MAGIS-100: Large Baseline Atom Interferometry

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Outline

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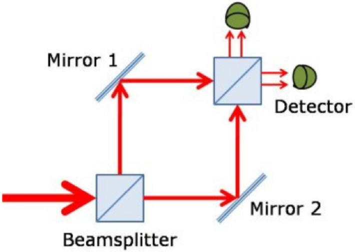
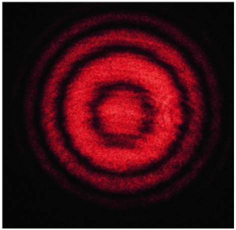
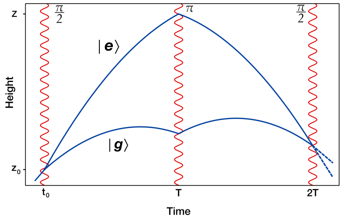
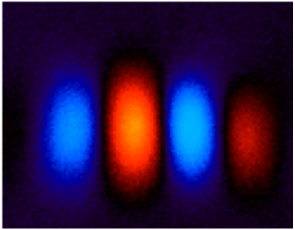


http://www.cobolt.se/interferometry.html



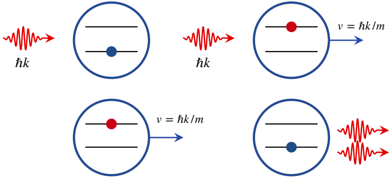
Laser interference

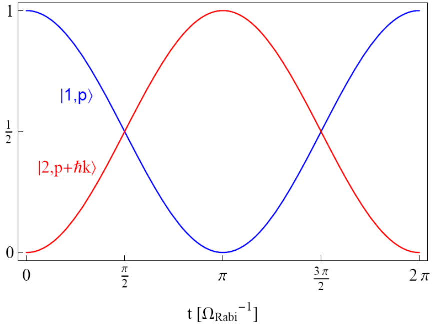




Matter-wave interference

- Atom optics implemented through stimulated absorption/emisison
- Raman scattering or Bragg scattering modes
- Two photon and single photon transitions





Atom Interferometry Phase

$$\Delta\phi = \Delta\phi_{\text{laser}} + \Delta\phi_{\text{propagation}} + \Delta\phi_{\text{separation}}$$

$$\Delta\phi_{\text{laser}} = \sum_{j}^{\text{upper}} \pm\phi_L(t_j, \mathbf{x}_u(t_j)) - \sum_{j}^{\text{lower}} \pm\phi_L(t_j, \mathbf{x}_l(t_j))$$

$$\Delta\phi_{\text{propagation}} = \sum_{\text{upper}} \left(\int_{t_i}^{t_f} (L_c - E_i)\,\mathrm{d}t\right) - \sum_{\text{lower}} \left(\int_{t_i}^{t_f} (L_c - E_i)\,\mathrm{d}t\right)$$

$$\Delta\phi_{\text{separation}} = \langle \mathbf{p} \rangle \cdot \Delta\mathbf{x}$$

$$\phi_L(t, \mathbf{x}(t)) = \mathbf{k}_{\text{eff}} \cdot \mathbf{x}(t) - \omega t + \delta$$

Matter-wave Atomic Gradiometer Interferometric Sensor (MAGIS-100)

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Current a 10 m EP tower and 10 m prototype

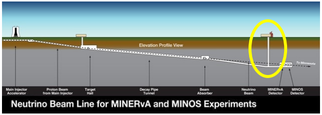
Science Reach

- Measured Earth's rotation with extreme precision
- Detected local gravity gradient of moving test-masses and testing Equivalence Principle ($< 10^{-12}$)
- Macroscopic separation of quantum states of atom ensembles (54 cm)

MAGIS-100 uses 100 m baseline with three atom sources

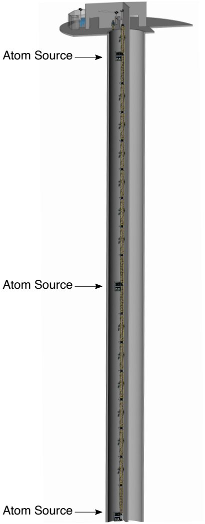
Science Goals

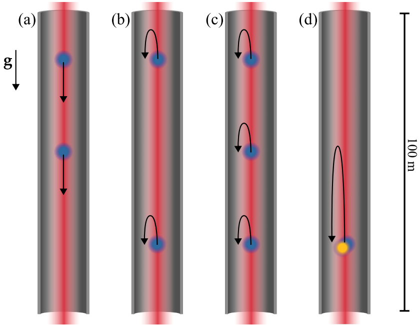
- Dark matter detection
- Gravitational wave detector prototype
- Extreme macroscopic quantum mechanics



