Superradiance and quantum states on black hole space-times

Elizabeth Winstanley

Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020)
Balakumar, Bernar & EW to appear

Consortium for Fundamental Physics
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QFT on curved space-time

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- Classical background space-time
- Quantum field on this background

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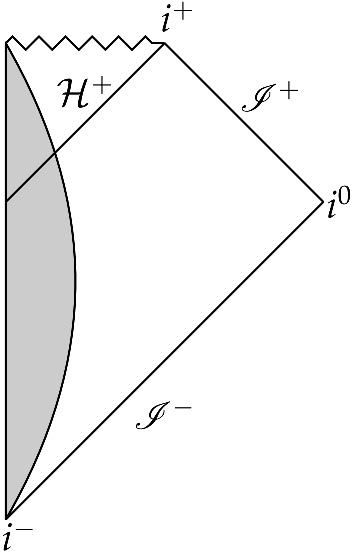
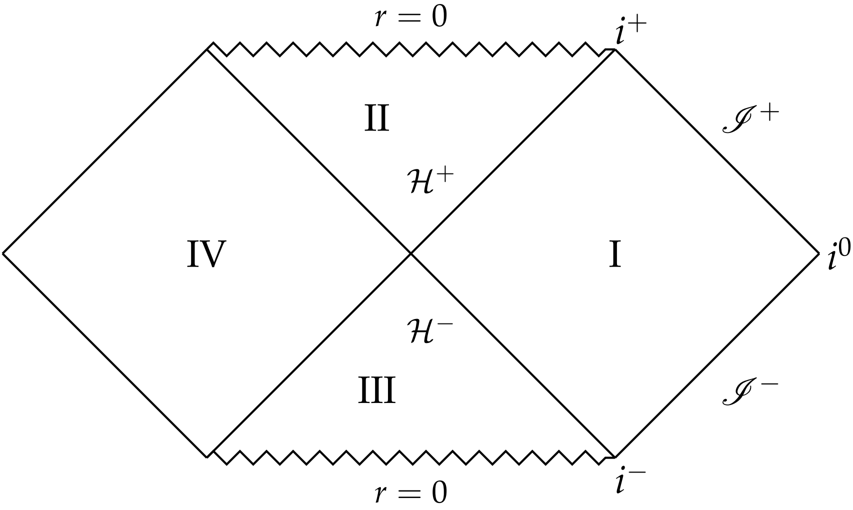
Hawking radiation

- Black hole formed by gravitational collapse
- Thermal flux at $\mathscr{I}^+$

$$T_H = \frac{\kappa}{2\pi}$$

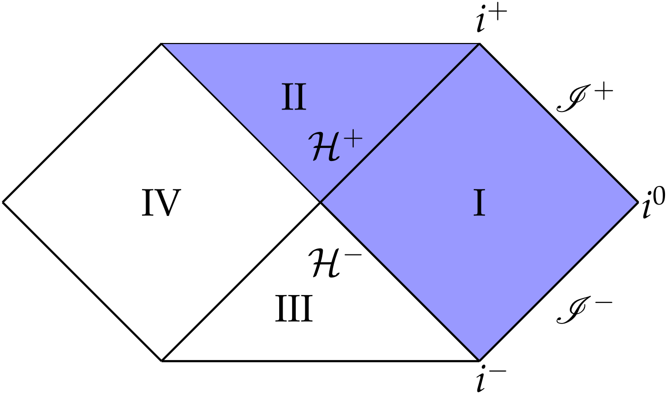


Schwarzschild black hole



Unruh state $|U\rangle$

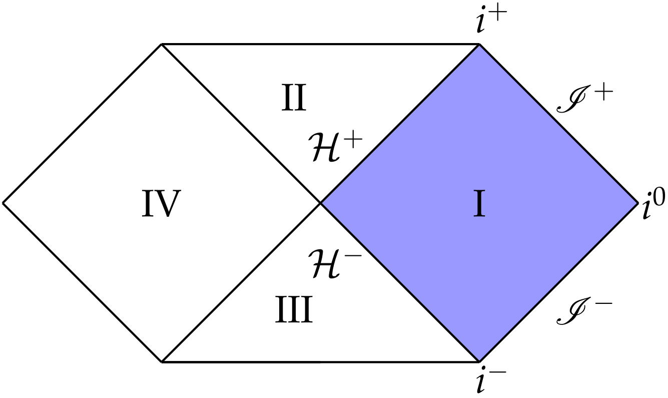
- Hawking flux at $\mathscr{I}^+$
- Regular at $\mathcal{H}^+$



