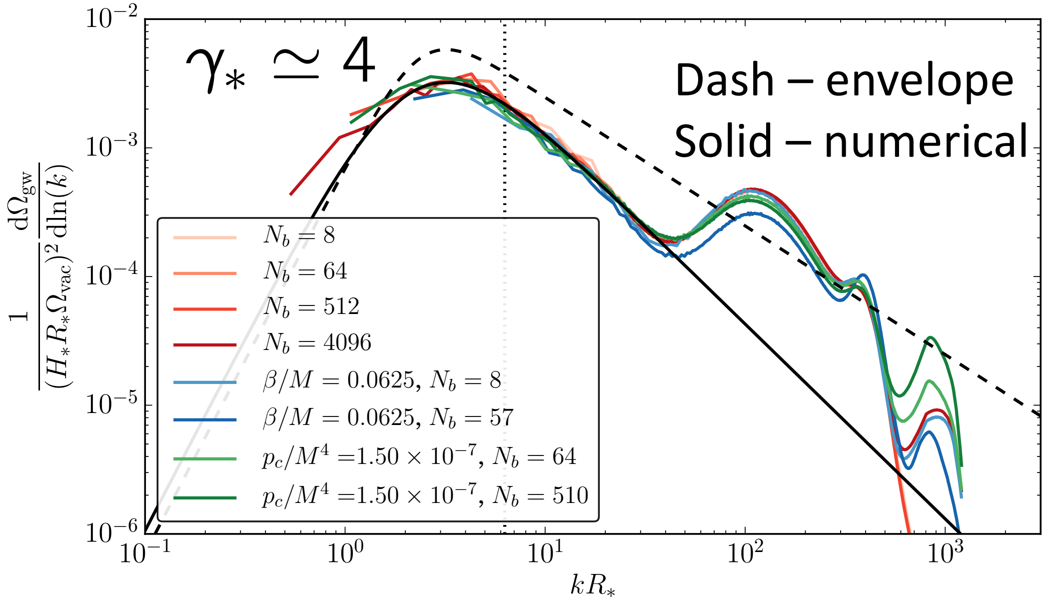
$$\tilde{\Omega}_{\rm gw} = O(10^{-2})$$

Estimate K (kinetic energy fraction) from self-similar hydro solution



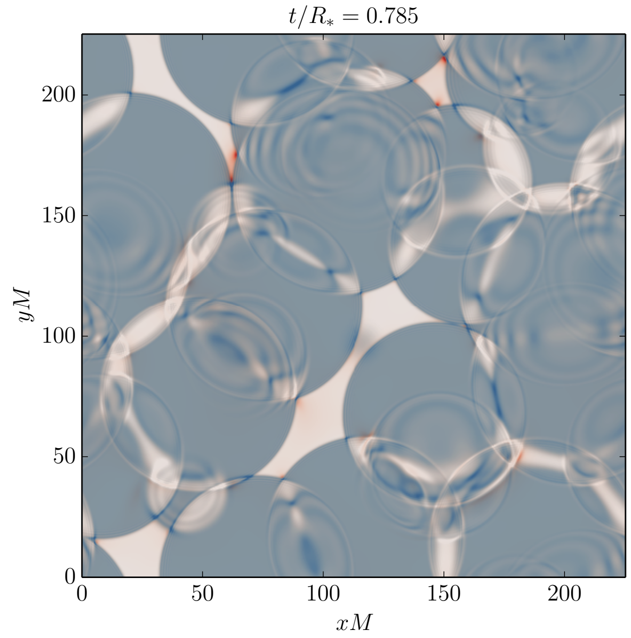
Scalar field only (runaway walls)

- History: envelope approximation
  Kamionkowski, Kosowsky, Turner 1994; Huber, Konstandin 2008
- Numerical simulations show differences
  Cutting, MH, Weir 2018



$$\frac{d\Omega_{\rm gw}^{\rm fit}}{d\ln k} = \Omega_{\rm p}^{\rm fit} \frac{(3+b)^c \tilde{k}^b k^3}{(b\tilde{k}^{(3+b)/c} + 3k^{(3+b)/c})^c}$$

$$\Omega_{\rm p}^{\rm fit} = (3.22 \pm 0.04) \times 10^{-3} \, (H_{\rm n} R_*)^2 \Omega_\phi^2,$$
$$\tilde{k} R_* = 3.20 \pm 0.04,$$
$$b = 1.51 \pm 0.04, \quad c = 2.18 \pm 0.15$$



LISA CWG party line 2019

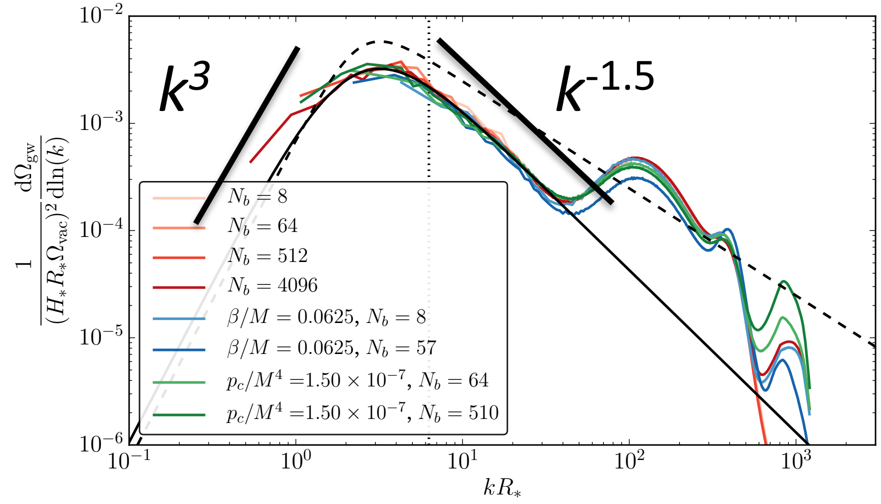
Caprini et al 2019

- Three contributions to total power:
  - Scalar field ϕ
  - Acoustic ac
  - Turbulent tu