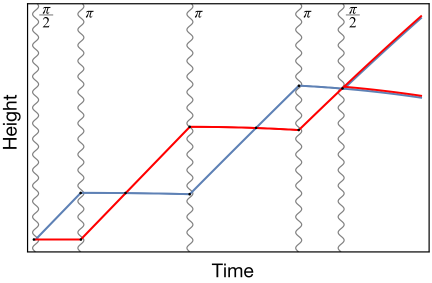
Abe, Mahiro, et al. QST (2021).

arXiv:2104.02835v1





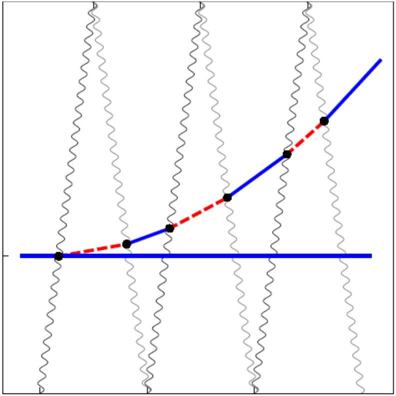
Multiple configurations



Large momentum transfer (LMT) and multi-loop atom interferometry

- Multiple laser pulses to accelerate one of the atom interferometer arms
- Coherently enhance differential phase measurement scales with n the number of LMT pulses

$$\Delta\phi \sim 2n\omega_A(L)$$



Graham, et al. PRL (2013)

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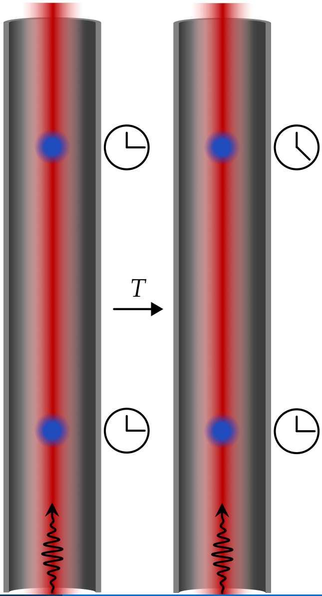
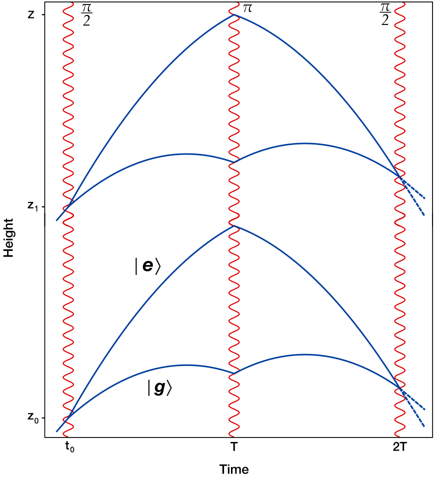
- Clocks are started simultaneously ticking with frequency ω_A
- Fluctuation of laser alters clock measurement of time $T \to T + \Delta T$

Initial States	$\frac{1}{\sqrt{2}}\lvert g\rangle + \frac{1}{\sqrt{2}}\lvert e\rangle$
Lower Source	$\frac{1}{\sqrt{2}}\lvert g\rangle + \frac{1}{\sqrt{2}}\lvert e\rangle\, e^{-i\omega_a T}$
Upper Source	$\frac{1}{\sqrt{2}}\lvert g\rangle + \frac{1}{\sqrt{2}}\lvert e\rangle\, e^{-i\omega_a (T+\Delta T)}$



- Simultaneous interferometer interference measurement
- Common laser allows common mode noise suppression

$$\frac{1}{\sqrt{2}}\left|g_0\right\rangle + \frac{1}{\sqrt{2}}\left|e_0\right\rangle e^{i\phi_L}$$
$$\frac{1}{\sqrt{2}}\left|g_1\right\rangle + \frac{1}{\sqrt{2}}\left|e_1\right\rangle e^{i\phi_L}$$

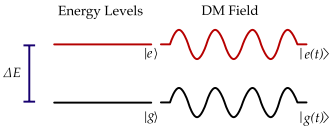


Dark Matter

- Most sensitive to general class of ultra-light scalar field dark matter
  - Time dependant fluctuation of fundamental constants (fine structure, electron mass, gravitational g)
  - Accelerations caused by gradients of field
- Includes QCD Axion, axion-like, and relaxion