[Unruh *PRD* **14** 870 (1976)]

Boulware state $|B\rangle$

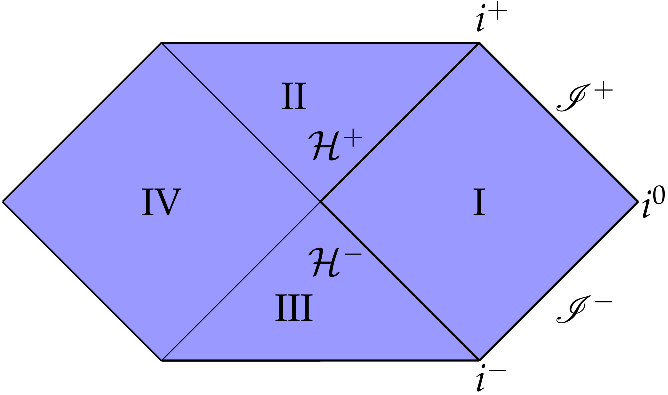
- State which is as empty as possible at infinity
- Diverges on the event horizon



[Boulware *PRD* **11** 1404 (1975)]

Hartle-Hawking state $|H\rangle$

- Represents a black hole in thermal equilibrium with a heat bath
- Regular on and outside event horizon



[Hartle & Hawking *PRD* **13** 2188 (1976), Israel *PLA* **57** 107 (1976)]

Canonical quantization of a neutral scalar field Φ

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Expand classical field in terms of orthonormal basis of field modes

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Positive frequency modes

$$\phi_j^+ \propto e^{-i\omega(t\pm x)} \qquad \omega > 0$$

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Promote expansion coefficients to operators $\hat{a}_j$, $\hat{a}_j^\dagger$ with

$$\left[\hat{a}_j, \hat{a}_k^\dagger\right] = \delta_{jk} \qquad \left[\hat{a}_j, \hat{a}_k\right] = 0 \qquad \left[\hat{a}_j^\dagger, \hat{a}_k^\dagger\right] = 0$$

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$\hat{a}_j$ - particle annihilation operators

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Define the vacuum state $|0\rangle$

$$\hat{a}_j |0\rangle = 0$$

Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\varphi^2$$

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Neutral scalar field modes

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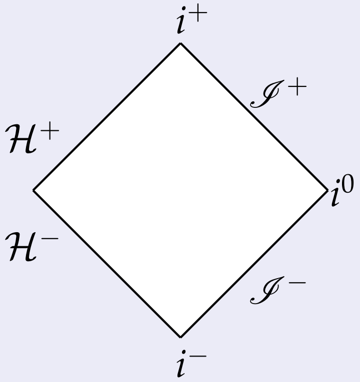
$$V_{\omega\ell}(r) \to \omega^2 \qquad r_* \to \pm\infty$$

“In” and “Up” modes

"In" and "Up" modes

"In" modes $R^{\text{in}}_{\omega\ell}$

$$\begin{cases} & r_* \to -\infty \\ & r_* \to \infty \end{cases}$$



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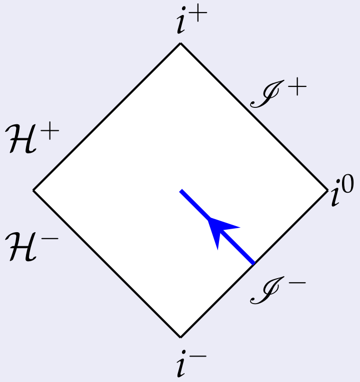
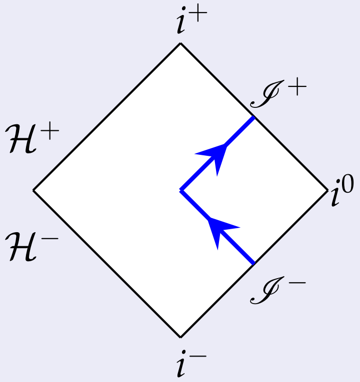


"In" and "Up" modes

"In" modes $R^{\rm in}_{\omega\ell}$

$$\begin{cases} & r_* \to -\infty \\ e^{-i\omega r_*} + A^{\rm in}_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \end{cases}$$



