$b \to sl^+l^-$ Anomalies and Y_3

by
Ben Allanach
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- $b \to sl^+l^-$ $B-$anomalies
- Third Family Hypercharge Models (BCA, Davighi, 1809.01158; ibid 1905.10327)
- Global fits (BCA, Camargo-Molina, Davighi, 2103.12056)
- $Z' \to \mu^+\mu^-$ search (BCA, Butterworth, Corbett, 1904.10954)

Cambridge Pheno Working Group

Where data and theory collide

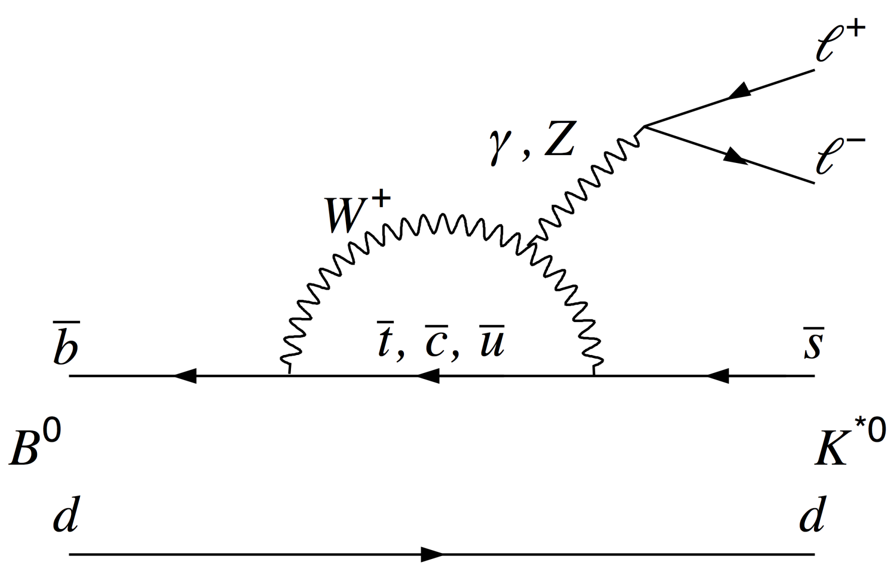
Science & Technology Facilities Council

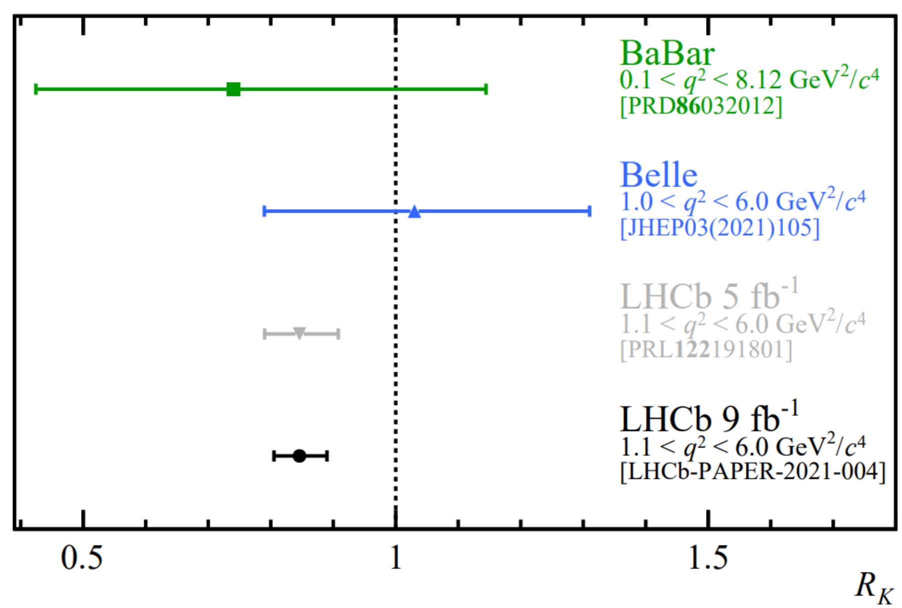
Strange b Activity

$R_K^{(*)}$ in Standard Model

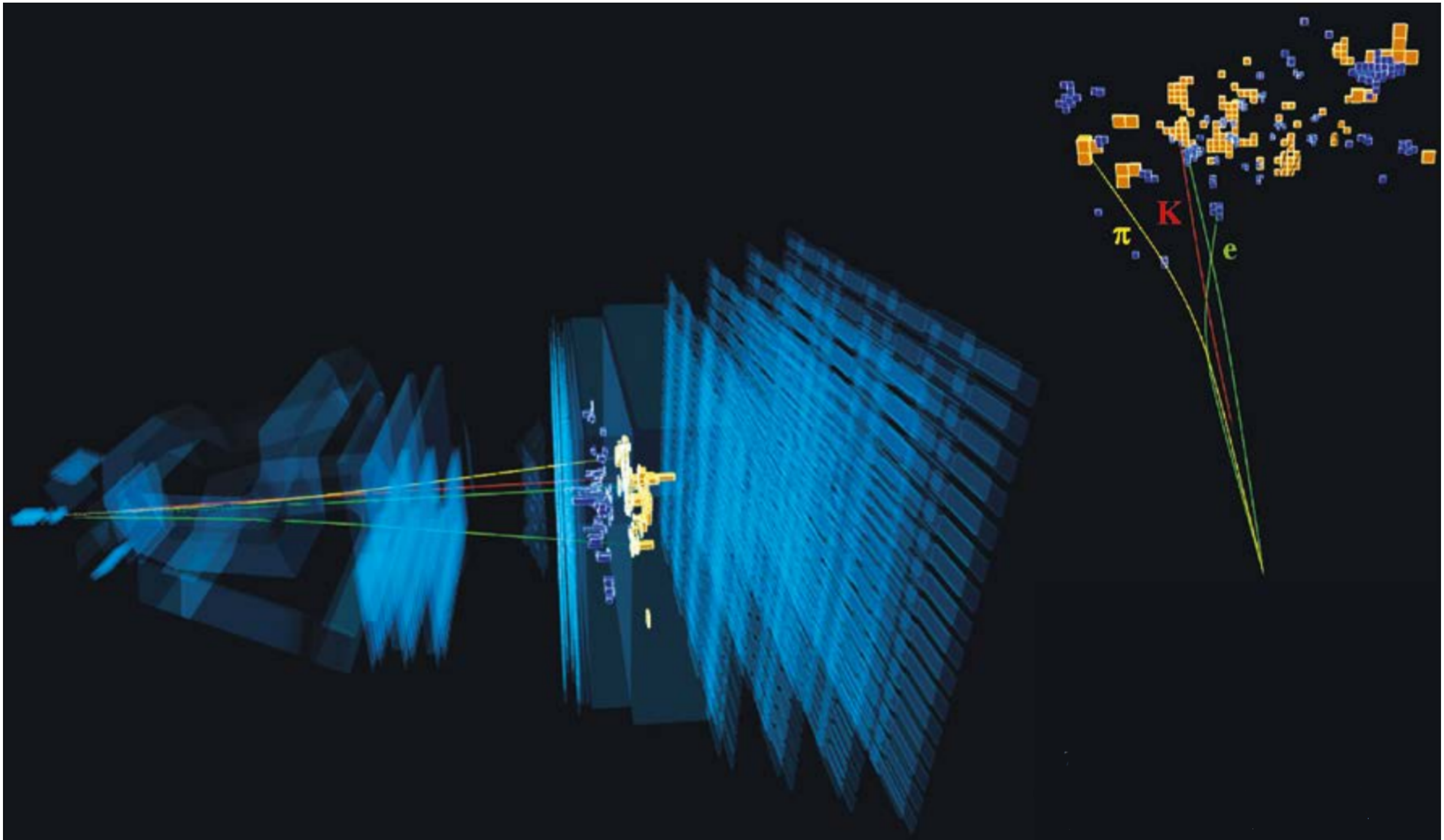
$$R_K = \frac{BR(B \to K\mu^+\mu^-)}{BR(B \to Ke^+e^-)}, \qquad R_{K^*} = \frac{BR(B \to K^*\mu^+\mu^-)}{BR(B \to K^*e^+e^-)}.$$

These are rare decays (each BR$\sim \mathcal{O}(10^{-7})$) because they are absent at tree level in SM+EW+CKM





LHCb $B^0 \to K^{0*}e^+e^-$ Event[1]

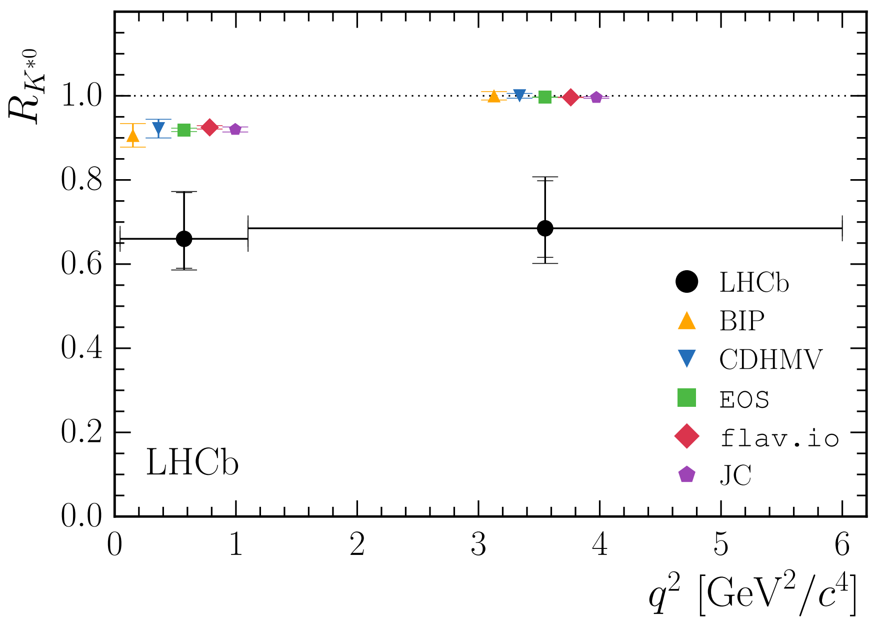


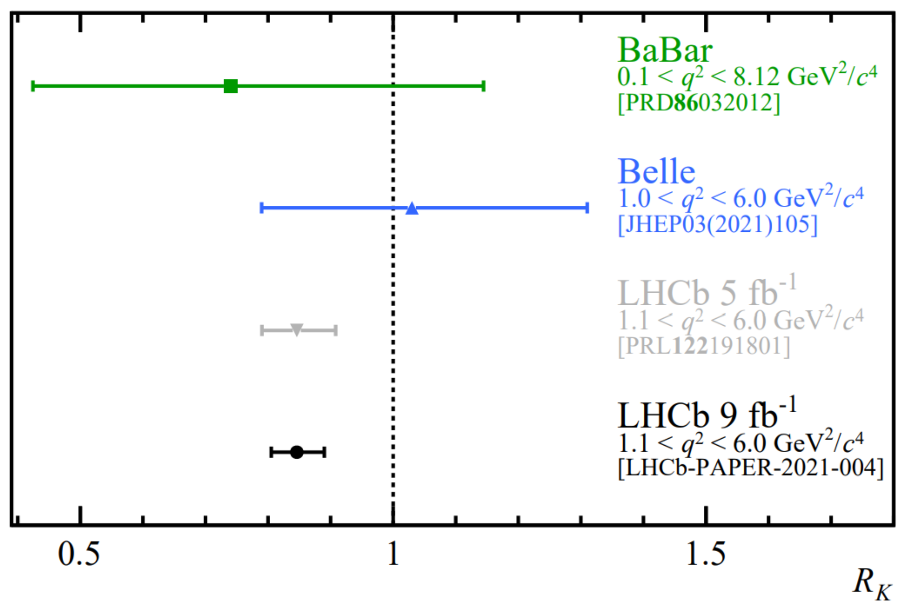
[1]Picture from CERN Courier April 2018

$R_{K^{(*)}}$

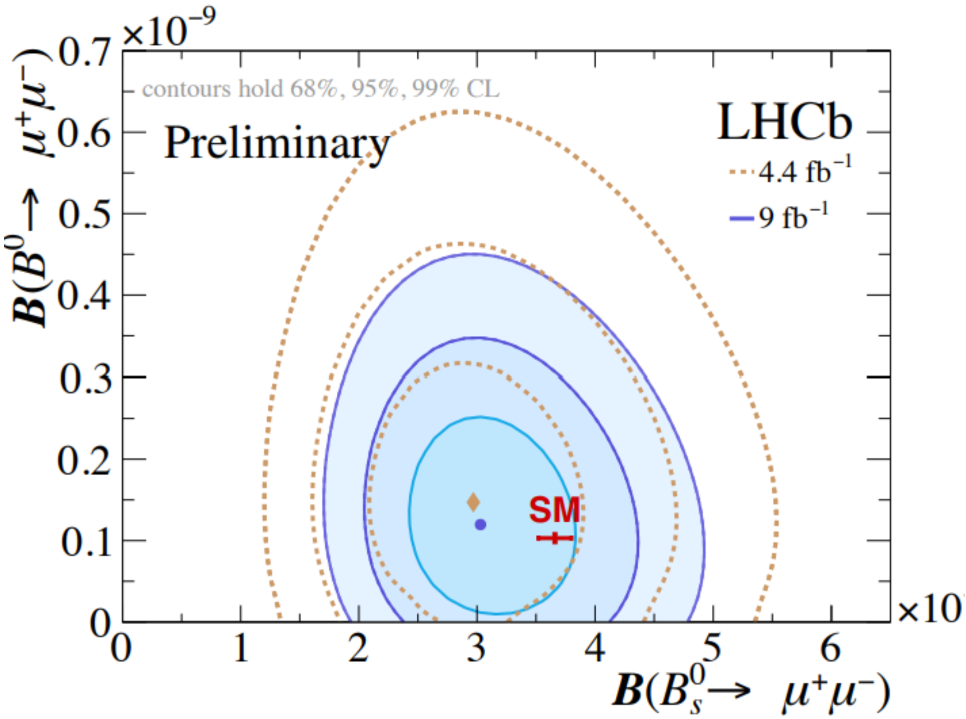
LHCb results: $q^2 = m_{ll}^2$.