Gravitational Waves

- Atoms interact with common laser beam, gravitational waves will alter the laser travel time between 2 atom clouds
- Gradiometer measurement of atoms in atomic clock modes will show phase difference



$$\Delta\phi \sim \omega_A(2L/c)$$

Dark Matter $\delta\omega_A$

Gravitational Waves $\delta L = hL$

MAGIS-100 Sensitivity

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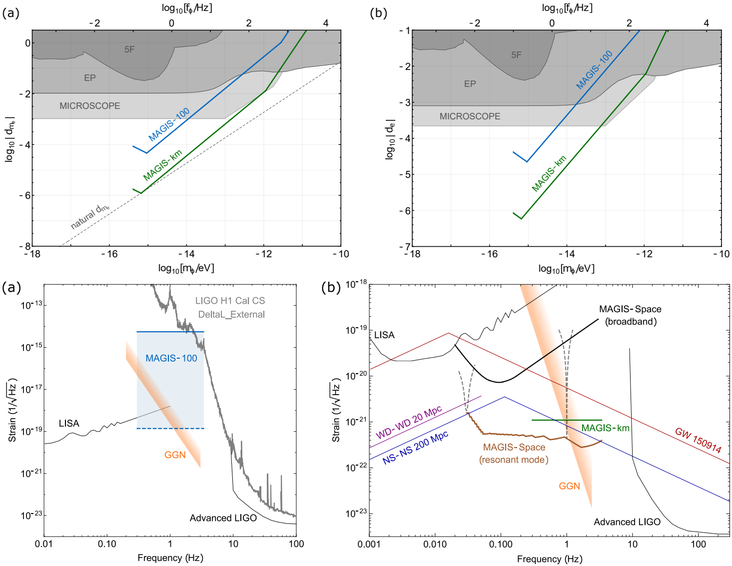

Abe, Mahiro, et al. QST (2021). arXiv:2104.02835v1

AION and MAGIS Integration

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- Atom Interferometry Observatory Network (AION) in the UK
- Plans to work complementary to MAGIS and network the detectors together for enhanced science goals
- Will use similar technologies as MAGIS
- Provides uncommon noise background suppression
- Adds to the verification of science measurements

MAGIS-100 Subsystems

Main subsystems to consider for systematics and noise sources

- Laser system
- Atom Sources
- Detector environment
- Detection and readout

Systematics and Simulation

1. Systematics and Error Analysis
   - Coriolis
   - Laser Wavefront Noise
   - Seismic Environment Characterization
   - Gravity Gradient Noise Analysis
2. Modeling and Simulation
   - Gravity Gradient Noise Model
   - MAGIS-100 simulation for R&D

Understanding Systematics and Noise

- Crucial for maximizing detector sensitivity
- For test bed systems like MAGIS-100 plays important R&D role for long baseline atom interferometers
- Goal is to understand leading order systematics and devise methods for mitigating them
- Interesting leading order effects: Coriolis effect, laser wavefront perturbations, gravity gradients, gravity gradient noise (GGN)
- Full list can be found in [Abe, Mahiro, et al. QST (2021). arXiv:2104.02835v1]

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Deflection caused by atoms in intertial free-fall reference frame embedded in accelerating frame of Earth

- Causes atoms to fall out of interferometry laser beam, leads to decrease in contrast
- Atoms pick up phase shift during propagation

$$\mathbf{a}_c = -2\mathbf{\Omega} \times \mathbf{v}. \tag{1}$$

$$\mathbf{\Omega} = \begin{pmatrix} 0 \\ \omega\cos(\phi) \\ \omega\sin(\phi) \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}, \tag{2}$$

$$\Delta\mathbf{r} = \omega \begin{pmatrix} v_y\sin(\phi)t^2 - \left[v_{0,z}t^2 - g\frac{t^3}{3}\right]\cos(\phi) \\ -v_x\sin(\phi)t^2 \\ v_x\cos(\phi)t^2 \end{pmatrix}. \tag{3}$$

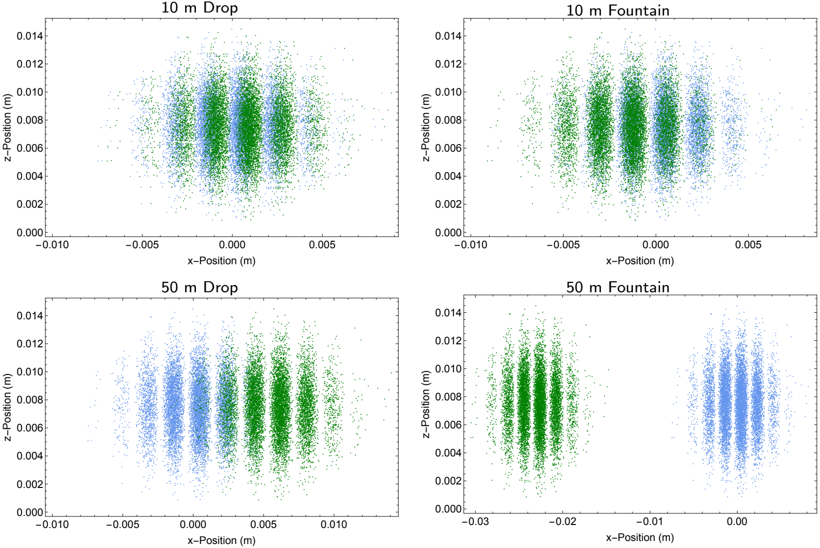
Table 1: Table of flight times for various configurations of atom launch schemes.