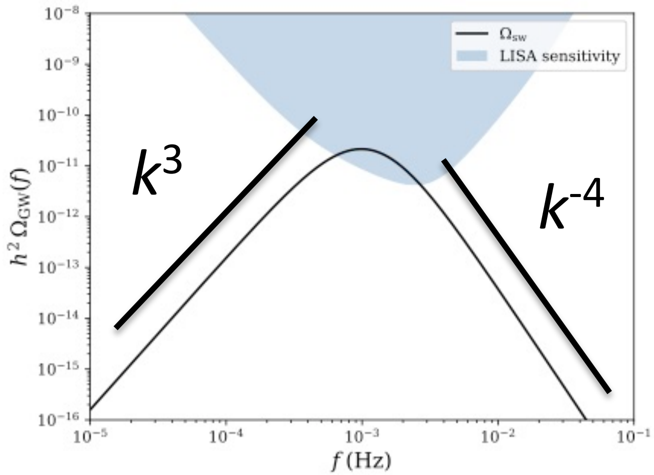
$$\Omega_{\rm gw} = \Omega_{\rm gw}^{\phi} + \Omega_{\rm gw}^{\rm ac} + \Omega_{\rm gw}^{\rm tu}$$

- Scalar field: bubble wall collisions
  - relevant only for runaway walls
- Acoustic production:
  - M.H. et al 2013, 2015, 2017, 2019
- Turbulent production:
  - Uncertain, probably subdominant around peak
  - Conservative estimate: neglect

Runaway



Constant v_w, $\alpha < 0.1$



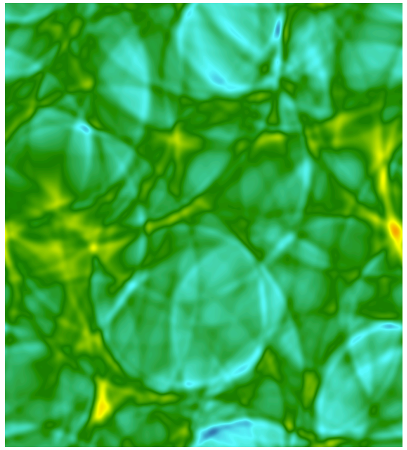
Sound shell model

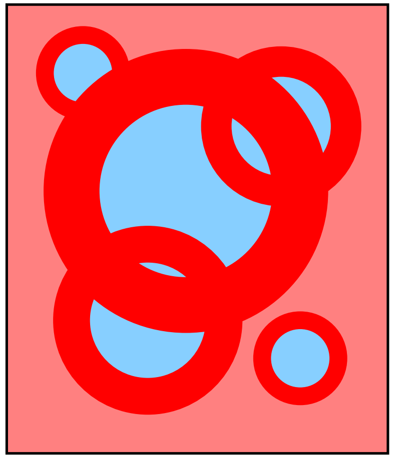
- Gaussian velocity field from weighted addition of sound shells $\mathbf{v}_{\mathbf{q}}(t_i)$

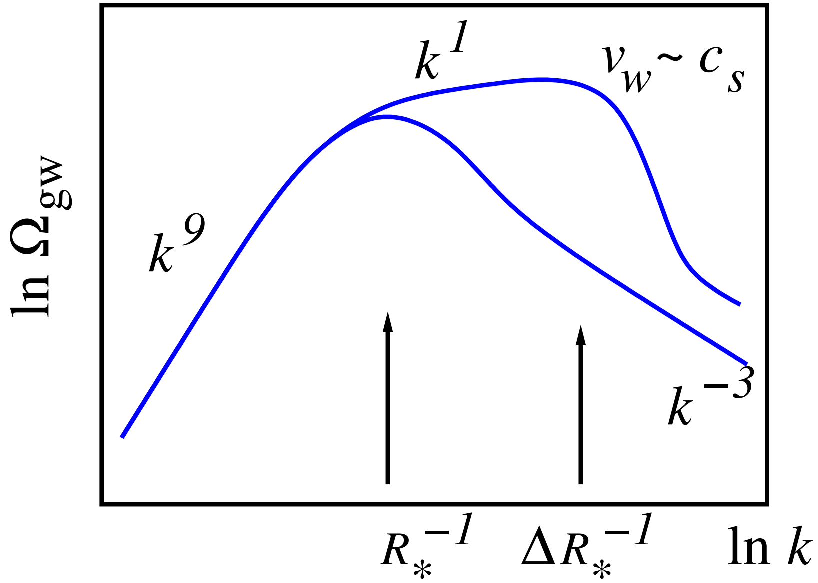
  MH 2017, MH, Hijazi (2019)

- Two length scales:
  - Bubble spacing R_*
  - Shell width $R_* \frac{|v_w - c_s|}{v_w}$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude proportional to:
  - Bubble spacing
  - Shear stress lifetime
  - (Kinetic energy)^2
- Similar: bulk flow model