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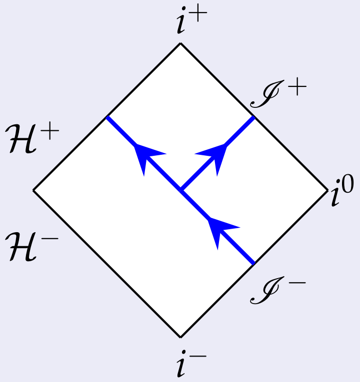

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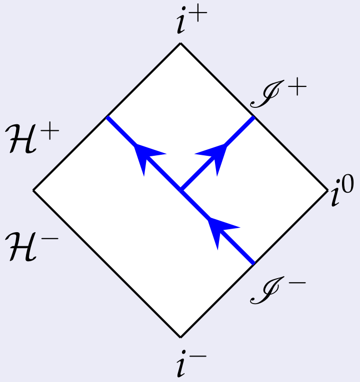
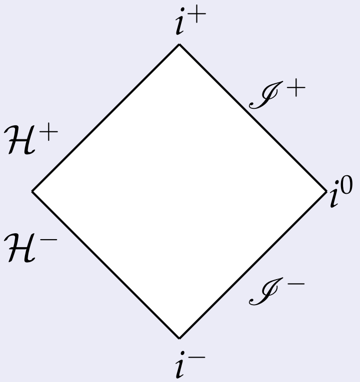


"Up" modes $R_{\omega\ell}^{\text{up}}$

$$\begin{cases} & r_* \to -\infty \\ & r_* \to \infty \end{cases}$$



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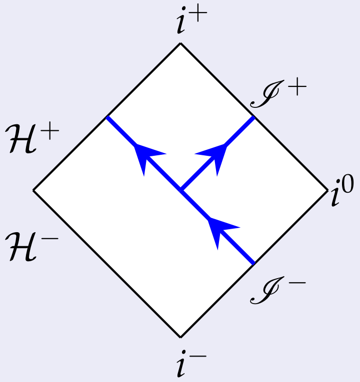

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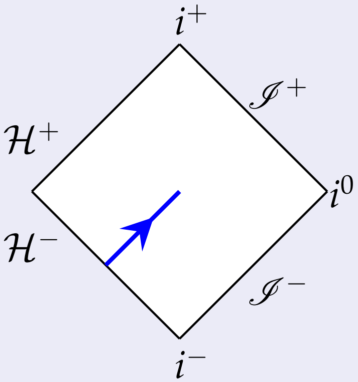

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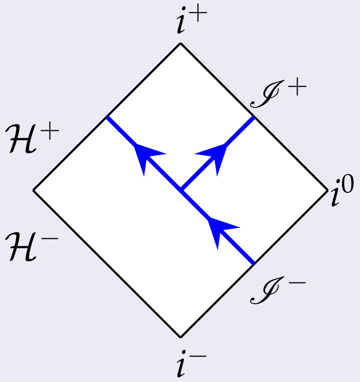

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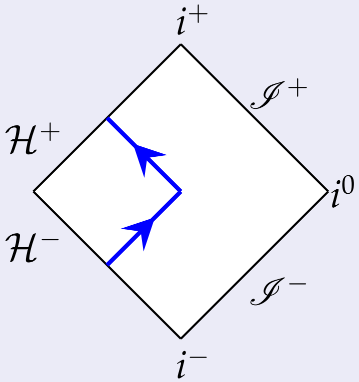

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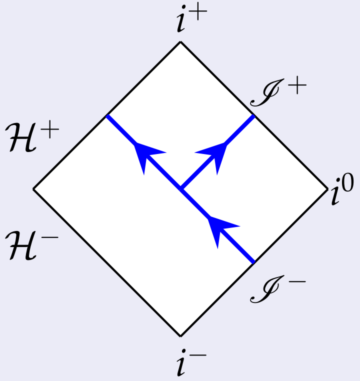

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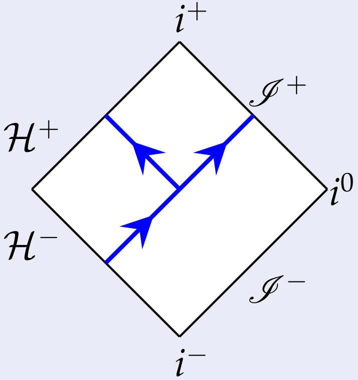

Boulware and Hartle-Hawking states

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Boulware state $|B\rangle$

Positive frequency with respect to Schwarzschild time

[Boulware *PRD* **11** 1404 (1975)]