Configuration	T	$2T$
100 m Drop	2.26 s	4.52 s
100 m Fountain	4.52 s	9.03 s
50 m Drop	1.595 s	3.19 s
50 m Fountain	3.19 s	6.38 s
10 m Drop	0.72 s	1.43 s
10 m Fountain	1.43 s	2.86 s

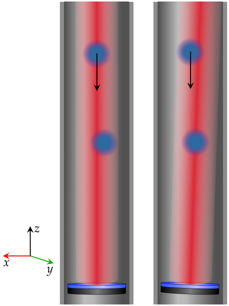
Table 2: The calculation was performed with $\omega = 7.27 \times 10^{-5}$ rad/s, $v_x = v_y = 1$ mm/s, and initial launch velocities for the fountain configurations of $v_0 = 44.29$ m/s, 31.32 m/s and 13.03 m/s for the 100 m, 50 m, and 10 m fountains respectively.

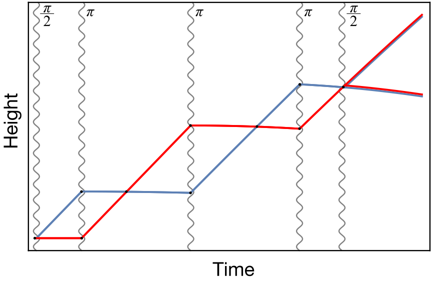
Configuration	Δx	Δz
100 m Drop	1.64 cm	1.1×10^{-6} m
100 m Fountain	−6.52 cm	4.4×10^{-6} m
50 m Drop	5.75 mm	5.5×10^{-7} m
50 m Fountain	−2.31 cm	2.2×10^{-6} m
10 m Drop	0.518 mm	1.1×10^{-7} m
10 m Fountain	−1.63 mm	4.4×10^{-7} m

Blue is unperturbed position of atom cloud during imaging, Green is position affected by unmitigated Coriolis



- Primary suppression from tip-tilt mirror rotating counter to Earth's rotation
- Multiloop sequences that can be designed for compensation of Coriolis phase shift terms





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- Amplitude perturbations of the general form $E(x, z) = u(x, z)e^{ikz}$
- We then consider the initial amplitude $u(x, 0) = 1 + \delta \cos(k_x x)$ and the propagated amplitude $u(x, z) = 1 + \delta \cos(k_x x)e^{i\frac{k_x^2}{2k}z}$
- Paraxial approximation $k_z = \sqrt{k^2 - k_x^2} \approx k - \frac{k_x^2}{2k}$

$$
\begin{aligned}
\text{Amplitude:} \quad & \delta \cos(k_x x) \cos\left(\frac{k_x^2}{2k} z\right), \\
\text{Phase:} \quad & \phi_w = \delta \cos(k_x x) \sin\left(\frac{k_x^2}{2k} z\right), \\
\delta k_x = \frac{\partial \phi_w}{\partial x} & = -k_x \delta \sin(k_x x) \sin\left(\frac{k_x^2}{2k} z\right), \\
\delta k_z = \frac{\partial \phi_w}{\partial z} & = \frac{k_x^2}{2k} \delta \cos(k_x x) \cos\left(\frac{k_x^2}{2k} z\right).
\end{aligned} \tag{4}
$$