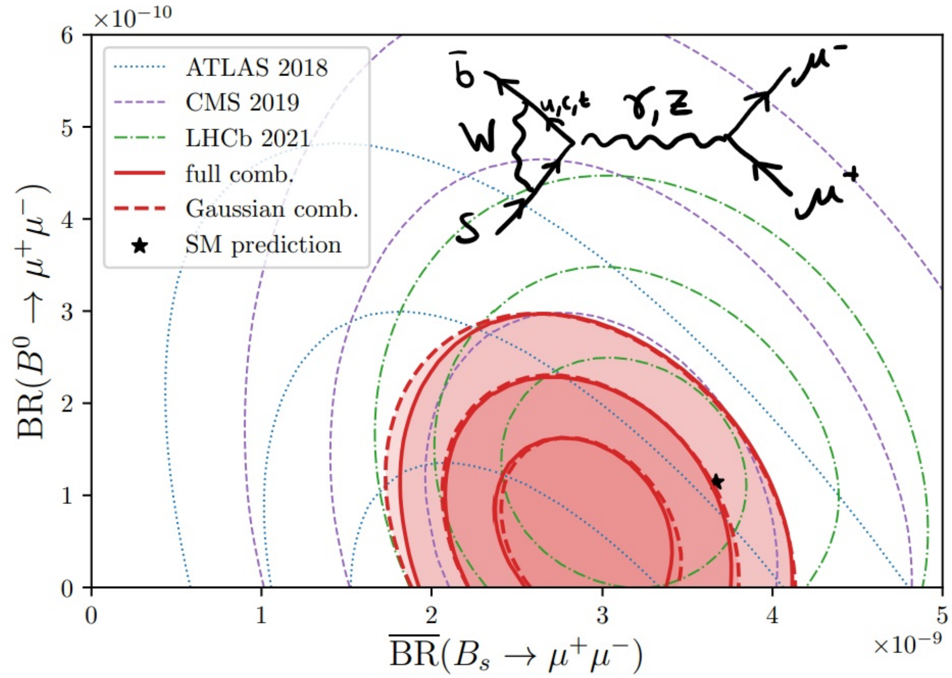
	q^2/GeV^2	SM	LHCb 3 fb^{-1}	σ
R_K(9 fb)	$[1.1, 6]$	1.00 ± 0.01	0.846 ± 0.04	3.1
R_{K^*}(3 fb)	$[0.045, 1.1]$	0.91 ± 0.03	$0.66^{+0.11}_{-0.07}$	2.2
R_{K^*}(3 fb)	$[1.1, 6]$	1.00 ± 0.01	$0.69^{+0.11}_{-0.07}$	2.5





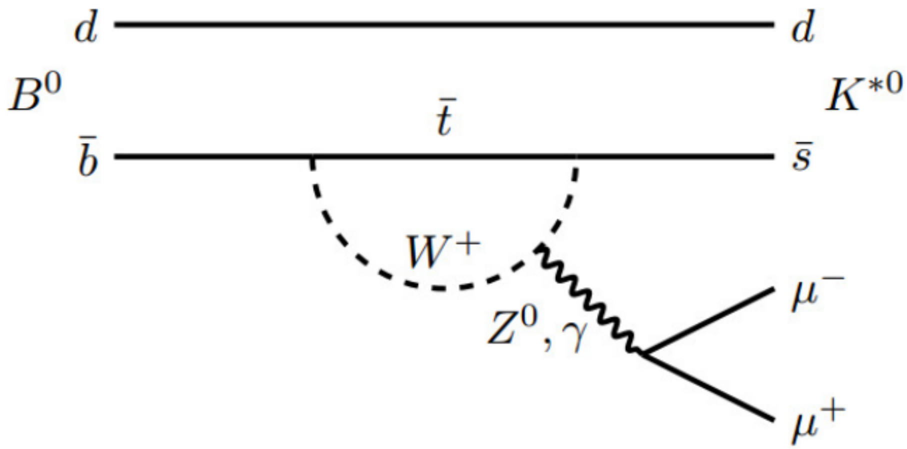
Latest $BR(B_s \to \mu^+\mu^-)$

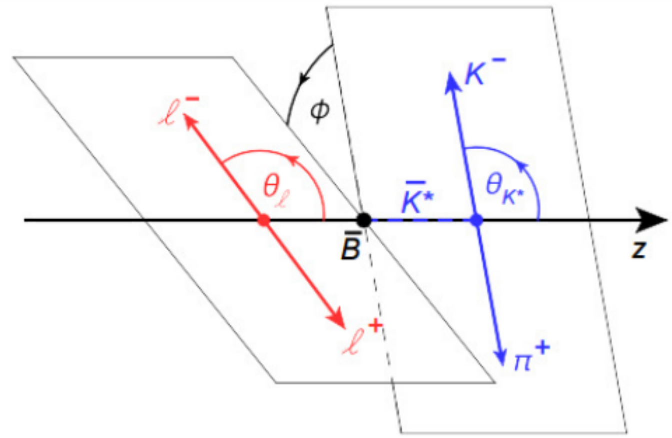




ATLAS + CMS + '21 LHCb measurements, multivariate Gaussian combination by Altmannshofer, Stangl 2103.13370

$B^0 \to K^{*0}(\to K^+\pi^-)\mu^+\mu^-$

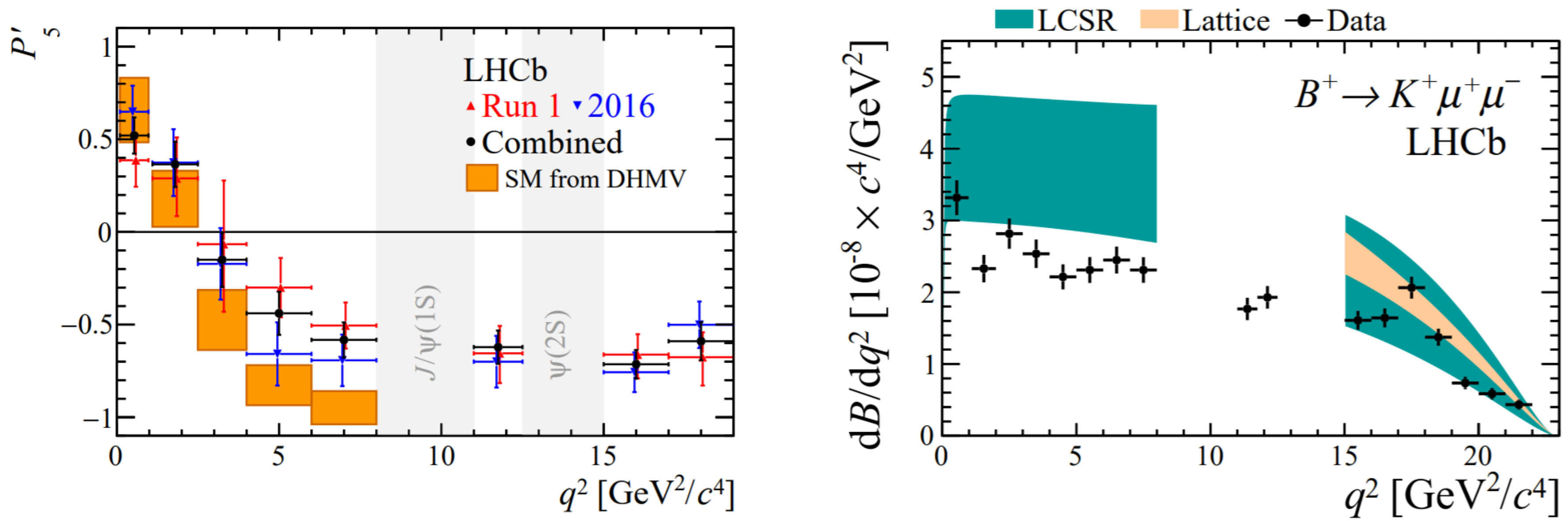




Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m^2_{\mu\mu}$

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_K + F_{\mathrm{L}}\cos^2\theta_K + \tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_{\mathrm{L}}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+ S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ \tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