Boulware and Hartle-Hawking states

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Unrenormalized expectation values

$$\langle \mathrm{B}|\hat{\mathrm{O}}|\mathrm{B}\rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^\infty d\omega \left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m} \right]$$

[Candelas *PRD* **21** 2185 (1980)]

Boulware and Hartle-Hawking states

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Hartle-Hawking state $|\mathrm{H}\rangle$

Positive frequency with respect to Kruskal time

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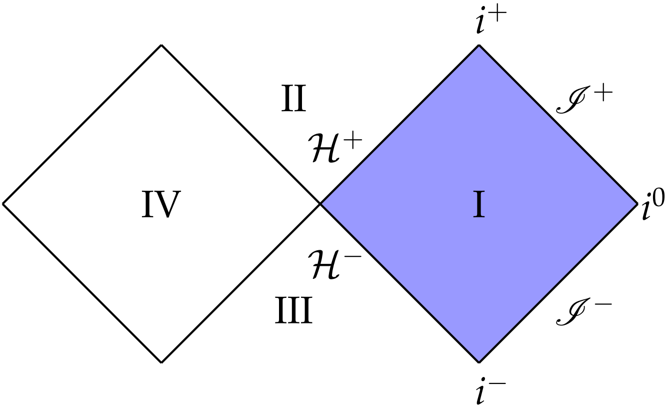
$$\langle \mathrm{H}|\hat{\mathrm{O}}|\mathrm{H}\rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^\infty d\omega \coth\left(\frac{\omega}{2T_H} \right) \left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m} \right]$$

[Candelas *PRD* **21** 2185 (1980)]

Properties of $|\mathrm{H}\rangle$ on Schwarzschild

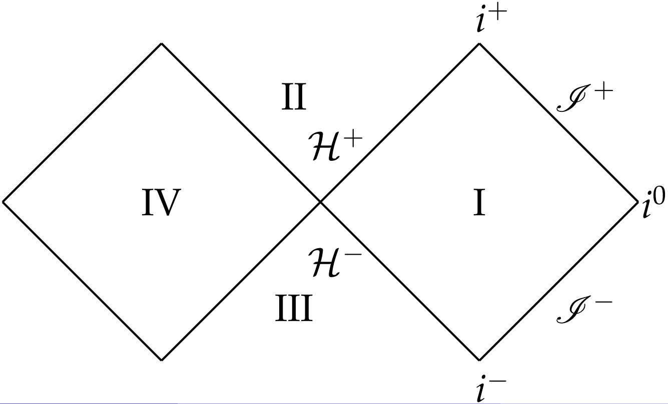
- Regular on and outside horizon
- Time-reversal symmetric
- Thermal state in region I



Kerr space-time

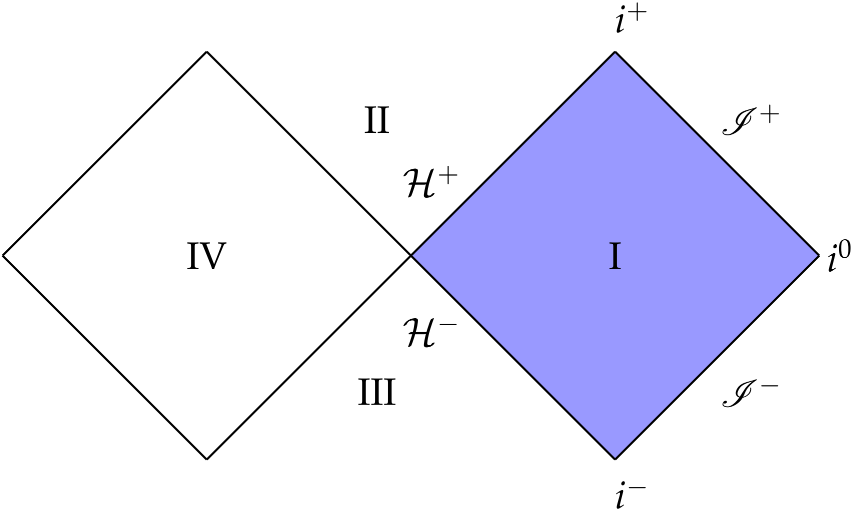
$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - a\sin^2\theta\, d\varphi\right]^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left[\left(r^2+a^2\right)d\varphi - a\,dt\right]^2$$

$$\Delta = r^2 - 2Mr + a^2 \qquad \Sigma = r^2 + a^2\cos^2\theta$$



The Kay-Wald theorem [Kay & Wald *Phys. Rept.* **207** 49 (1991)]