Considered effective momentum kicks because terms scale as

$$
\frac{\hbar \delta k_i^2}{2m} \qquad i = x, z \tag{5}
$$

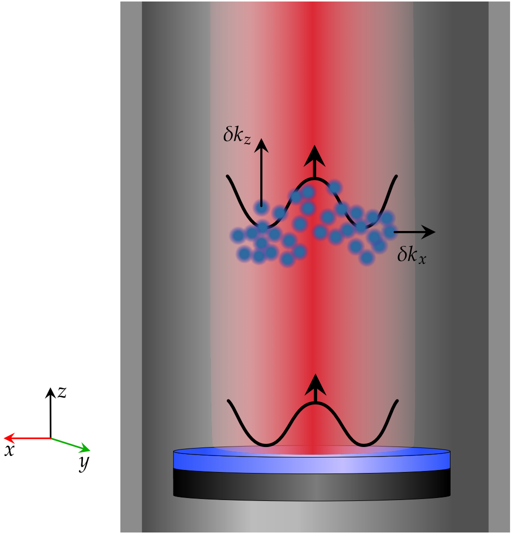
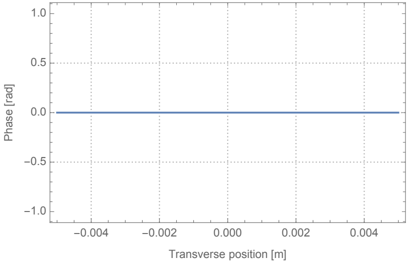

δk_z
δk_x
z
x
y

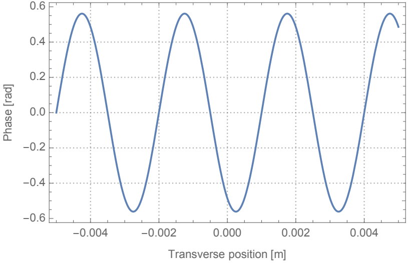
Table 3: Selection of example values for wavefront phase shifts at the first and second order in δ.

Term	Phase shift	Notes
1	$n\delta\cos(k_t x_i)\sin\left(\frac{k_t^2}{2k}H\right)$	First Order in δ
1	$(0.325\,\text{rad})\left(\frac{n}{100}\right)\left(\frac{\delta}{0.005}\right)\left(\frac{\sin\left(k_t^2 H/2k\right)}{0.65}\right)$	
2	$-\frac{k_t^4 n^2\delta^2 T\hbar\cos(k_t x_i)\cos\left(\frac{k_t^2}{2k}H\right)\cos(k_t T v_t + k_t x_i)\cos\left(\frac{g k_t^2 T^2}{4k}-\frac{k_t^2 T v_z}{2k}-\frac{k_t^2}{2k}H\right)}{8k^2 m}$	Longitudinal Kicks
2	$\left(-1.25\times10^{-11}\,\text{rad}\right)\left(\frac{n}{100}\right)\left(\frac{\delta}{0.005}\right)\left(\frac{\cos\left(k_t^2 H/2k\right)}{0.7}\right)$	
3	$\frac{k_t^2 n^2\delta^2 T\hbar\sin(k_t x_i)\sin\left(\frac{k_t^2}{2k}H\right)\sin(k_t T v_t + k_t x_i)\sin\left(\frac{g k_t^2 T^2}{4k}-\frac{k_t^2 T v_z}{2k}-\frac{k_t^2}{2k}H\right)}{2m}$	Transverse Kicks
3	$\left(-4.33\times10^{-9}\,\text{rad}\right)\left(\frac{n}{100}\right)\left(\frac{\delta}{0.005}\right)\left(\frac{\sin\left(k_t^2 H/2k\right)}{-0.2}\right)$	

Laser Wavefront Mitigation

- Apply laser beam tilt of $d_{\mathrm{atom}}\theta$ to final pulse from top or bottom tip-tilt mirror
- Allows measuring first derivative of wavefront via spatially resolved imaging
- Apply transverse initial velocity to atom cloud for information about wavefront in direction of kick

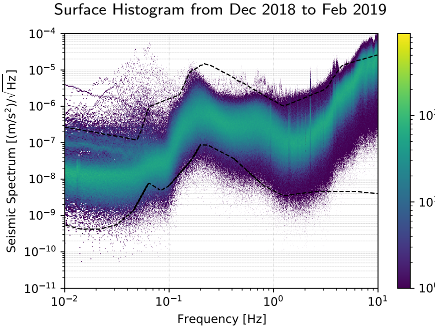


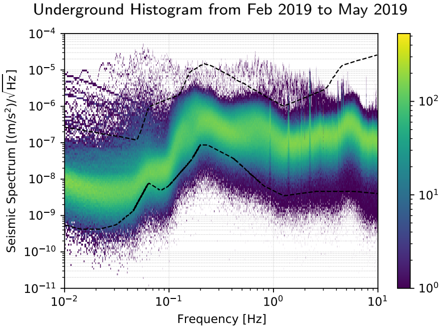


- To understand noise limits at Fermilab a state-of-the-art seismometer was purchased and installed on site
- Data was taken over the course of two years at one location underground and four surface locations
- Power spectra from the ambient seismic environment were made to understand vibrational noise sources as well as gravity gradient noise