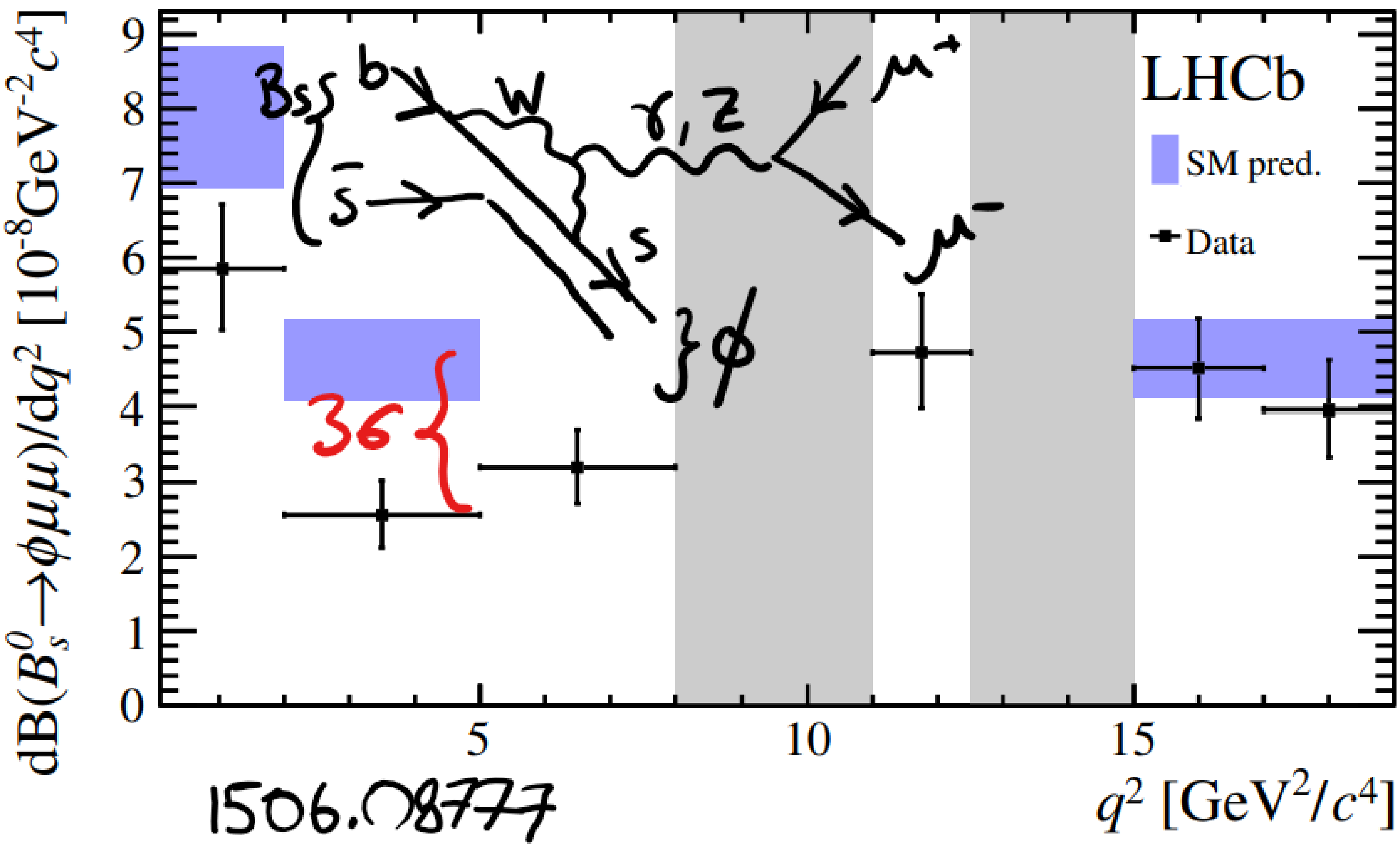
P'_5

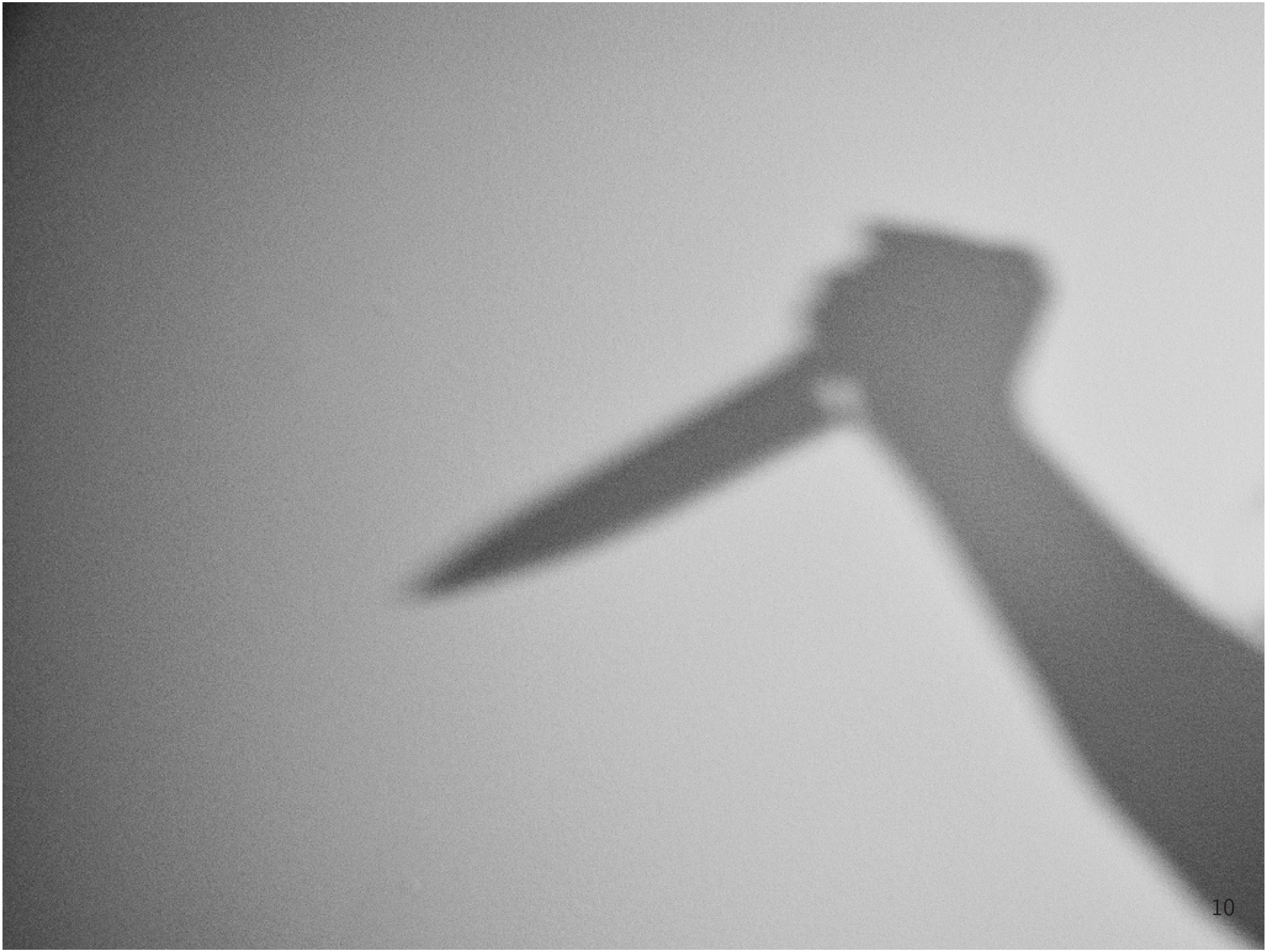


$P'_5 = S_5/\sqrt{F_L(1-F_L)}$, leading form factor uncertainties cancel [2]

[2] LHCb, 2003.04831

$B_s \to \phi\mu^+\mu^-$



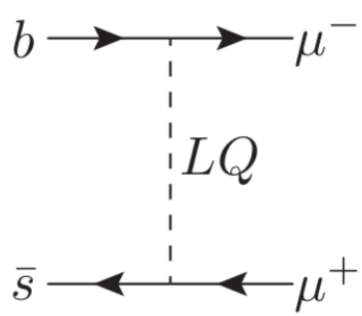


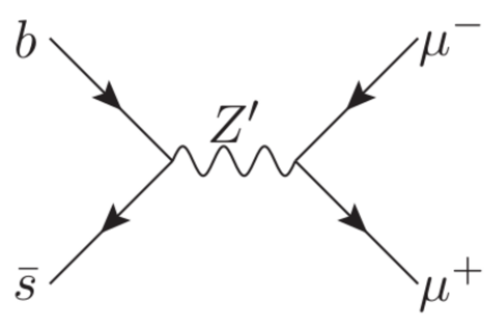
Simplified Models for NCBAs

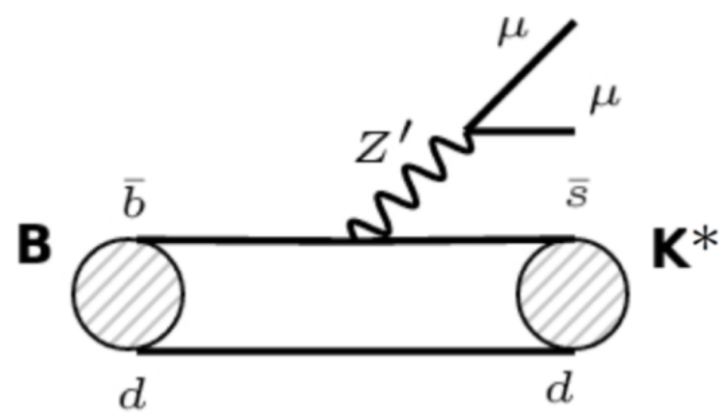
$2-4\sigma$ Discrepancies with SM predictions in (2021):

- R_K, R_{K^*}
- $BR(B_s \to \mu^+\mu^-)$
- $BR(B_s \to \phi\mu^+\mu^-)$
- Angular distributions of $B \to K^*\mu^+\mu^-$, etc

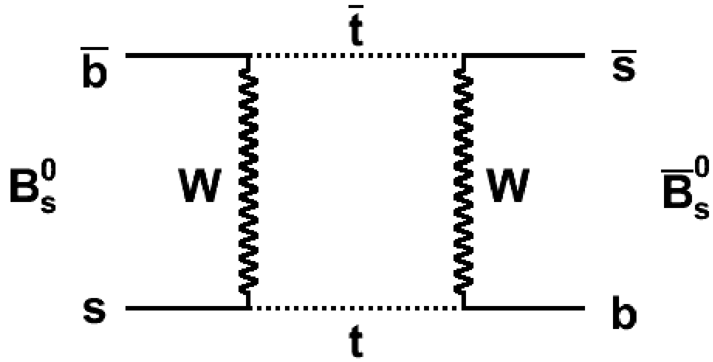
We have tree-level flavour changing new physics options:

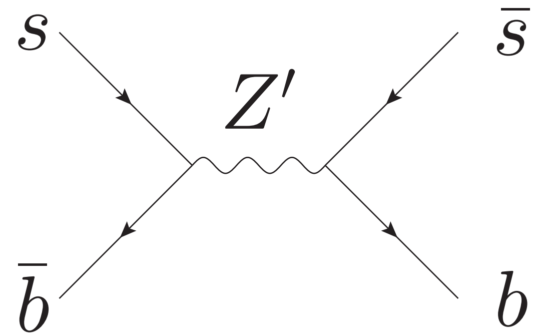






$B_s - \bar{B}_s$ Mixing





$$\bar{g}_L^{sb} \lesssim \frac{M_{Z'}}{194 \text{ TeV}}$$

from QCD sum rules and lattice[3]. Weaker on LQs (see later).

[3] King, Lenz, Rauh, arXiv:1904.00940

During the 1990s

We wanted to be the Grand Architects, searching for **the** string model to rule them all

During the 2020s

We are happy with **any** beyond the Standard Model roof

A Simple Z' Model

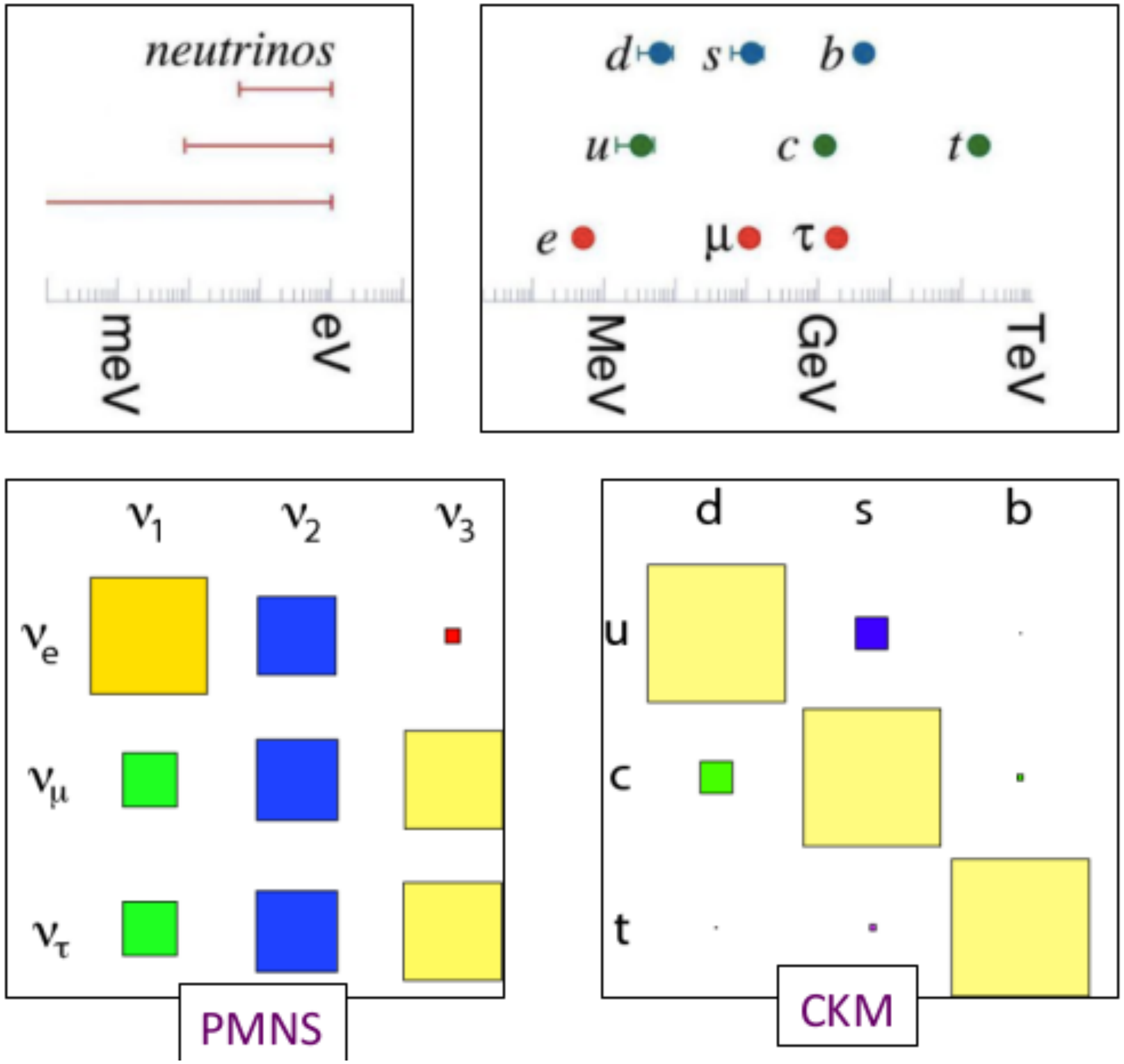
BCA, Davighi, 1809.01158: Add complex SM-singlet scalar 'flavon' $\theta_{X\neq 0}$ which breaks gauged $U(1)_X$:

$$SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_X$$
$$\Big\downarrow \langle\theta\rangle \sim \text{Several TeV}$$
$$SU(3) \times SU(2)_L \times U(1)_Y$$
$$\Big\downarrow \langle H\rangle \sim 246 \text{ GeV}$$
$$SU(3) \times U(1)_{em}$$

- SM fermion content
- Zero X charges for first two generations
- Solve anomaly cancellation for $U(1)_X$

The Flavour Problem

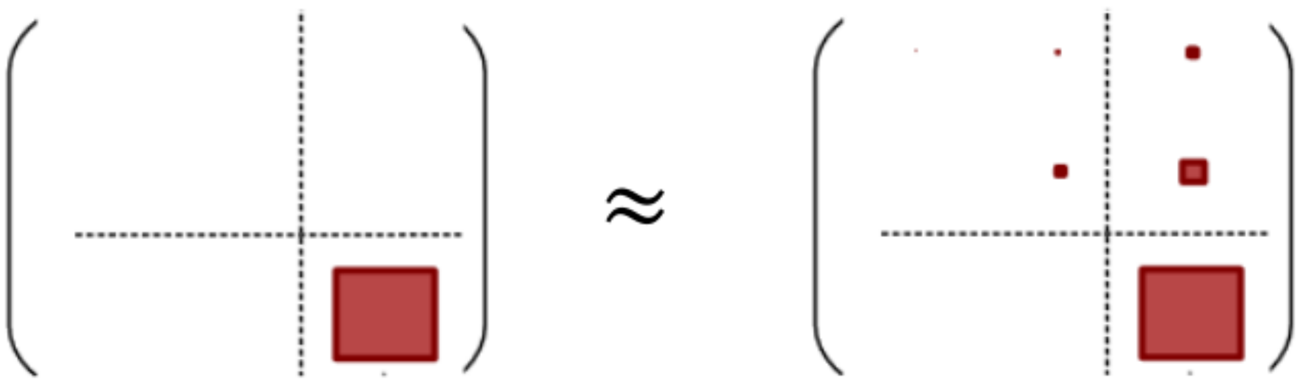
The Flavour Problem



Unique Solution: $X = Y_3$

$$\boxed{\begin{array}{cccc} X_{Q'_{1,2}} = 0 & X_{u_{R'_{1,2}}} = 0 & X_{d_{R'_{1,2}}} = 0 & X_{L'_{1,2}} = 0 \\ X_{e_{R'_{1,2}}} = 0 & X_H = -1/2 & X_{Q'_3} = 1/6 & X_{u'_{R3}} = 2/3 \\ X_{d'_{R3}} = -1/3 & X_{L'_3} = -1/2 & X_{e'_{R3}} = -1 & X_\theta \neq 0 \end{array}}$$

$$\mathcal{L} = Y_t \overline{Q_3{}'_L} H t'_R + Y_b \overline{Q'_{3L}} H^c b'_R + Y_\tau \overline{L_3{}'_L} H^c \tau'_R + H.c.,$$