No Hartle-Hawking state exists for a quantum scalar field on Kerr



Massless scalar field on Kerr space-time

Scalar field modes

$$\phi_{\omega\ell m}(t,r,\theta,\varphi) = \frac{1}{\mathcal{N}} \frac{1}{(r^2+a^2)^{\frac{1}{2}}} e^{-i\omega t} e^{im\varphi} S_{\omega\ell m}(\cos\theta) R_{\omega\ell m}(r)$$

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$S_{\omega\ell m}(\cos\theta)$: spheroidal harmonics

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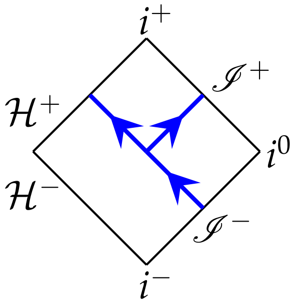
$$V_{\omega\ell m}(r) = \begin{cases} \widetilde{\omega}^2 = (\omega - m\Omega_H)^2 & \text{as } r_* \to -\infty \\ \omega^2 & \text{as } r_* \to \infty \end{cases}$$

Classical superradiance

$$R^{\text{in}}_{\omega\ell m}(r) = \begin{cases} B^{\text{in}}_{\omega\ell m} e^{-i\widetilde{\omega} r_*} & r_* \to -\infty \\ e^{-i\omega r_*} + A^{\text{in}}_{\omega\ell m} e^{i\omega r_*} & r_* \to \infty \end{cases}$$



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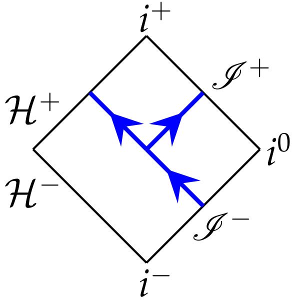

$$\omega \left[1 - \left| A^{\text{in}}_{\omega\ell m} \right|^2 \right] = \widetilde{\omega} \left| B^{\text{in}}_{\omega\ell m} \right|^2$$

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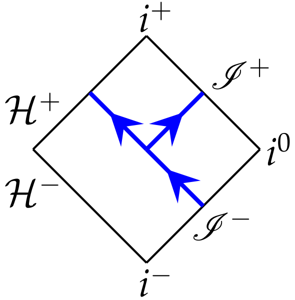

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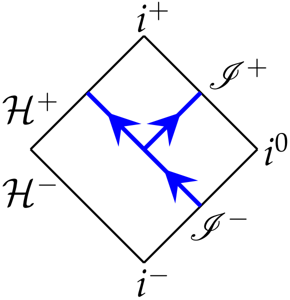
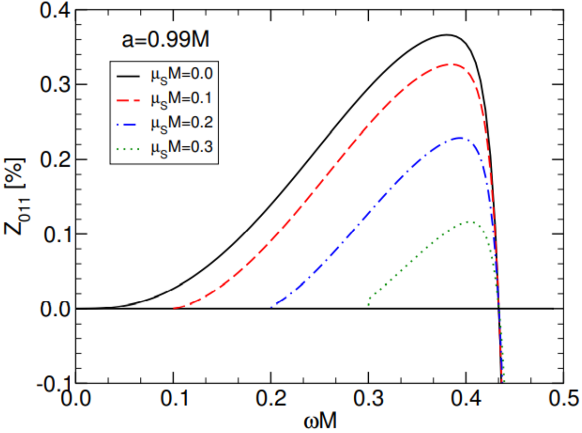
$$\left| A^{\text{in}}_{\omega\ell m} \right|^2 > 1 \text{ if } \omega\widetilde{\omega} < 0$$

Classical superradiance

$$R^{\rm in}_{\omega\ell m}(r) = \begin{cases} B^{\rm in}_{\omega\ell m} e^{-i\tilde{\omega} r_*} & r_* \to -\infty \\ e^{-i\omega r_*} + A^{\rm in}_{\omega\ell m} e^{i\omega r_*} & r_* \to \infty \end{cases}$$