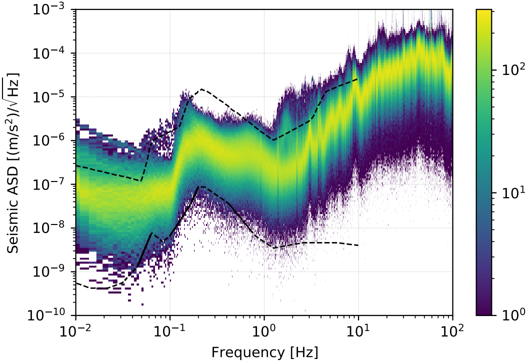
Ambient Seismic Noise

$$
\begin{aligned}
PSD\{x(t)\} &= \frac{2(\Delta t)^2}{T}|x(\omega)|^2, \\
x(\omega) = DFT\{x(t)\} &= \sum_{t=0}^{N-1} x(t)e^{2\pi i t/N}.
\end{aligned}
\tag{6}
$$





Measurements at proposed location of initial laser transport pedestal



Vibrational Noise Estimates

- Vibration couples to lasers and initial atom ensemble kinematics

$$\delta\phi_{\text{vibration}} \sim \left(10^{-9}\,\text{rad}/\sqrt{\text{Hz}}\right)\left(\frac{n}{100}\right)\left(\frac{\Delta v}{10\,\mu\text{m/s}}\right)\left(\frac{T}{1\,\text{s}}\right)\left(\frac{\delta a}{10^{-4}\,\text{m/s}^2/\sqrt{\text{Hz}}}\right) \quad (7)$$

$$\delta\phi_{\text{RGGV}} \sim \left(2\times 10^{-6}\,\text{rad}/\sqrt{\text{Hz}}\right)\left(\frac{n}{100}\right)\left(\frac{\Delta v_x}{1\,\mu\text{m/s}/\sqrt{\text{Hz}}}\right)\left(\frac{T}{1\,\text{s}}\right) \quad (8)$$

Can be mitigated by vibrational isolation on laser tables, large stable pedestal mounts for the laser transport system

Gravity Gradient Noise (GGN)

- Gravity gradient noise is second order effect where seismic surface waves (Rayleigh waves) couple to gravitational potential field
- Perturbations in potential field lead to acceleration perturbations
- Atom trajectories are perturbed under free fall and pickup phase shift

$$\delta h_{\text{ggn}} = \frac{\pi G \gamma_\nu \rho_0}{2LT\omega_{ggn}^3} \langle \delta\xi_z \rangle \left(e^{-\sqrt{2}L\omega_{ggn}/c_R} - 1 \right) \sin(\omega_{ggn} T) \cos(\phi_{ggn}) \tag{9}$$

$$\delta h_{\text{ggn}} \sim \left(10^{-19} / \sqrt{\text{Hz}}\right) \left(\frac{100\,\text{m}}{L}\right) \left(\frac{1.5\,\text{s}}{T}\right) \left(\frac{1\,\text{Hz}}{\omega_{ggn}}\right)^3 \left(\frac{\delta\xi_z}{1\,\mu\text{m}}\right) \left(\frac{300\,\text{m/s}}{c_R}\right) \tag{10}$$

- Atom interferometer phase response calculated by semi-classical analytical solver using the GGN phase term $\delta\phi_{\text{ggn}}$

$$\delta\phi_{\text{ggn}} = \frac{\pi G \gamma_\nu \rho_0}{\omega_{ggn}} \langle \delta\xi_z \rangle \, n k_{eff} \, T e^{-\sqrt{2} k_l z} \sin(\omega_{ggn} T) \cos(\phi_{ggn}). \tag{11}$$

$$L_c = \frac{1}{2} m \dot{\mathbf{r}}^2 - m\phi(\mathbf{r}) - m\delta\phi_{\text{ggn}} \tag{12}$$

GGN Analysis

Stochastically simulated 1000 measurements and compute the RMS measurement to be fit