  Jinno, Konstandin, Rubira 2020



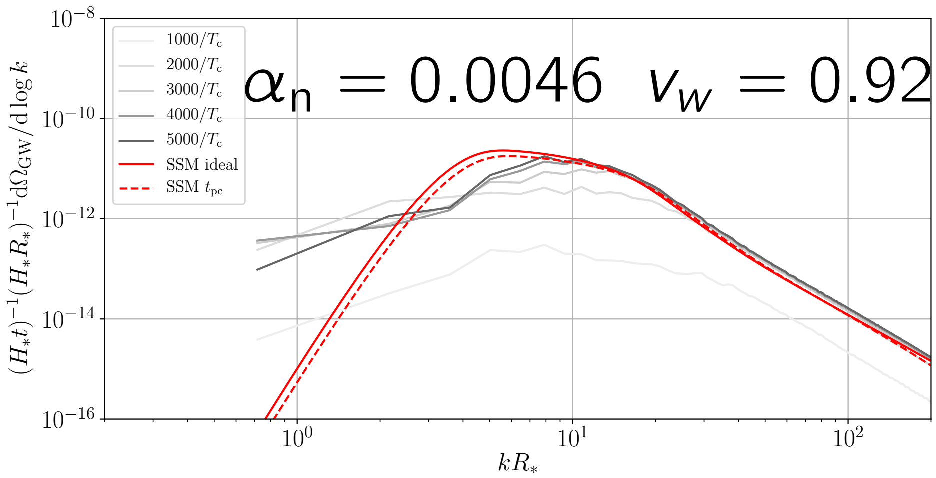


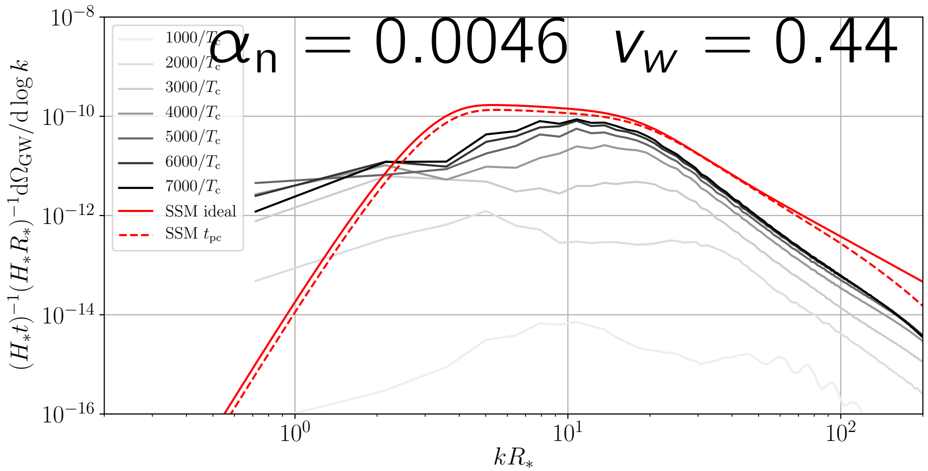


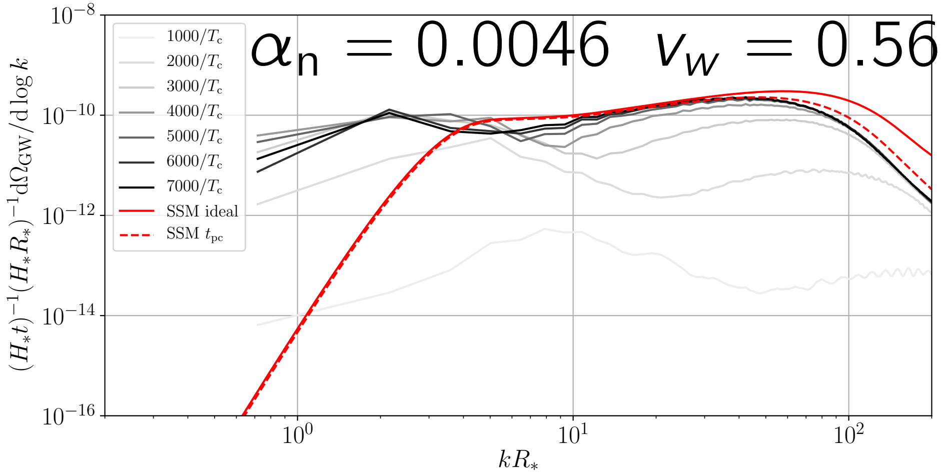
Sound shell model vs. simulations P_{gw}

- Solid: self-similar sound shell
- Dash: evolving sound shell at peak collision time
- Simultaneous nucleation