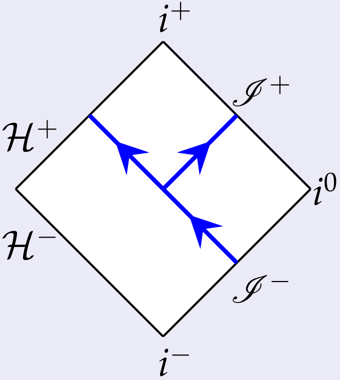
$$\left| A^{\rm in}_{\omega\ell m} \right|^2 > 1 \text{ if } \omega\tilde{\omega} < 0$$

[Brito, Cardoso & Pani *LNP* **905** (2015)]

“In” and “Up” modes

“In” and “Up” modes

“In” modes



“In” and “Up” modes

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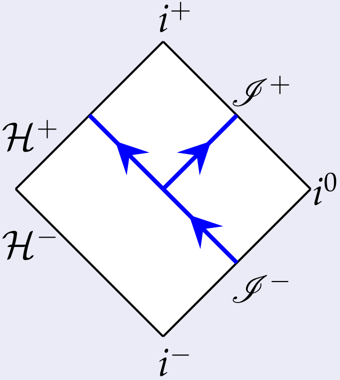
Positive frequency for

$$\omega > 0$$



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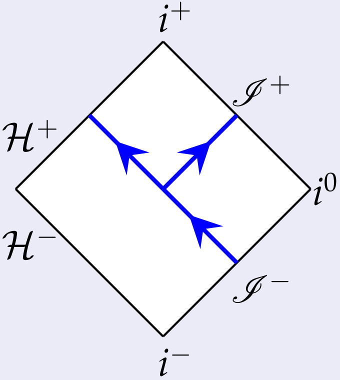

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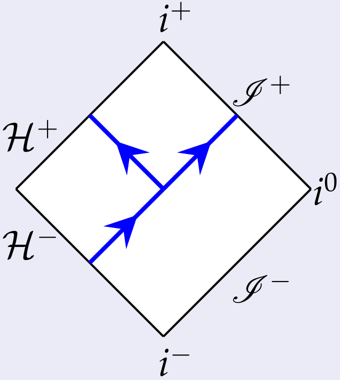

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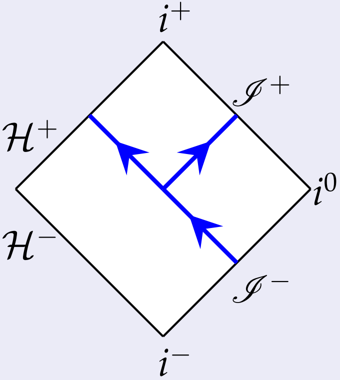
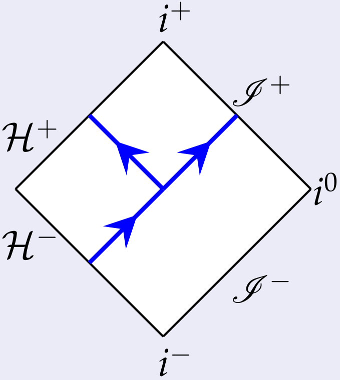


"Up" modes

Positive frequency for

$$\tilde{\omega} = \omega - m\Omega_H > 0$$



A Hartle-Hawking state for Kerr?

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$|\mathrm{CCH}\rangle$ [Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981)]

$$\langle \mathrm{CCH}|\hat{O}|\mathrm{CCH}\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_0^\infty d\omega \coth\left(\frac{\omega}{2T_H}\right) o^{\mathrm{in}}_{\omega\ell m} + \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_0^\infty d\widetilde{\omega} \coth\left(\frac{\widetilde{\omega}}{2T_H}\right) o^{\mathrm{up}}_{\omega\ell m}$$

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- Regular outside the event horizon

[Ottewill & Winstanley *PRD* **62** 084018 (2000)]

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- Regular outside the event horizon
- Does not represent an equilibrium state

[Ottewill & Winstanley *PRD* **62** 084018 (2000)]

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- Potentially an equilibrium state
- Divergent everywhere except on the axis of rotation

[Ottewill & Winstanley *PRD* **62** 084018 (2000)]

Charged scalar field on RN space-time

Charged scalar field on RN space-time

Reissner-Nordström black hole

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

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Charged scalar field equation

$$D_\mu D^\mu \Phi = 0 \qquad D_\mu = \nabla_\mu - iqA_\mu \qquad A_0 = -\frac{Q}{r}$$

Charged scalar field on RN space-time

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Charged scalar field modes

$$\phi_{\omega\ell m}(t,r,\theta,\varphi) = \frac{1}{\mathcal{N}r} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