<u>Exponential Model</u>

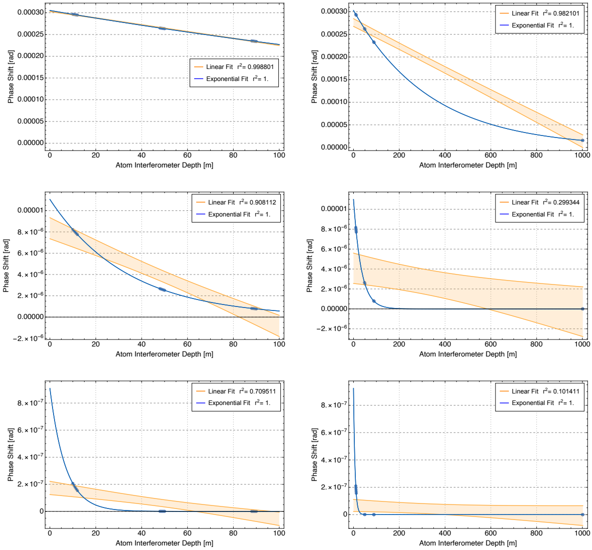
$$\phi(z) = A\exp(-\alpha z)$$

<u>Linear Model</u>

$$\phi(z) = Cz$$

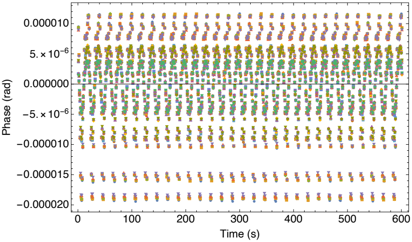
Assumed Rayleigh wave frequencies:

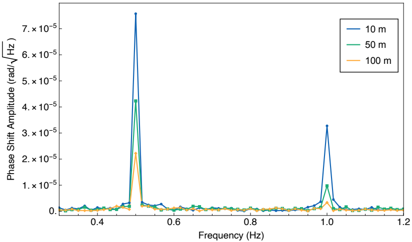
0.1 Hz
1 Hz
5 Hz



Ongoing GGN Analysis

- GGN analysis using Fourier components simulated from time sampled phase measurements
- Allows for correlation between atom sources for extraction of exponential character
- No loss of shot-to-shot information by averaging
- Possibly more sensitive than absolute phase measurements





GGN Suppression and Mitigation

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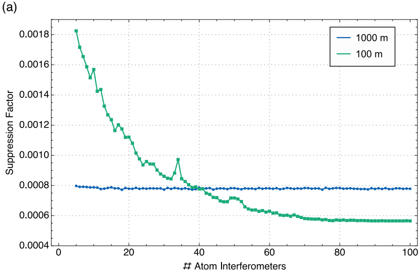
- String-of-pearls characterization
- Residual linear strain noise interferes with GW and DM measurements
- Suppression factor defined by ratio of linear fitting and simulated phase difference along the baseline
- Correlated seismometer array and *in situ* calibration data



- Modeled gravity gradient noise (GGN)
  - Test mass acceleration perturbation caused by density perturbations of ground and atmosphere
  - Important systematic that limits MAGIS-100 maximum sensitivity
- Simulation package developed for MAGIS-100
  - Generates realistic wavefunctions and simulated CCD images
  - Used for R&D and evaluating parameter choice effects on phase extraction precision

Short wavelength approximation:

$$h_{GGN} = \frac{G\rho_0}{2\pi f^2 L} \langle \xi_z \rangle$$

Long wavelength approximation:

$$h_{GGN} = \frac{G\rho_0}{f c_R} \langle \xi_z \rangle$$

Using simple model convert between measured seismic displacements into inferred GGN strain

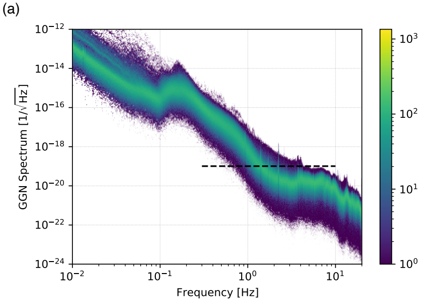


Figure 1: Inferred GGN strain spectrum (a) from surface measurements, (b) from underground measurements. Black dashed line is strain sensitivity after advancements.

- Began as case study into optimizing the optical system parameters
- Evolved into a simulation package for generating images after an atom interferometric measurement