Y_3 Consequences

- Flavour changing TeV-scale Z' to do NCBAs: couples dominantly to third family quarks and second family leptons
- First two fermion families massless at renormalisable level
- Their masses and fermion mixings generated by small non-renormalisable operators

This explains the hierarchical heaviness of the third family and small CKM angles

$Z - Z'$ mixing

Because $Y_3(H) = 1/2$, $B - W^3 - X$ bosons mix:

$$\mathcal{M}_N^2 = \frac{1}{4}\begin{pmatrix} g'^2v^2 & -gg'v^2 & g'g_Xv^2 \\ -gg'v^2 & g^2v^2 & -gg_Xv^2 \\ g'g_Xv^2 & -gg_Xv^2 & 4g_X^2\langle\theta\rangle^2\left(1+\frac{\epsilon^2}{4}\right) \end{pmatrix} \begin{matrix} -B_\mu \\ -W_\mu^3 \\ -(X)_\mu \end{matrix}$$

- $v \approx 246$ GeV is SM Higgs VEV,
- $\langle\theta\rangle$ ~TeV. $M_{Z'} = g_X\langle\theta\rangle$.
- $g_X = U(1)_X$ gauge coupling
- $\epsilon \equiv v/\langle\theta\rangle \ll 1$

$$
\begin{aligned}
\mathcal{L}_{X\psi} &= g_X \left(\frac{1}{6}\overline{\mathbf{u_L}}\Lambda^{(u_L)}\gamma^\rho \mathbf{u_L} + \frac{1}{6}\overline{\mathbf{d_L}}\Lambda^{(d_L)}\gamma^\rho \mathbf{d_L} - \right. \\
&\quad \frac{1}{2}\overline{\mathbf{n_L}}\Lambda^{(n_L)}\gamma^\rho \mathbf{n_L} - \frac{1}{2}\overline{\mathbf{e_L}}\Lambda^{(e_L)}\gamma^\rho \mathbf{e_L} + \\
&\quad \frac{2}{3}\overline{\mathbf{u_R}}\Lambda^{(u_R)}\gamma^\rho \mathbf{u_R} - \\
&\quad \left. \frac{1}{3}\overline{\mathbf{d_R}}\Lambda^{(d_R)}\gamma^\rho \mathbf{d_R} - \overline{\mathbf{e_R}}\Lambda^{(e_R)}\gamma^\rho \mathbf{e_R} \right) Z'_\rho,
\end{aligned}
$$

$$
\Lambda^{(I)} \equiv V_I^\dagger \xi V_I, \qquad \xi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

Z' couplings, $I \in \{u_L, d_L, e_L, \nu_L, u_R, d_R, e_R\}$

A simple limiting case

$$V_{u_R} = V_{d_R} = V_{e_R} = 1$$

for simplicity and the ease of passing bounds.

$$V_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{pmatrix}, \qquad V_{e_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$\Rightarrow V_{u_L} = V_{d_L} V^\dagger_{CKM}$ and $V_{\nu_L} = V_{e_L} U^\dagger_{PMNS}$.

Important Z' Couplings

$$g_X \left[\frac{1}{6} (\overline{d_L}\ \overline{s_L}\ \overline{b_L}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2\theta_{23} & \frac{1}{2}\sin 2\theta_{23} \\ 0 & \frac{1}{2}\sin 2\theta_{23} & \cos^2\theta_{23} \end{pmatrix} \not{Z}' \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \right.$$

$$\left. -\frac{1}{2} (\overline{e_L}\ \overline{\mu_L}\ \overline{\tau_L}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \not{Z}' \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \right]$$

$Z - Z'$ mixing

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- $v \approx 246$ GeV is SM Higgs VEV,
- $\langle\theta\rangle$ $\sim$TeV. $M_{Z'} = g_X \langle\theta\rangle$.
- $g_X = U(1)_X$ gauge coupling
- $\epsilon \equiv v/\langle\theta\rangle \ll 1$

$Z - Z'$ mixing angle

$$\sin\alpha_z \approx \frac{g_X}{\sqrt{g^2 + g'^2}} \left(\frac{M_Z}{M'_Z}\right)^2 \ll 1.$$

This gives small non-flavour universal couplings to the Z boson propotional to g_X and:

$$Z_\mu = \cos\alpha_z \left(-\sin\theta_w B_\mu + \cos\theta_w W^3_\mu\right) + \sin\alpha_z X_\mu,$$

Important Z' Couplings

$$g_X \left[\frac{1}{6} (\overline{d_L}\ \overline{s_L}\ \overline{b_L}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2\theta_{23} & \frac{1}{2}\sin 2\theta_{23} \\ 0 & \frac{1}{2}\sin 2\theta_{23} & \cos^2\theta_{23} \end{pmatrix} \not{Z}' \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \right.$$

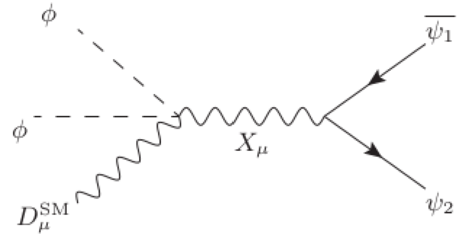
$$\left. -\frac{1}{2} (\overline{e_L}\ \overline{\mu_L}\ \overline{\tau_L}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \not{Z}' \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \right]$$

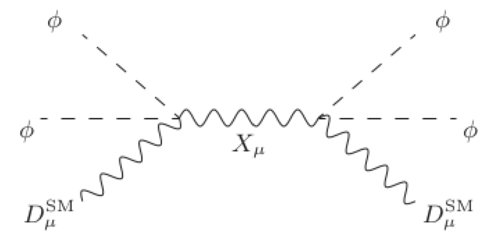
B/EW Observables

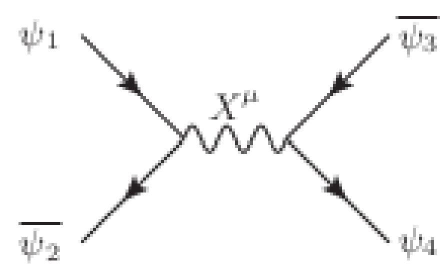
SMEFT$(M_{Z'})$ $\rightarrow$`smelli`$\rightarrow$WET(M_W) $\rightarrow$obs(m_B)

In units of g_X^2/M_X^2:

WC	value	WC	value
C_{ll}^{2222}	$-\frac{1}{8}$	$(\mathbf{C}_{\mathbf{lq}}^{(\mathbf{1})})^{22ij}$	$\frac{1}{12}\Lambda_{\xi\ ij}^{(d_L)}$
$(C_{qq}^{(1)})^{ijkl}$	$\Lambda_{\xi\ ij}^{(d_L)}\Lambda_{\xi\ kl}^{(d_L)}\frac{\delta_{ik}\delta_{jl}-2}{72}$	C_{ee}^{3333}	$-\frac{1}{2}$
C_{uu}^{3333}	$-\frac{2}{9}$	C_{dd}^{3333}	$-\frac{1}{18}$
C_{eu}^{3333}	$\frac{2}{3}$	C_{ed}^{3333}	$-\frac{1}{3}$
$(C_{ud}^{(1)})^{3333}$	$\frac{2}{9}$	C_{le}^{2233}	$-\frac{1}{2}$
C_{lu}^{2233}	$\frac{1}{3}$	C_{ld}^{2233}	$-\frac{1}{6}$
C_{qe}^{ij33}	$\frac{1}{6}\Lambda_{\xi\ ij}^{(d_L)}$	$(C_{qu}^{(1)})^{ij33}$	$-\frac{1}{9}\Lambda_{\xi\ ij}^{(d_L)}$
$(C_{qd}^{(1)})^{ij33}$	$\frac{1}{18}\Lambda_{\xi\ ij}^{(d_L)}$	$(C_{\phi l}^{(1)})^{22}$	$\frac{1}{4}$
$(C_{\phi q}^{(1)})^{ij}$	$-\frac{1}{12}\Lambda_{\xi\ ij}^{(d_L)}$	$C_{\phi e}^{33}$	$\frac{1}{2}$
$C_{\phi u}^{33}$	$-\frac{1}{3}$	$C_{\phi d}^{33}$	$\frac{1}{6}$
$C_{\phi D}$	$-\frac{1}{2}$	$C_{\phi\Box}$	$-\frac{1}{8}$







`smelli` observables

- 167 quarks: P_5', $BR(B_s \to \mu^+\mu^-)$ and others with significant theory errors
- 21 LFU FCNCs: $R_K, R_{K^\star}$, $B \to$di-tau decays
- 31 EWPOs from LEP not assuming lepton flavour universality

Theory uncertainties modelled as multi-variate Gaussians: approximated to be independent of new physics.

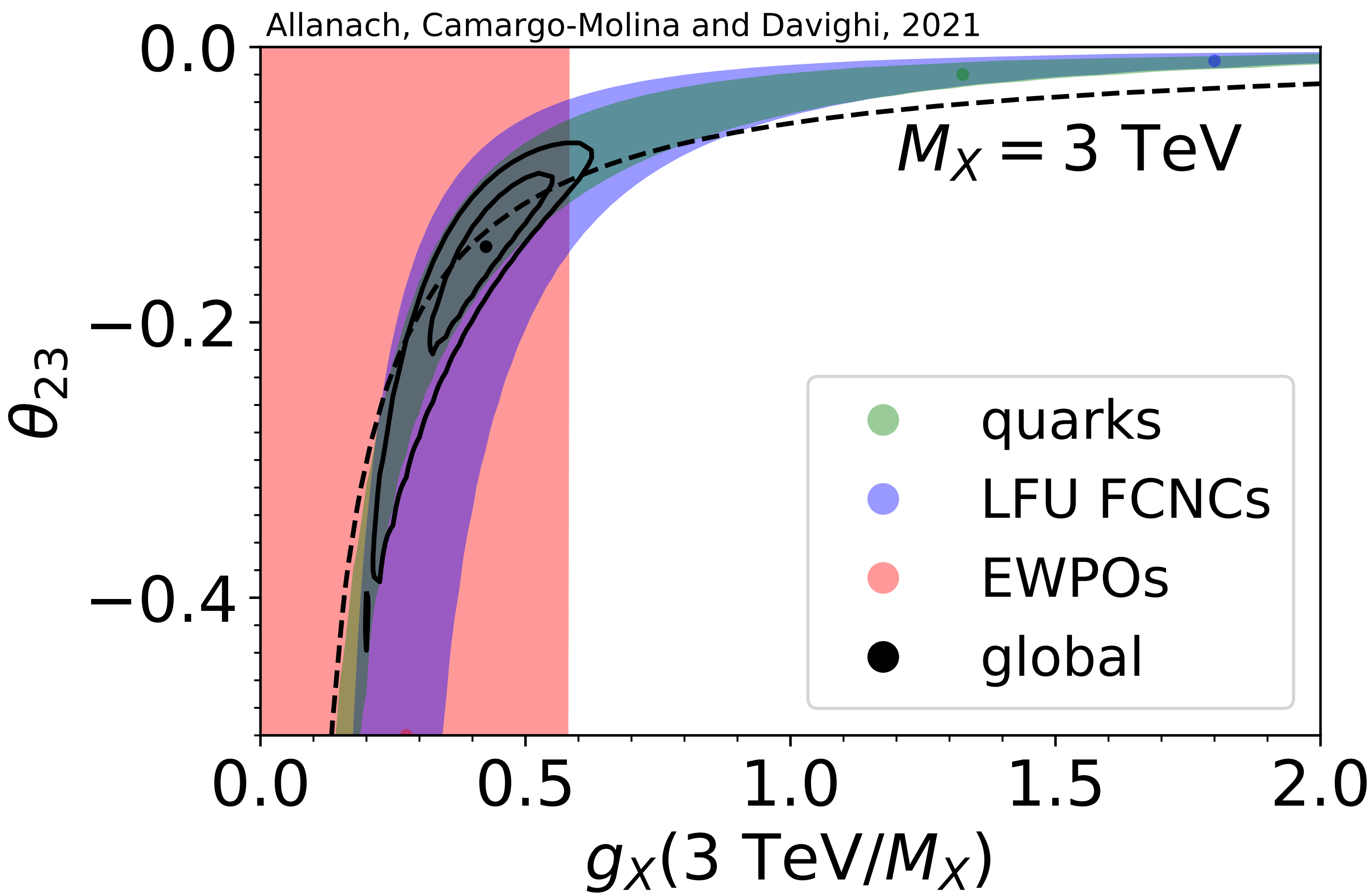
SM:

data set	χ^2	n	$p-$value
quarks	221.6	167	.003
LFU FCNCs	35.3	21	.026
EWPOs	35.7	31	.26
global	292.6	219	.00065