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$$0 = \left[\frac{d^2}{dr_*^2} + V_{\omega\ell}(r) \right] R_{\omega\ell}(r) \qquad \frac{dr_*}{dr} = \frac{1}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}$$

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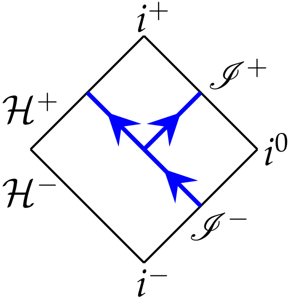
$$V_{\omega\ell}(r) = \begin{cases} \widetilde{\omega}^2 = \left(\omega - \frac{qQ}{r_+} \right)^2 & \text{as } r_* \to -\infty \\ \omega^2 & \text{as } r_* \to \infty \end{cases}$$

Charge superradiance

$$R^{\text{in}}_{\omega\ell}(r) = \begin{cases} B^{\text{in}}_{\omega\ell} e^{-i\widetilde{\omega} r_*} & r_* \to -\infty \\ e^{-i\omega r_*} + A^{\text{in}}_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \end{cases}$$



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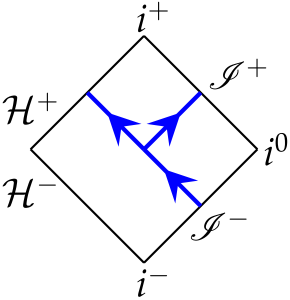

$$\omega \left[1 - \left| A^{\text{in}}_{\omega\ell} \right|^2 \right] = \widetilde{\omega} \left| B^{\text{in}}_{\omega\ell} \right|^2$$

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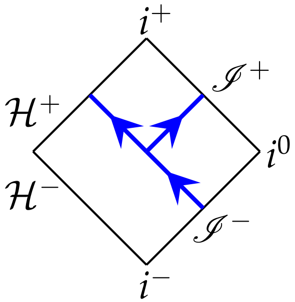

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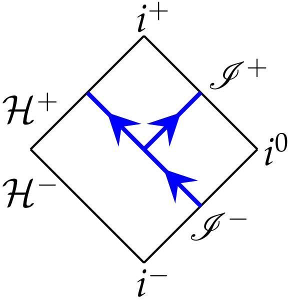

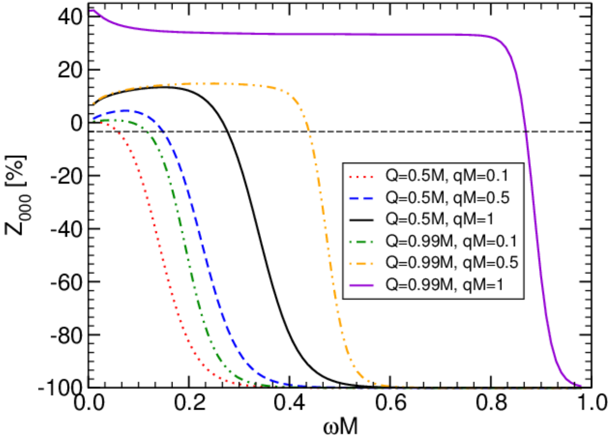

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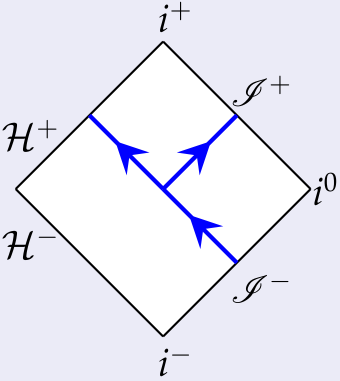
“In” and “Up” modes

"In" and "Up" modes

"In" modes



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Positive frequency for

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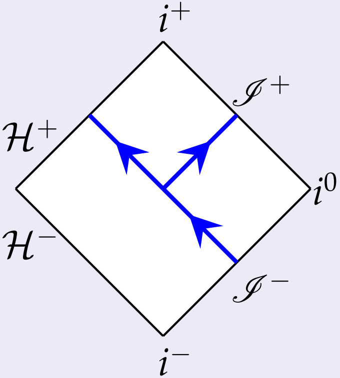