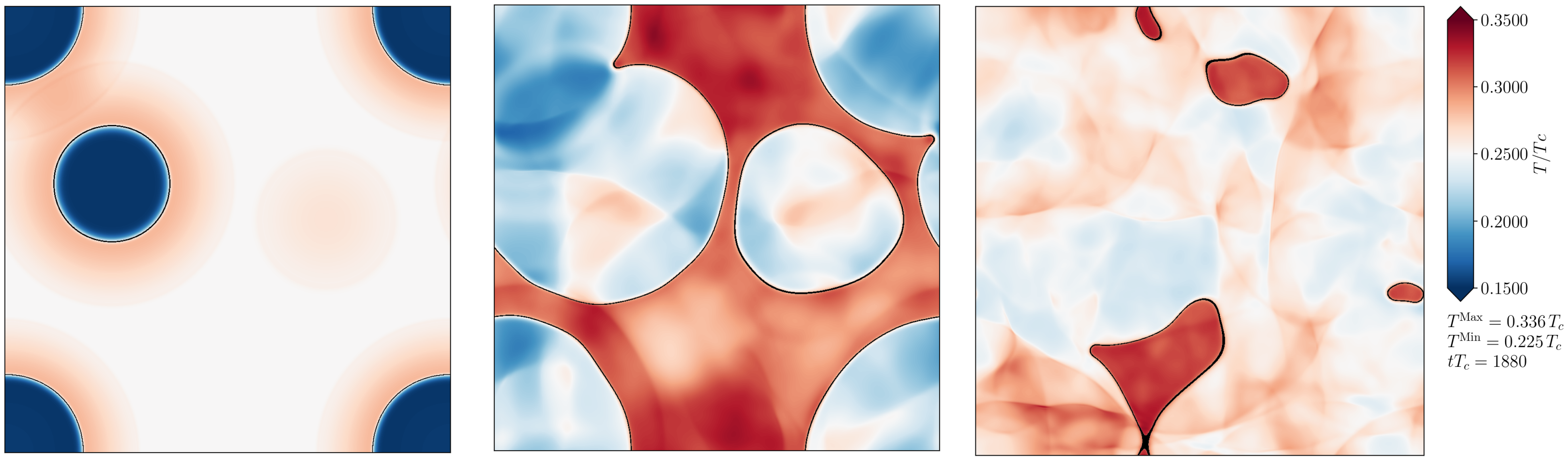
MH et al in prep





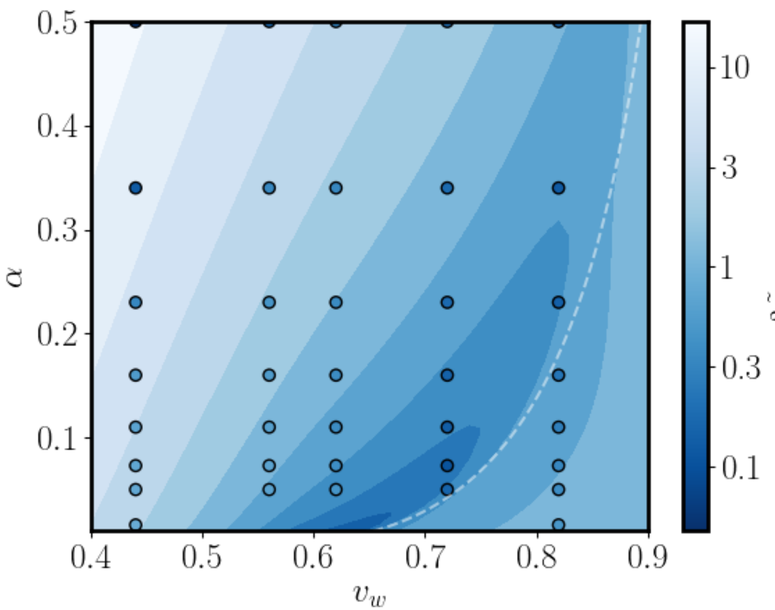


Kinetic energy suppression



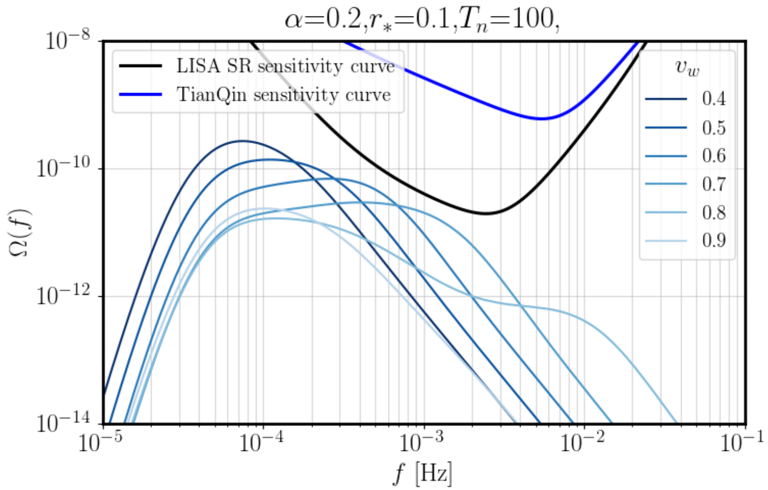
Cutting, MH, Weir, 2020

- **Deflagrations**: heat up fluid in front
- Pressure in front of wall increases, walls slow down
- Less transfer into kinetic energy, more into heat.
- Effect not yet included in SSM
- More simulations & thought needed