→ 3.4

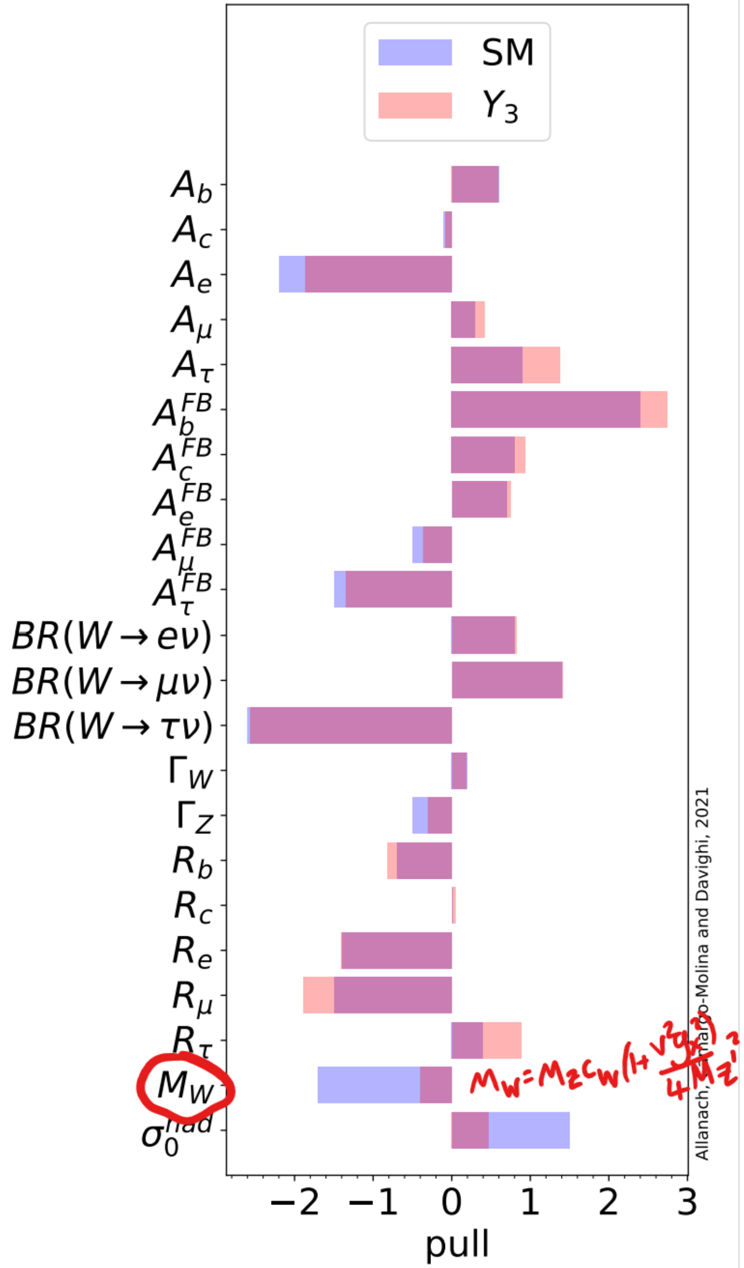
$B_d \to \tau\tau$

Y_3 Fit: 95% CL



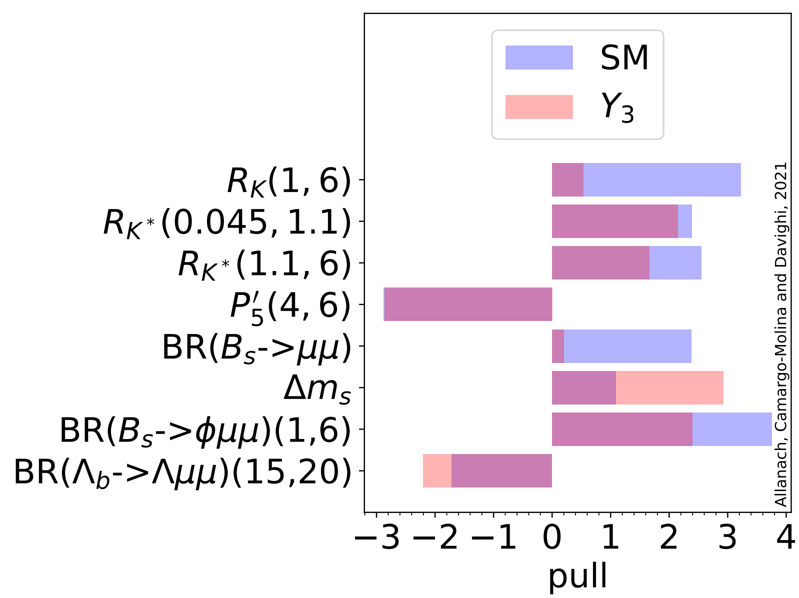
Relies on: `smelli-2.2.0` (Aebischer, Kumar, Stangl, Straub, `1810.07698`), `flavio-2.2.0` (Straub, `1810.08132`), `Wilson` (Aebischer *et al*, `1712.05298`)

Global Fits $M_{Z'} = 3$ TeV



data set	χ^2	n	$p-$value
quarks	221.3	167	.0031
LFU FCNCs	35.3	21	.026
EWPOs	35.7	31	.26
global	292.3	219	.00067

data set	χ^2	n	$p-$value
quarks	192.8	167	.068
LFU FCNCs	21.0	21	.34
EWPOs	36.0	31	.17
global	249.9	219	.062



Other Z' Decay Modes

Mode	BR	Mode	BR	Mode	BR
$t\bar{t}$	0.42	$b\bar{b}$	0.12	$\nu\bar{\nu}'$	0.08
$\mu^+\mu^-$	0.08	$\tau^+\tau^-$	0.30	other $f_i f_j$	$\sim \mathcal{O}(10^{-4})$

LEP LFU

$$g_X^2 \left(\frac{M_Z}{M_{Z'}}\right)^2 \leq 0.004 \Rightarrow g_X \leq \frac{M_{Z'}}{1.3\ \text{TeV}}.$$

It's worth chasing $BR(B \to K^{(*)}\tau^\pm\tau^\mp)$.

$Z' \to \mu\mu$ ATLAS 13 TeV 139 fb^{-1}

ATLAS analysis: look for two track-based isolated μ, $p_T > 30$ GeV. One reconstructed primary vertex. Keep only highest scalar sum p_T pair[4]

$$m^2_{\mu_1\mu_2} = (p_1^\mu + p_2^\mu)\left(p_{1\mu} + p_{2\mu}\right)$$

CMS also have released[5] a 139 fb^{-1} analysis.

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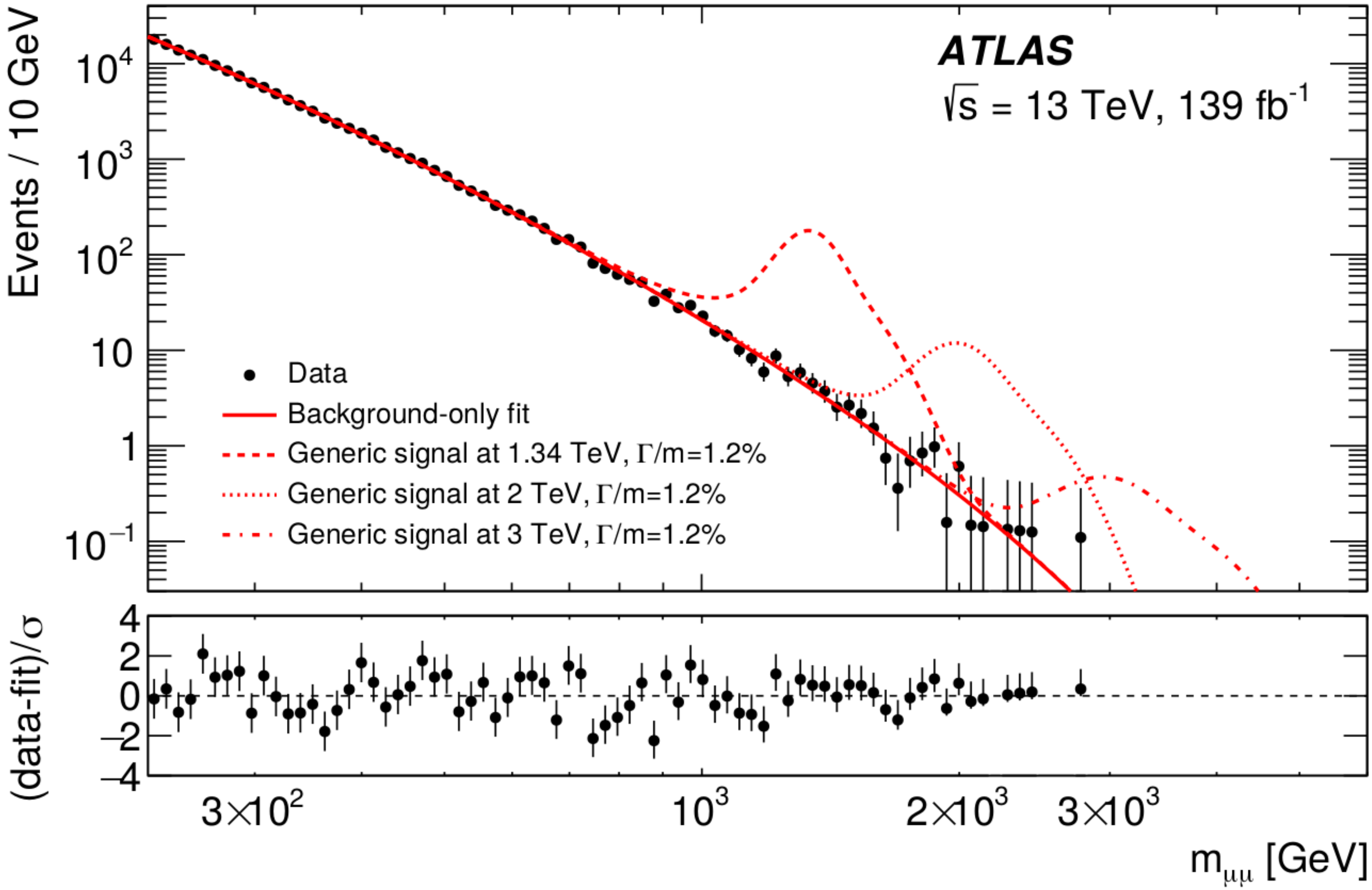
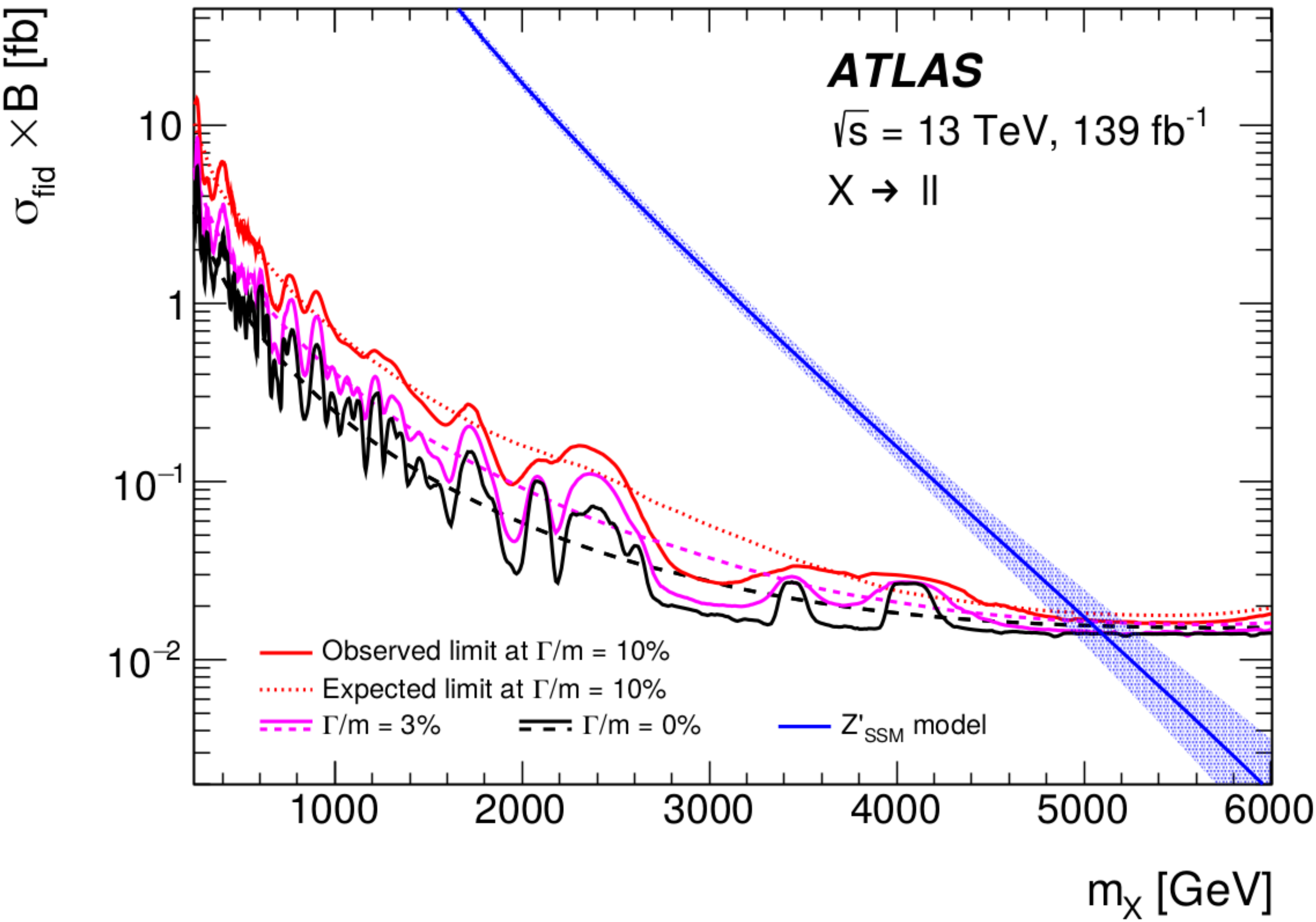
[4] 1903.06248
[5] 2103.02708