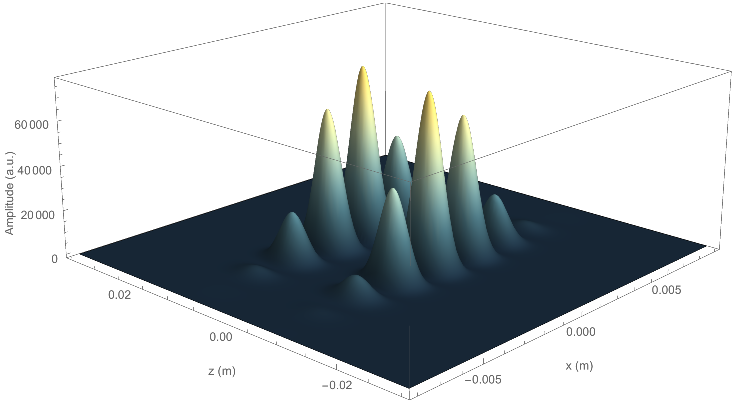


Figure 2: Plot of analytical 2 port wavefunction. Parameters used for strontium atoms with interferometry time $T = 3\,\mathrm{s}$, extra drift time $T_{\mathrm{extra}} = 0.15\,\mathrm{s}$, cloud radius $w_0 = 0.002\,\mathrm{m}$, port separation velocity $\Delta v_z = \hbar k_{z,spread}/m = 0.10\,\mathrm{m/s}$, and a fringe shear $k_{\mathrm{fringe}} = 1/0.0003\ \mathrm{m}^{-1}$ with a relative phase $\phi = \pi/2$ between the ports.

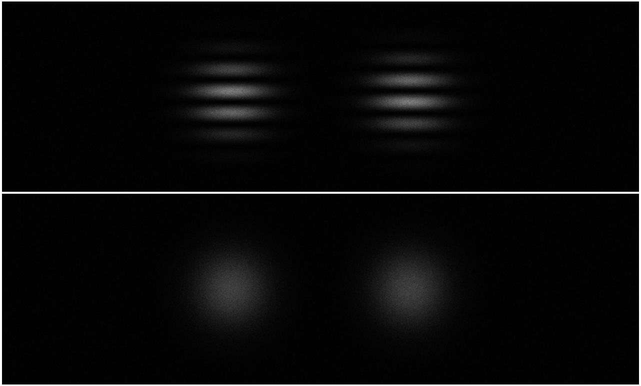


Figure 3: Simulated atom cloud interference pattern pixelated images. Images, going from the top to the bottom, are of the xz projection where z runs horizontally, yz projection with z running horizontally, and xy projection from top down view.

Outlook

- GGN and environmental noise analysis and site characterization for AION
- Characterization and measurement of atmospheric Newtonian noise
- Modeling of deep underground GGN for kilometer length baselines

Summary

- Leading systematics have been studied and analyzed for MAGIS-100
- Mitigation strategies will ensure crucial precision of MAGIS-100 when operation begins
- Helped develop MAGIS simulation and analysis tools that will become part of the MAGIS-100 data analysis pipeline

Collaborators and Thanks

MAGIS-100 Collaboration
Fermilab Team: Rob Plunkett and others
Stanford: Jason Hogan and his students
Northwestern: Tim Kovachy and his students
Liverpool: Jon Coleman and his students

Support from the Kavli Foundation

Backup Slides

- Gravitational waves will cause acceleration difference between freely falling atoms
- Measure differential acceleration where atoms are proof masses
- Light flight time is mass separation
- Reference is atomic phase

$$\mathrm{d}s^2 = \mathrm{d}t^2 - (1 + h\sin(\omega(t-z)))\,\mathrm{d}x^2 - (1 - h\sin(\omega(t-z)))\,\mathrm{d}y^2 - \mathrm{d}z^2$$

$$L(1 + h\sin(\omega t))$$

$$\delta\phi_{gw} = 2k_{eff}\,hL\sin^2\left(\frac{\omega_{gw}T}{2}\right)\sin(\phi_0),$$

Laser Wavefront Limits

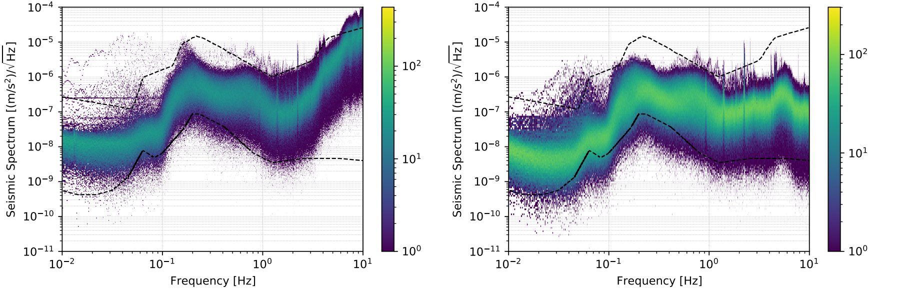
UNIVERSITY OF CAMBRIDGE

- Our analysis of laser wavefront provides the required specs to achieve maximum sensitivity
- More suppression can be gained by averaging effects across the atom cloud during large momentum transfer
- Mitigation strategies include:
  - Tilted laser pulse sequences to image the wavefront across the atom cloud
  - Advanced array of image sensors to image the wavefront from reconstruction

$$\Delta x \sim 0.1\,\mu\text{m}$$

$$\delta \sim 3 \times 10^{-4}$$

Seismic Plots



Laser Transport Vibrations