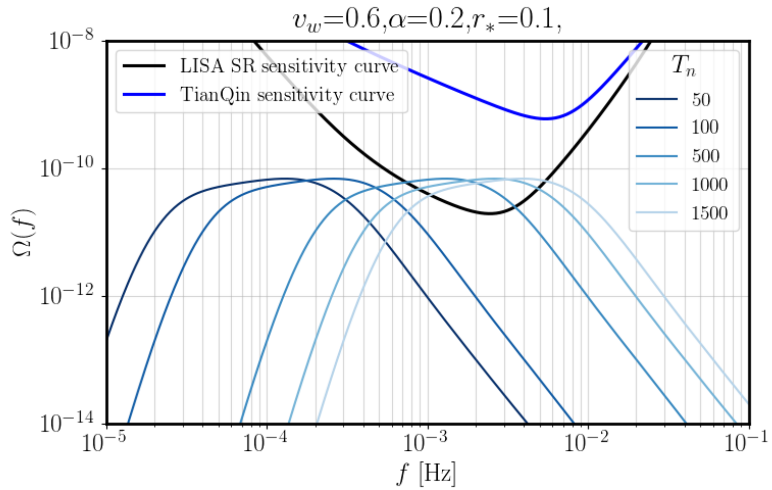


Gowling, MH, to appear

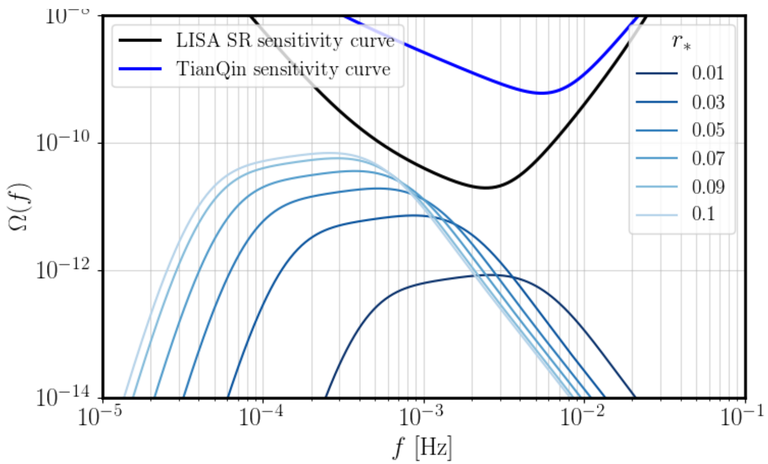
Parameters and power spectra



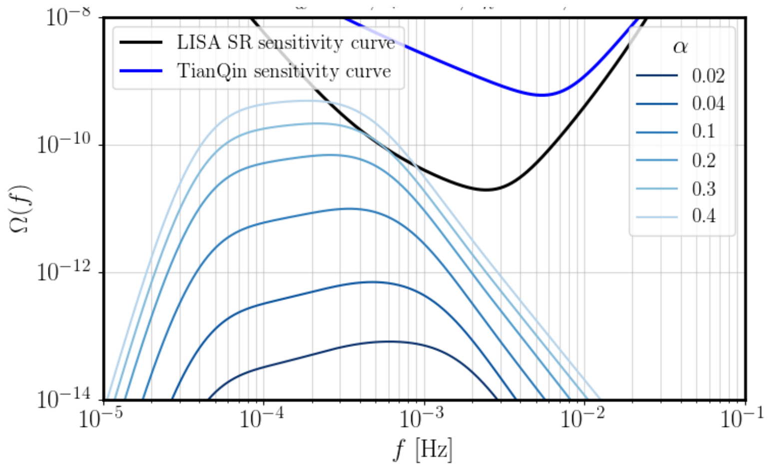
Vary wall speed



Vary transition temp



Vary bubble separation

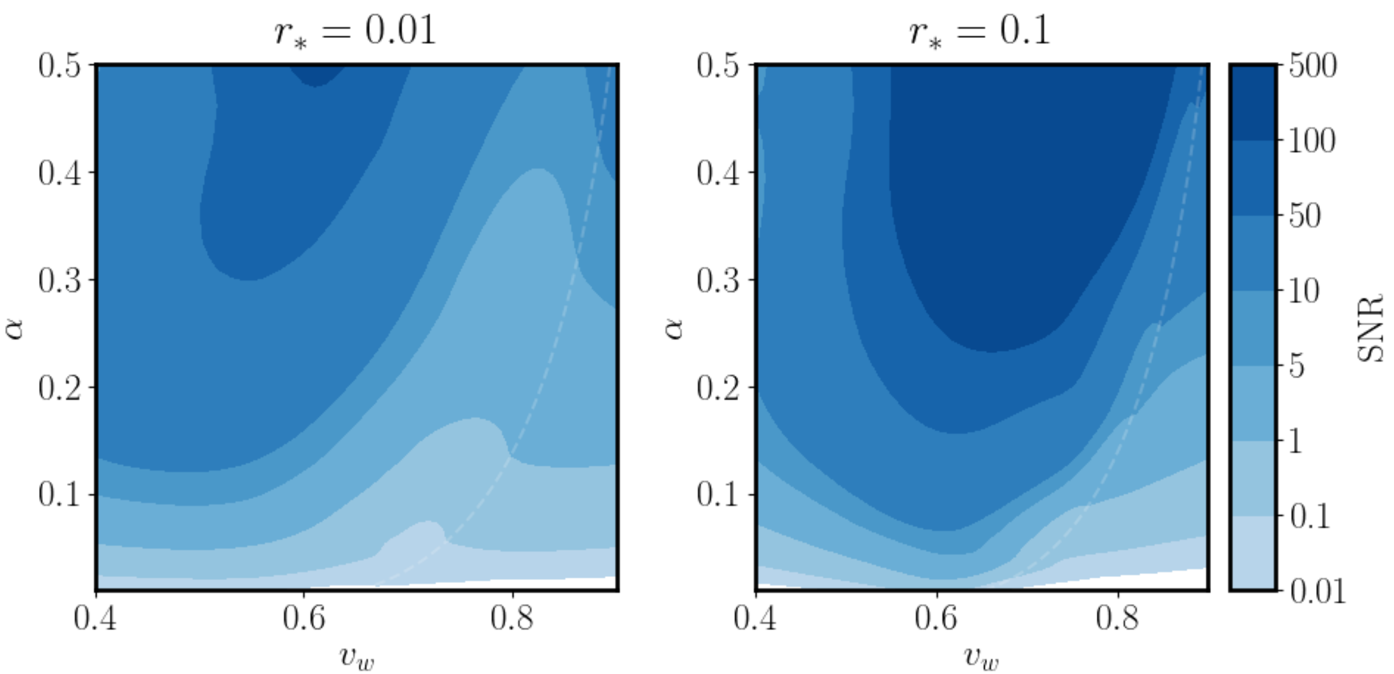


Vary transition strength

Sound shell model predictions

Gowling, MH (in prep)

Signal-to-noise ratios (LISA)