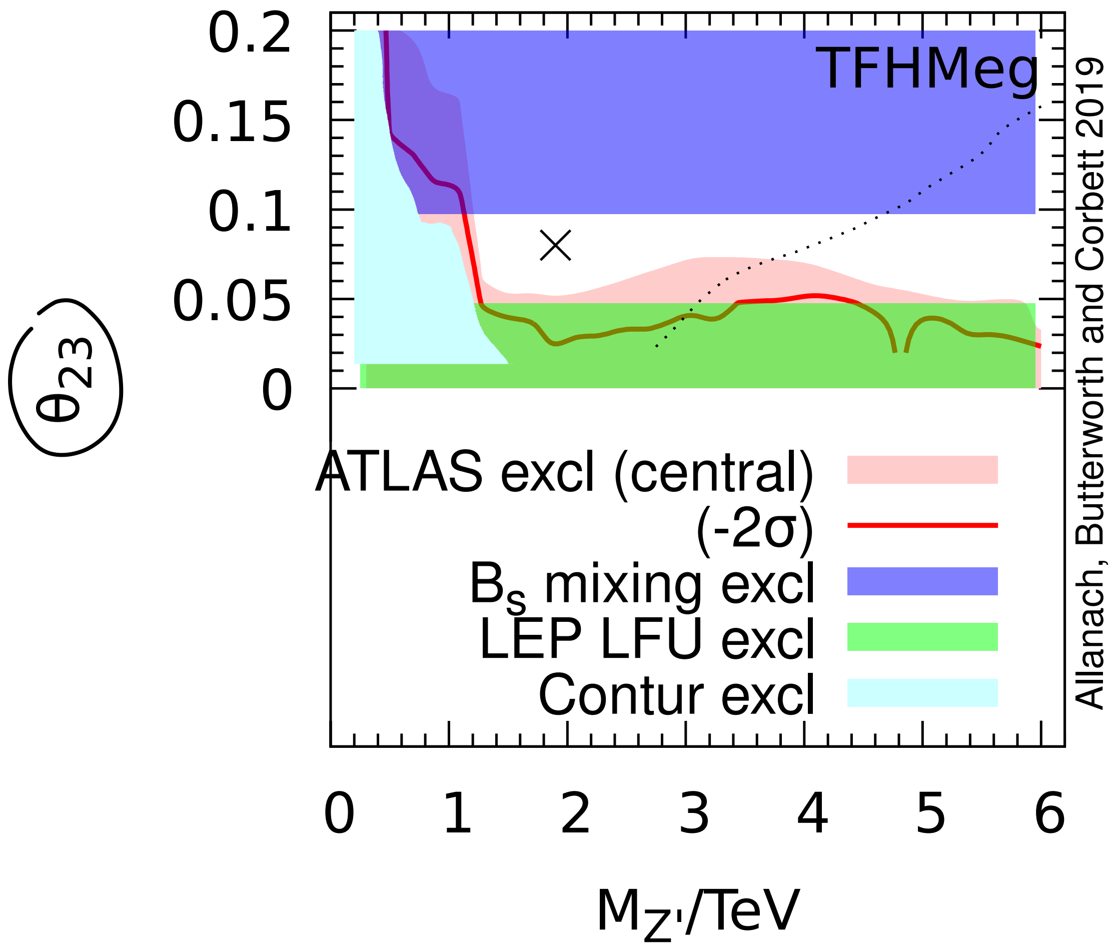
ATLAS
√s = 13 TeV, 139 fb-1
Events / 10 GeV
Data
Background-only fit
Generic signal at 1.34 TeV, Γ/m=1.2%
Generic signal at 2 TeV, Γ/m=1.2%
Generic signal at 3 TeV, Γ/m=1.2%
(data-fit)/σ
3×10^2
10^3
2×10^3
3×10^3
m_μμ [GeV]

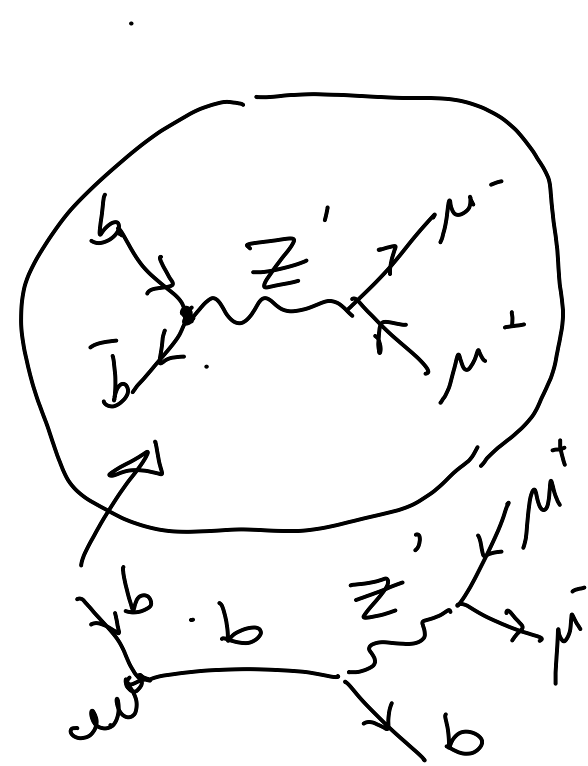
ATLAS l^+l^- limits



1903.06248

$$g_X \propto M_{Z'}/\sqrt{\sin 2\theta_{23}}$$ [6]

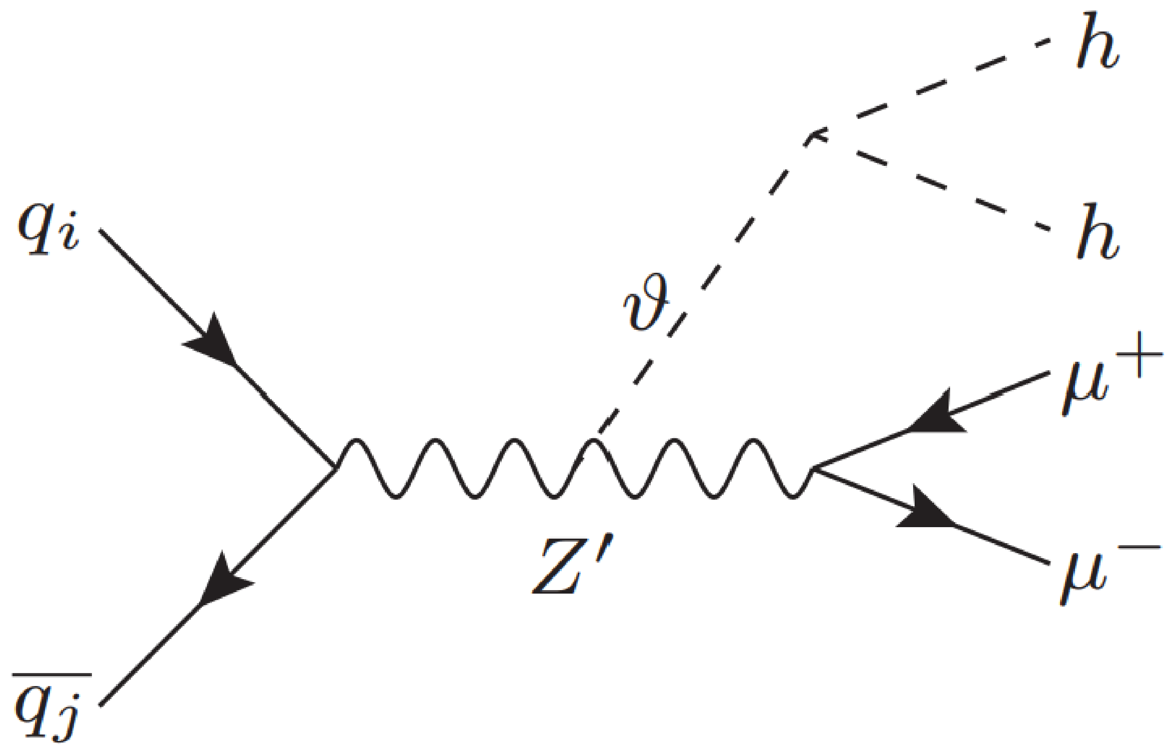




[6]BCA, Butterworth, Corbett, `1904.10954`, *doesn't include 2021 LHCb data*

Flavonstrahlung[7]

Models of this ilk possess $\mathcal{L} = \lambda H H^\dagger \theta\theta^\dagger \Rightarrow$ a *flavonstrahlung* signature:



[7] BCA, 2009.02197

Summary

The Third Family Hypercharge Model is a simple and successful model. Global 2-parameter fits to 217 electroweak and neutral current $B-$anomalies data:

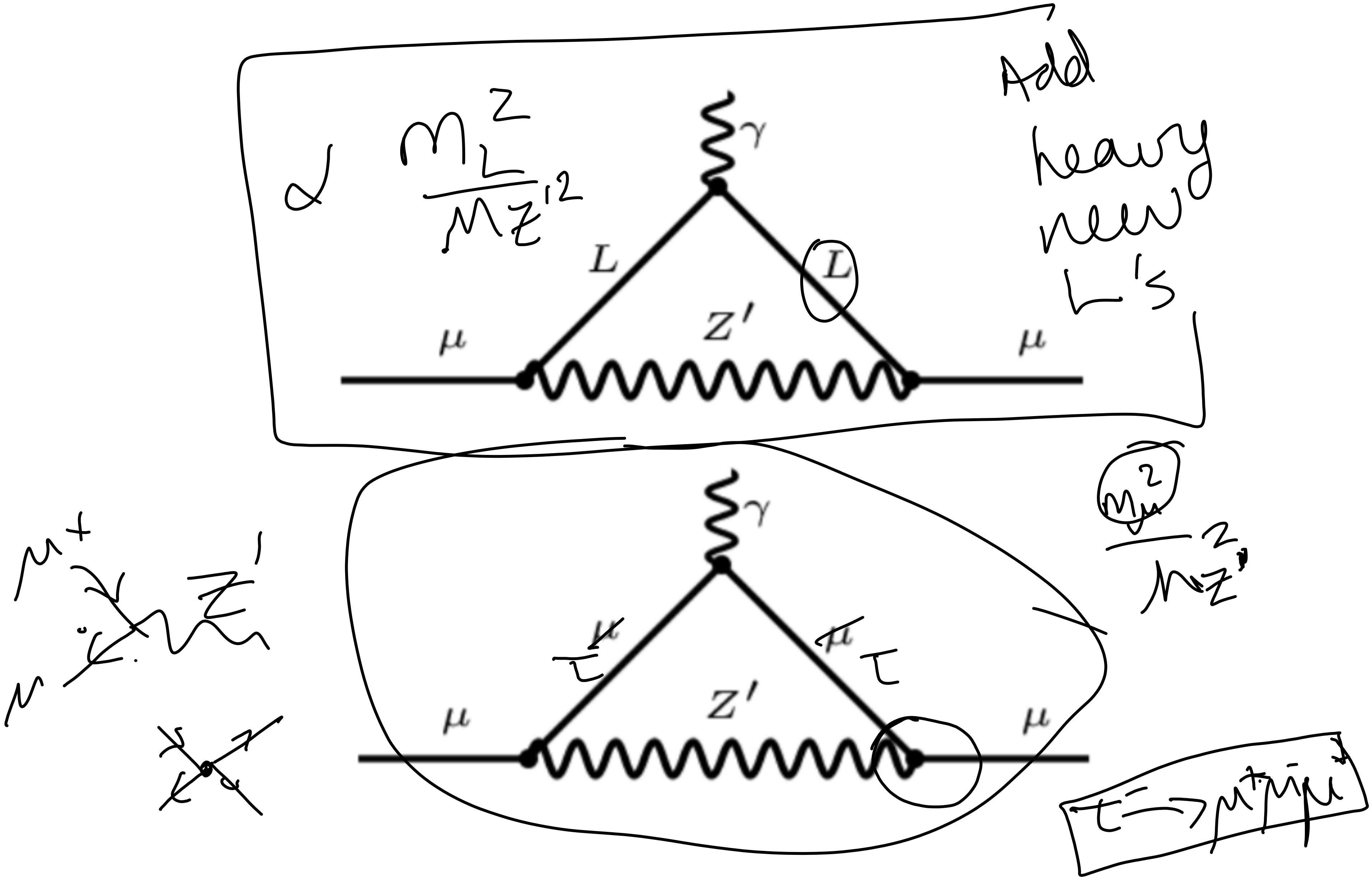
model	$p-$value	$\sqrt{\chi^2_{SM} - \chi^2}$
SM	.00065	0
Y_3	.062	6.5σ

NB perturbativity $\Rightarrow M_{Z'} < 8$ TeV.

The answers to the questions raised by the $B-$anomalies may provide a direct experimental probe into the flavour problem.