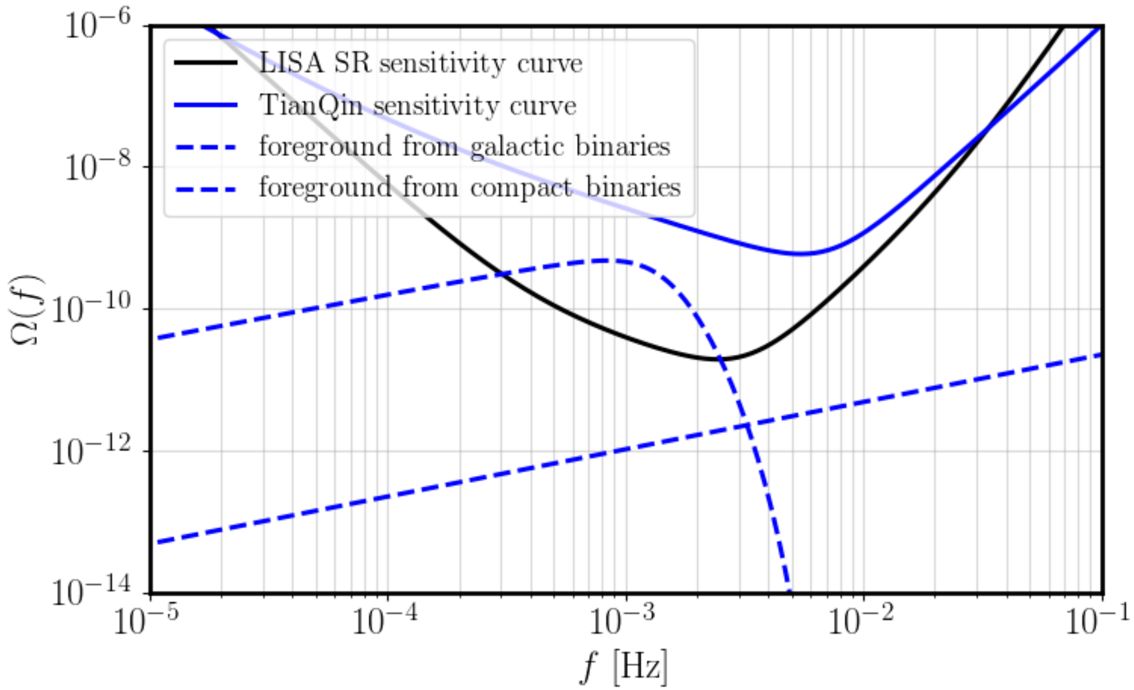
- T_c = 100 GeV

Gowling, MH (in prep)

- Signal-to-noise ratio ρ (t_{obs} = 4 years)
- Includes kinetic energy suppression
- No foregrounds
- "LISA science requirements" noise

$$\rho^2(\vec{\theta}) = t_{\rm obs} \int df \left(\frac{\Omega_{\rm gw}(f;\vec{\theta})}{\Omega_{\rm noise}(f)} \right)^2$$

Foregrounds in stochastic signal

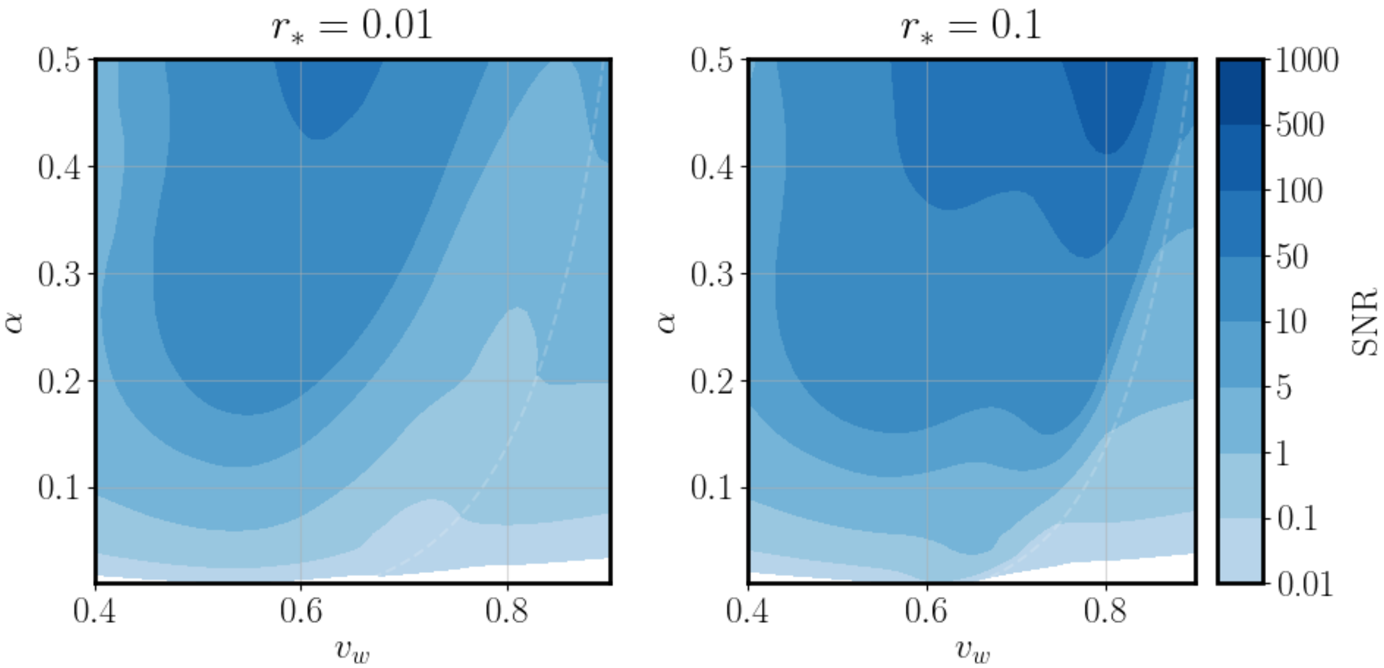


Gowling, MH (in prep)

After detected objects are removed from signal:

- Unresolved white dwarf binaries in our galaxy (~ 20 million)
- Unresolved extra-galactic compact binaries
  - Mostly stellar origin black hole binaries ("LIGO-type")
- What can we hope to see? Pieroni, Barausse 2019

Signal-to-noise ratios (LISA)



- T_c = 100 GeV

Gowling, MH (in prep)

- Signal-to-noise ratio ρ (t_{obs} = 4 years)
- Includes kinetic energy suppression
- “Worst case” galactic binary foreground (annual variation)
- “LISA science requirements” noise

$$\rho^2(\vec{\theta}) = t_{\rm obs} \int df \left(\frac{\Omega_{\rm gw}(f;\vec{\theta})}{\Omega_{\rm noise}(f)} \right)^2$$

Connection to fundamental theory

- Scalar effective potential $V(\phi,T)$
- Scalar-fluid coupling $\eta(\phi,T,v)$ → T_n, α, β, c_s, g_{eff} (equilibrium)

→ v_w (non-equilibrium)

Phase transition parameters :

T_n = nucleation temperature
g_{eff} = effective d.o.f.
α = (scalar V)/(thermal energy)
c_s = sound speed
v_w = bubble wall speed
β = transition rate

$H_n(T_n, g_{\text{eff}})$ (Hubble rate)

$K(v_w, \alpha)$ (kinetic energy fraction)

$R_*(\beta, v_w)$ (mean bubble separation)

GW power spectrum parameters :

Ω_p = peak amplitude
f_p = peak frequency
σ_i = shape parameters

→ *How are they related?*

Theory challenges

- Scalar effective potential $V(\phi,T)$
  - Breakdown of perturbation theory at high T
  - Non-perturbative methods:
    - Dimensional reduction — Croon et al 2020
    - Functional renormalization group — Einhorn et al 2020,
  - Strongly interacting fields
    - Lattice + Polyakov — Huang et al 2020,
    - Holography — Ares et al 2020
- Scalar-fluid coupling $\eta(\phi,T,v)$
  - O(1) perturbative estimates for SM and MSSM — Moore, Prokopec 1994; John, Schmidt 2000
- Connection to particle physics phenomenology

Caprini et al 2019; …., Kozaczuk et al 2015; Ellis, Lewicki, No 2018; Fairbain et al 2019; …

A holographic phase transition

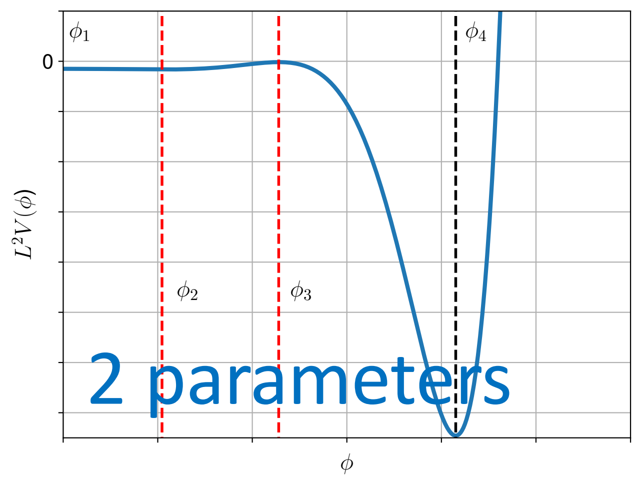
Ares, MH, Hoyos, Jokela 2020

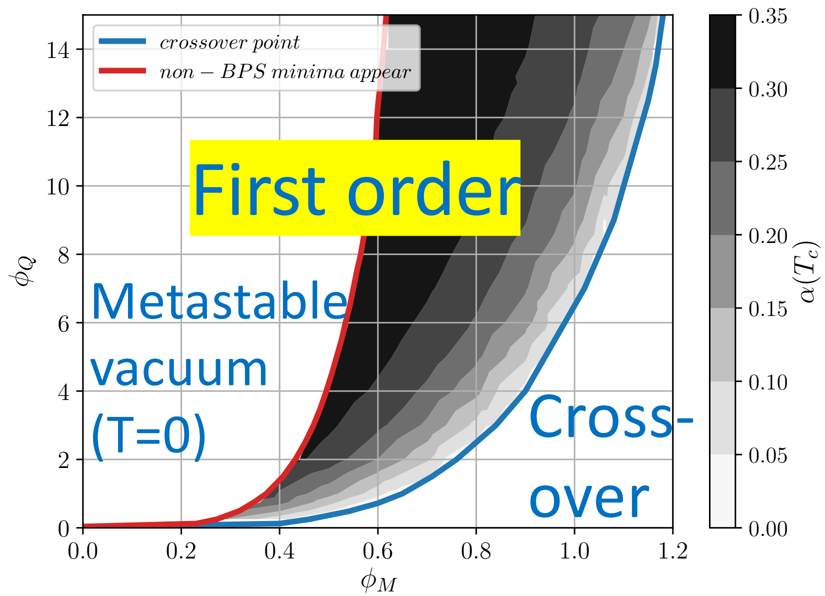
- Bottom-up model: phase transition in a strongly-interacting sector

$$S_{\text{non-reg}} = \frac{2}{\kappa_5^2}\int \mathrm{d}^5x\sqrt{-g}\left(\frac{\mathcal{R}}{4} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

Bea, Mateos 2018

- *T*, *s* of field theory = Horizon surface gravity/area
- Bulk field dual to scalar operator vev in field theory
- "Intermediate" strength phase transitions are generic ($\alpha \sim 0.1 - 0.3$)
- Sound speed $c_s < 1/\sqrt{3}$





A theory skeleton in the cupboard

- Is nucleation theory correct?
  - ^{3}He A/B transition rate puzzle

Kaul Kleinert 1980, Bailin, Love 1980, Leggett 1984, Tye Wohns 2011

- Cahn/Hilliard, Langer theory of nucleation rate:

Coleman 1974, Linde 1983

$$\frac{\Gamma}{\mathcal{V}} = \frac{\sqrt{V_T''(0)}}{\pi}\left[\frac{\det(-\vec{\nabla}^2 + V_T''(0))}{|\det'(-\vec{\nabla}^2 + V_T''(\bar{\phi}))|}\right]^{1/2}\left(\frac{\beta E_{\rm c}}{2\pi}\right)^{3/2} e^{-\beta E_{\rm c}}$$