Backup

$(g-2)_\mu$



Trident Neutrino Process

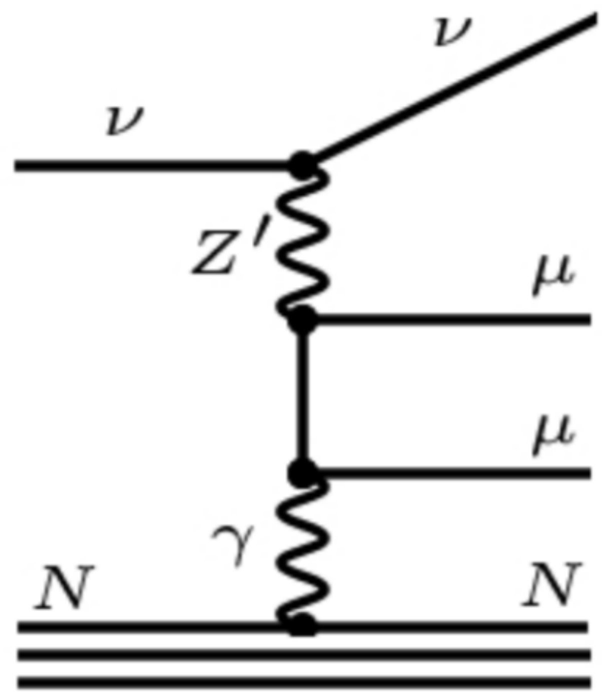
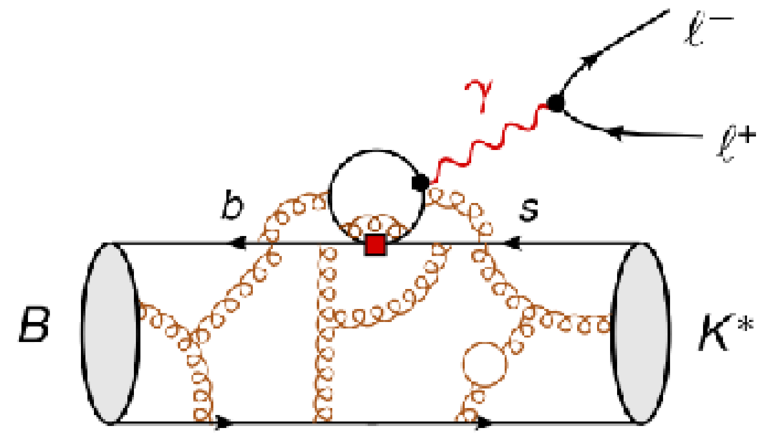


FIG. 10. Neutrino trident process that leads to constraints on the Z^{μ} coupling strength to neutrinos-muons, namely $M_{Z'}/g_{v\mu} \gtrsim 750$ GeV.

Hadronic Uncertainties

- Hadronic effects like charm loop are photon-mediated ⇒ vector-like coupling to leptons just like C_9



- How to disentangle NP ↔ QCD?
  - Hadronic effect can have different q^2 dependence
  - Hadronic effect is lepton flavour universal ($\to R_K$!)

Wilson Coefficients c^l_{ij}

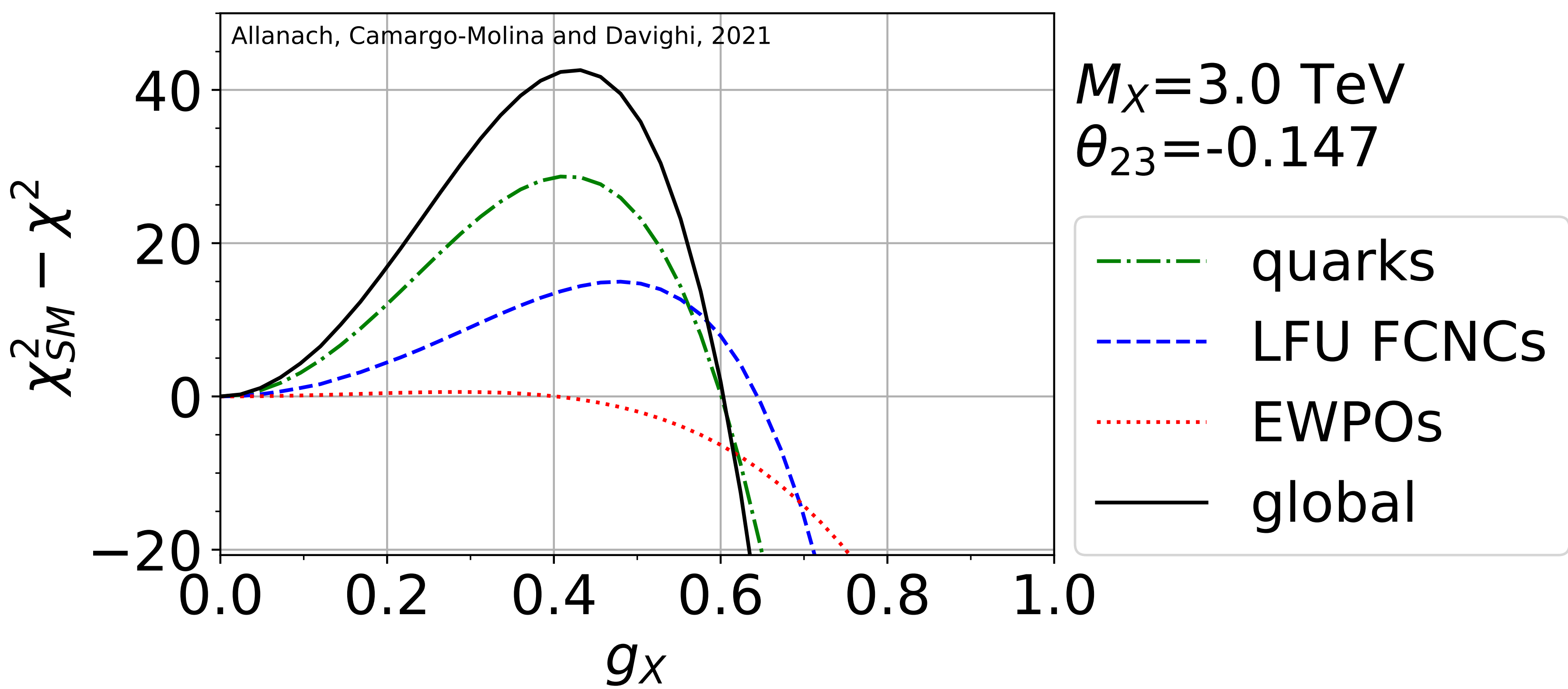
In SM, can form an EFT since $m_B \ll M_W$:

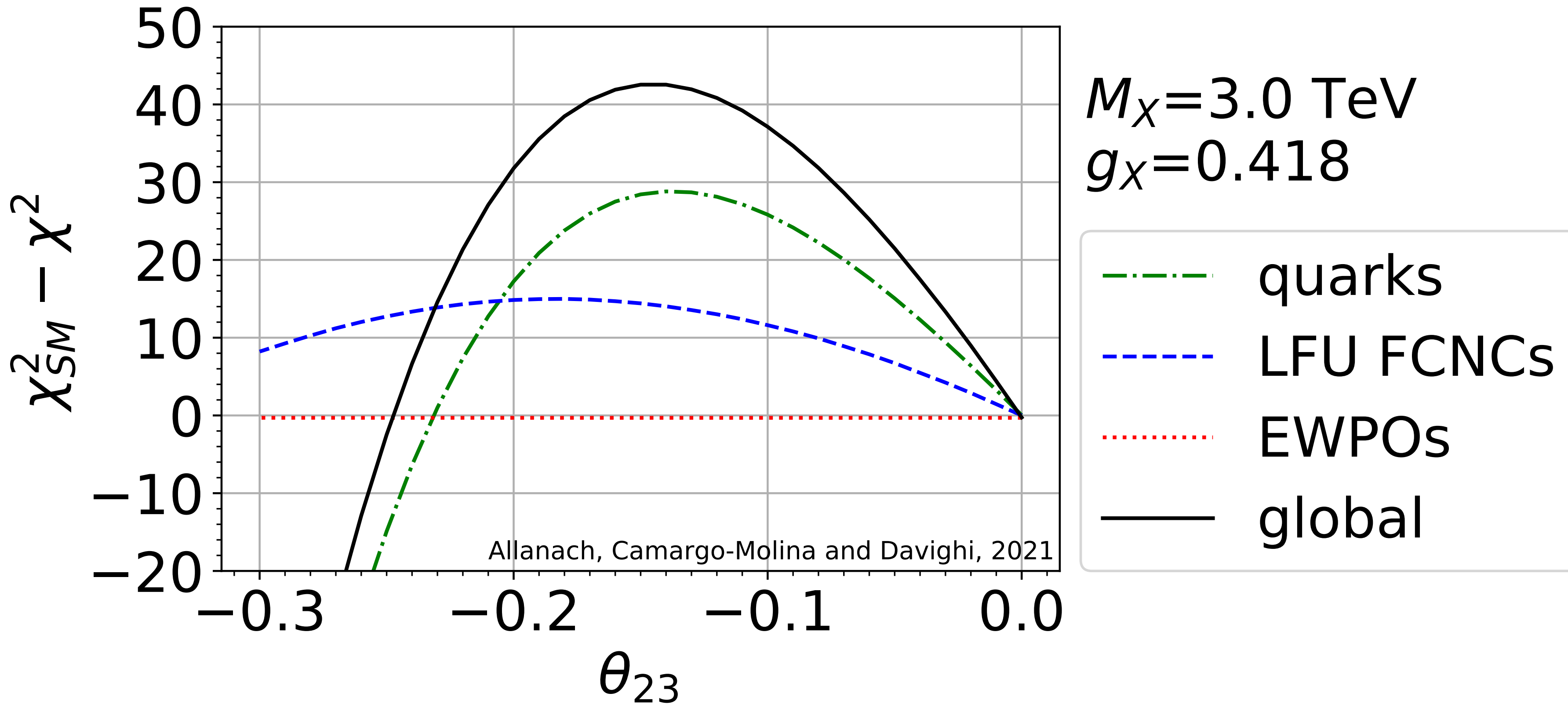
$$\mathcal{L}_{\text{eff}} = \frac{1}{(36\ \text{TeV})^2} c^l_{ij} (\bar{s}\gamma^\mu P_i b)(\bar{l}\gamma_\mu P_j l) \qquad (1)$$

One loop weak interactions give $c^l_{ij} \sim \pm\mathcal{O}(1)$ in SM. $(1/36\ \text{TeV})^2 = V_{tb}V^*_{ts}\alpha/(4\pi v^2)$.

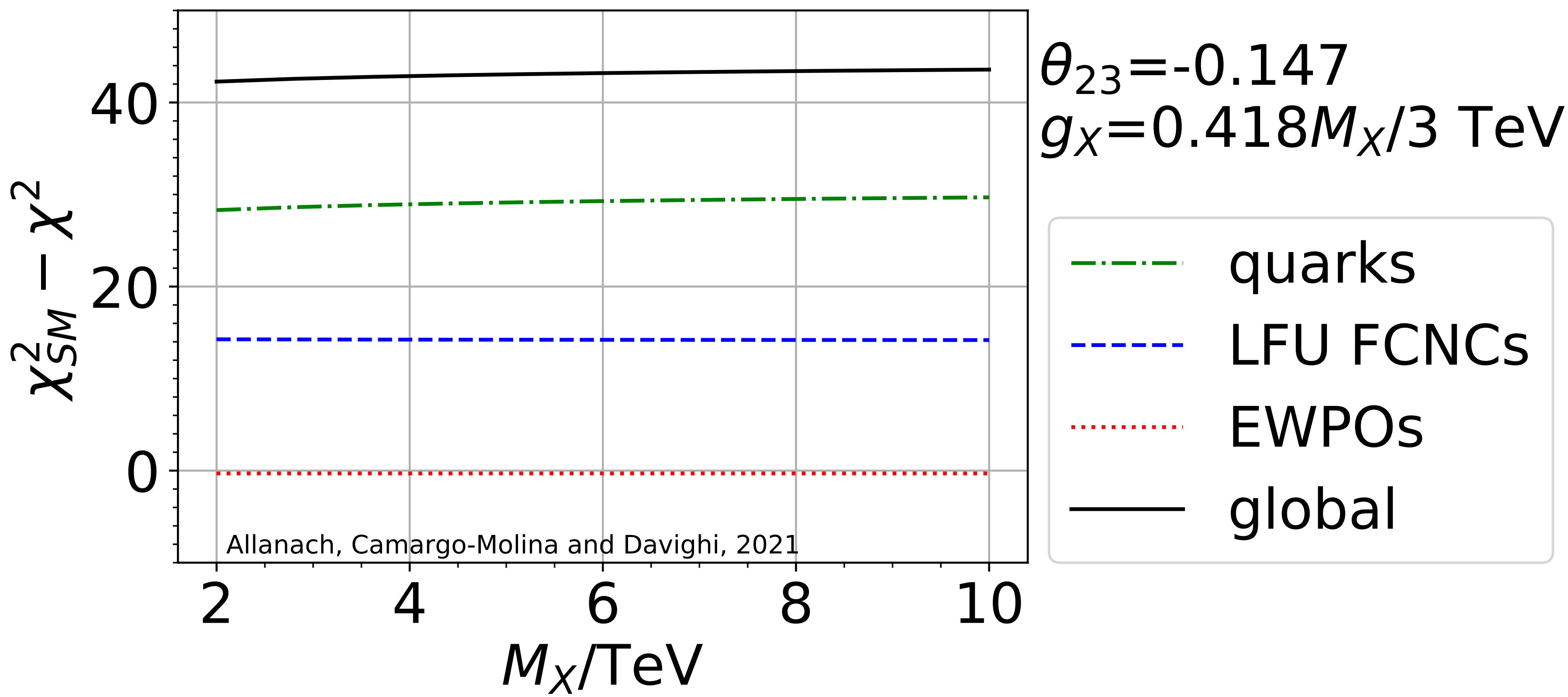
From now on, c^l_{ij} refer to *beyond* SM contribution.

TFHM Near best-fit point





M_X=3.0 TeV
g_X=0.418
quarks
LFU FCNCs
EWPOs
global
Allanach, Camargo-Molina and Davighi, 2021
$\chi^2_{SM} - \chi^2$
θ_{23}
50
40
30
20
10
0
−10
−20
−0.3
−0.2
−0.1
0.0



θ_{23}=-0.147
g_X=0.418M_X/3 TeV
quarks
LFU FCNCs
EWPOs
global
Allanach, Camargo-Molina and Davighi, 2021
$\chi^2_{SM} - \chi^2$
M_X/TeV

Which Ones Work?

Options *for a single BSM operator*:

- c^e_{ij} operators fine for $R_{K^{(*)}}$ but are disfavoured by global fits including other observables.
- c^μ_{LR} disfavoured: predicts *enhancement* in both R_K and R_{K^*}
- c^μ_{RR}, c^μ_{RL} disfavoured: they pull R_K and R_{K^*} in *opposite directions*.
- $c^\mu_{LL} = -1.06$ fits well globally[8].

[8] D'Amico et al, 1704.05438; Aebischer et al 1903.10434.

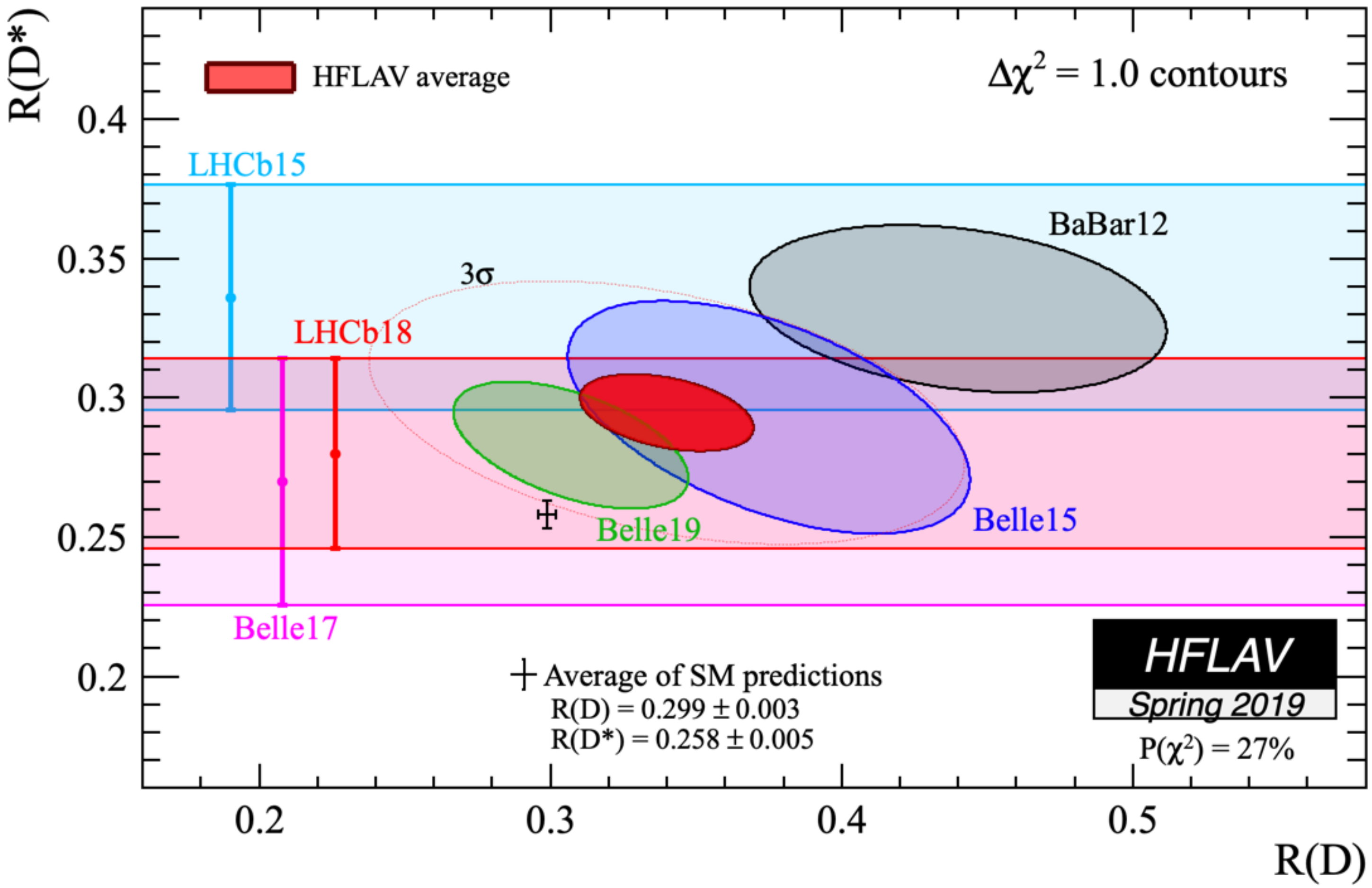
Invisible Width of Z Boson

$\Gamma_{\text{inv}}^{(\text{exp})} = 499.0 \pm 1.5$ MeV, whereas $\Gamma_{\text{inv}}^{(\text{SM})} = 501.44$ MeV.

$$\Rightarrow \Delta\Gamma^{(\text{exp})} = \Gamma_{\text{inv}}^{(\text{exp})} - \Gamma_{\text{inv}}^{(\text{SM})} = -2.5 \pm 1.5 \text{ MeV}.$$

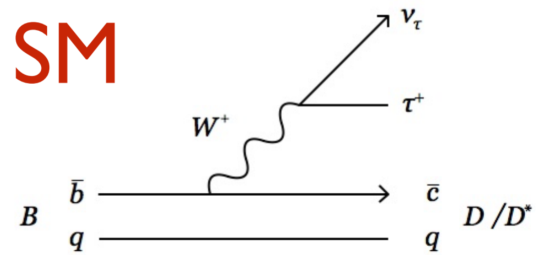
$$\begin{aligned}
\mathcal{L}_{\bar{\nu}\nu Z} &= -\frac{g}{2\cos\theta_w}\overline{\nu'_{Le}}\not{Z}P_L\nu'_{Le} \\
&\quad -\overline{\nu'_{L\mu}}\left(\frac{g}{2\cos\theta_w} + \frac{5}{6}g_F\sin\alpha_z\right)\not{Z}\nu'_{L\mu} \\
&\quad -\overline{\nu'_{L\tau}}\left(\frac{g}{2\cos\theta_w} - \frac{8}{6}g_F\sin\alpha_z\right)\not{Z}\nu'_{L\tau}.
\end{aligned}$$

$$R_{D^{(*)}} = BR(B^- \to D^{(*)}\tau\nu)/BR(B^- \to D^{(*)}\mu\nu)$$



$R_{D^{(*)}}$: BSM Explanation

... has to compete with



$$\mathcal{L}_{eff} = -\frac{2}{\Lambda^2}\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\tau}_L\gamma_\mu\nu_{\tau L}\right) + H.c.$$

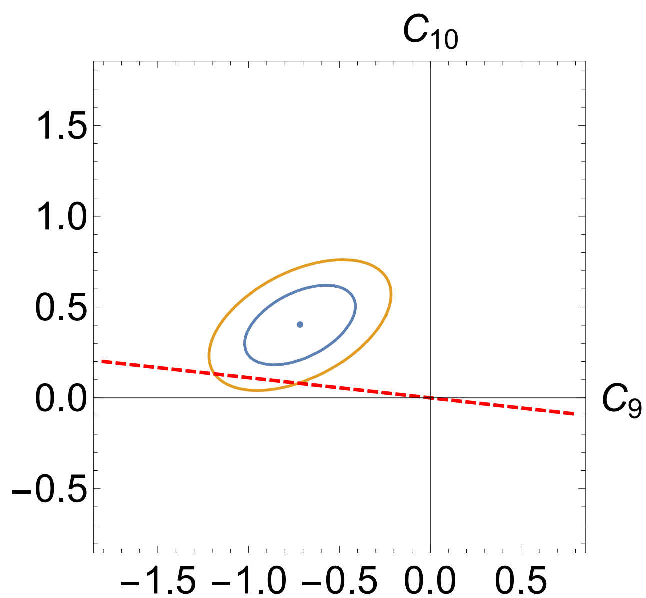
$$\Lambda = 3.4\ \mathrm{TeV}$$

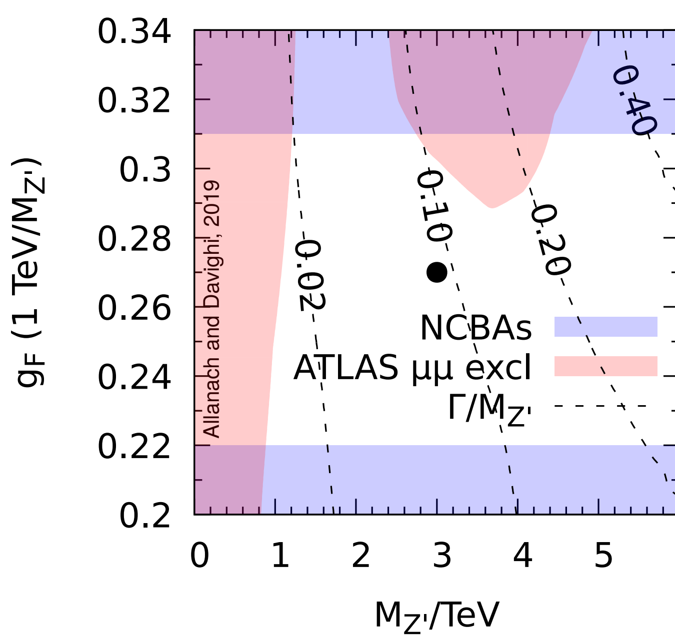
A factor 10 lower than required for $R_{K^{(*)}} \Rightarrow$ different explanation?

Deformed TFHM

$F_{Q'_i} = 0$	$F_{u_{R'_i}} = 0$	$F_{d_{R'_i}} = 0$	$F_H = -1/2$
$F_{e_{R'_1}} = 0$	$F_{e_{R'_2}} = 2/3$	$F_{e_{R'_3}} = -5/3$	
$F_{L'_1} = 0$	$F_{L'_2} = 5/6$	$F_{L'_3} = -4/3$	
$F_{Q'_3} = 1/6$	$F_{u'_{R3}} = 2/3$	$F_{d'_{R3}} = -1/3$	$F_\theta \neq 0$

$$\mathcal{L} = Y_t \overline{Q_3'}_L H t'_R + Y_b \overline{Q'_{3L}} H^c b'_R + H.c.,$$





Neutrino Masses

At dimension 5:

$$\mathcal{L}_{SS} = \frac{1}{2M}({L'_3}^T H^c)(L'_3 H^c),$$

but if we add RH neutrinos, then integrate them out

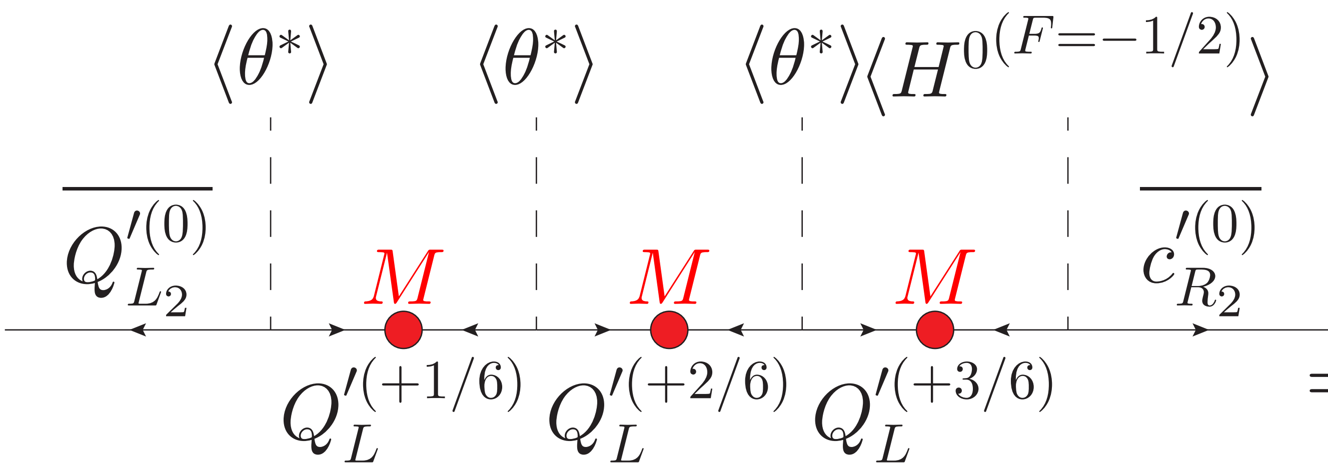
$$\mathcal{L}_{SS} = 1/2 \sum_{ij} (L'_i H^c)(M^{-1})_{ij}(L'_j H^c),$$

where now $(M^{-1})_{ij}$ may well have a non-trivial structure. If $(M^{-1})_{ij}$ are of same order, large PMNS mixing results.

Froggatt Neilsen Mechanism[9]

A means of generating the non-renormalisable Yukawa terms, e.g. $X_\theta = 1/6$:

$$Y_c \overline{Q'_{L2}}^{(F=0)} H^{(F=-1/2)} {c'_R}^{(F=0)} \sim \mathcal{O}\left[\left(\frac{\langle\theta\rangle}{M}\right)^3 \overline{Q'_{L2}} H c'_R\right]$$



eg $\left(\frac{\langle\theta\rangle}{M}\right) \sim 0.2$

$\Rightarrow Y_c/Y_t \sim 1/100$

[9] C Froggatt and H Neilsen, NPB**147** (1979) 277