E_c – energy of critical droplet/bubble

- ^{3}He A/B theory prediction: $\beta E_c \sim 10^6$
- Lab: metastable ^{3}He A lasts minutes.
- QTFP project aims to resolve the puzzle

QUEST DMC

Summary

- Good understanding of GWs (sound shell model)
  - near-linear flows ($\alpha \leq 0.1$)
  - fast walls: $v_w > 0.4$
  - for sub-Hubble bubble separations ($r^* << 1$)
- Dominant source sound (kinetic energy K)
- Total power estimate:

$$\Omega_{\text{gw},0} \simeq F_{\text{gw},0} \frac{r_*}{1 + \sqrt{K}/r_*} K^2 \tilde{\Omega}_{\text{gw}}$$

Standard cosmology:

$$F_{\text{gw},0} = 3.6 \times 10^{-5} \left(\frac{100}{g_{\text{eff}}} \right)^{\frac{1}{3}}$$

Numerical simulations, SSM:

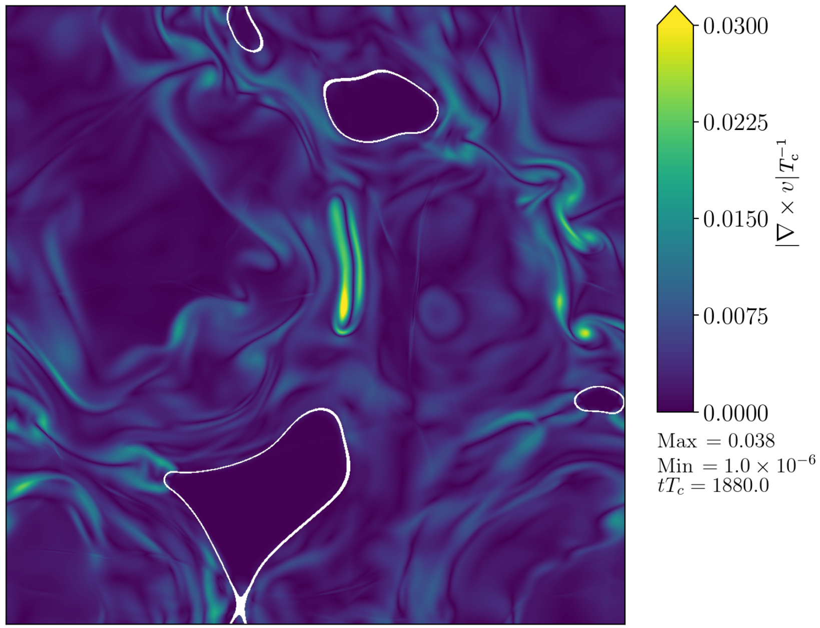
$$\tilde{\Omega}_{\text{gw}} = \text{O}(10^{-2})$$

- Naïve extrapolation:
  an upper bound on GWs from PTs:

$$\Omega_{\text{gw},0} \lesssim 10^{-7}$$

Future challenges

- Realistic equations of state
  - Sound speed is important Giese et al 2020
- Non-linear evolution of fluid
  - Longitudinal/compression modes
    - Kinetic energy suppression
    - Shocks, wave turbulence
  - Transverse/rotational modes
    - Vorticity generation
    - Turbulence
    - Turbulence less efficient at producing GWs? Roper pol et al 2019



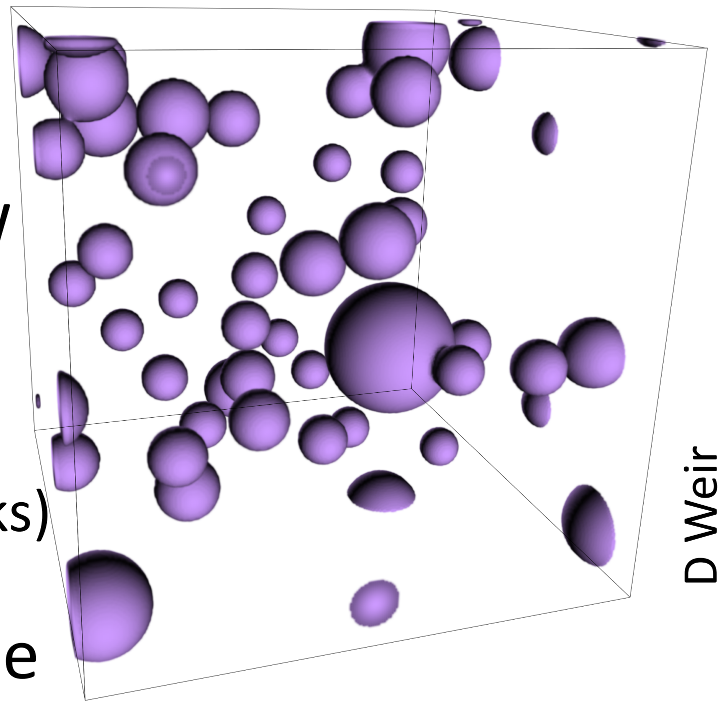
Vorticity, strong transition

Cutting, MH, Weir 2019

- Accurate computation of phase transition parameters from fundamental theory Croon et al 2020

Conclusions

- GWs probe physics at very high energy
  - LISA and other missions will probe physics of Higgs transition from 2034
  - Measure/constrain phase transition parameters
- Towards accurate calculations of GW power spectrum from parameters
  - Understanding of acoustic production, probably dominant
  - Non-linear evolution (turbulence, shocks) not well understood
- Ambition: make GWs as good a probe of the electroweak era as CMB is for the decoupling era



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