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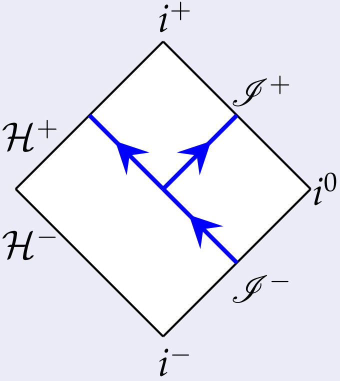
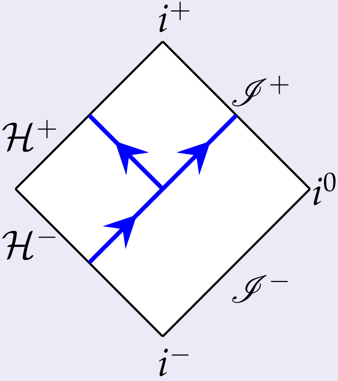


“Up” modes



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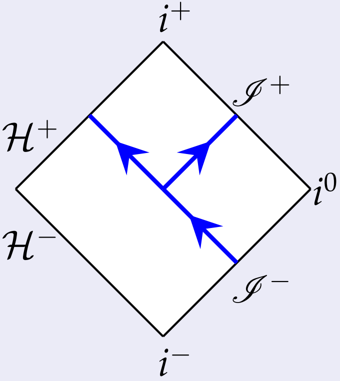
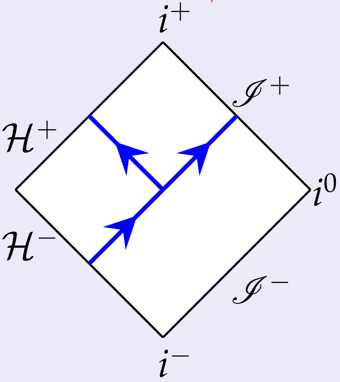


"Up" modes

Positive frequency for

$$\tilde{\omega} = \omega - \frac{qQ}{r_+} > 0$$



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Fluxes

$$\langle \hat{J}^{r} \rangle = -\frac{\mathcal{K}}{r^2} \qquad \langle \hat{T}^{r}_{t} \rangle = -\frac{\mathcal{L}}{r^2} + \frac{4\pi Q\mathcal{K}}{r^3}$$

- Flux of charge $\mathcal{K}$
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Unruh state $|\mathrm{U}\rangle$

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“Boulware”-like state on RN

[Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020)]

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- Positive frequency with respect to Schwarzschild time t

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[Bernar, Balakumar & EW]

"Boulware"-like state on RN

$$\langle \mathrm{B}|\hat{O}|\mathrm{B}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{ \int_{-\infty}^{\infty} d\omega \left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m}\right] - 2\int_{\min\{0,\frac{qQ}{r_+}\}}^{\max\{0,\frac{qQ}{r_+}\}} d\omega \, o^{\mathrm{up}}_{\omega\ell m} \right\}$$

[Bernar, Balakumar & EW]

“Boulware”-like state on RN

$$\langle \mathrm{B}|\hat{\mathrm{O}}|\mathrm{B}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{\int_{-\infty}^{\infty} d\omega\,\left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m}\right] - 2\int_{\min\{0,\frac{qQ}{r_+}\}}^{\max\{0,\frac{qQ}{r_+}\}} d\omega\, o^{\mathrm{up}}_{\omega\ell m}\right\}$$

Properties of the state $|\mathrm{B}\rangle$

[Bernar, Balakumar & EW]

"Boulware"-like state on RN

$$\langle \mathrm{B}|\hat{\mathrm{O}}|\mathrm{B}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{\int_{-\infty}^{\infty} d\omega\,\left[o_{\omega\ell m}^{\mathrm{in}} + o_{\omega\ell m}^{\mathrm{up}}\right] - 2\int_{\min\{0,\frac{qQ}{r_+}\}}^{\max\{0,\frac{qQ}{r_+}\}} d\omega\, o_{\omega\ell m}^{\mathrm{up}}\right\}$$

Properties of the state $|\mathrm{B}\rangle$

- Equilibrium state

$$\mathcal{K}_{\mathrm{B}} = 0 \qquad \mathcal{L}_{\mathrm{B}} = 0$$

[Bernar, Balakumar & EW]

"Boulware"-like state on RN

$$\langle \mathrm{B}|\hat{O}|\mathrm{B}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{\int_{-\infty}^{\infty} d\omega \, \left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m}\right] - 2\int_{\min\{0,\frac{qQ}{r_+}\}}^{\max\{0,\frac{qQ}{r_+}\}} d\omega \, o^{\mathrm{up}}_{\omega\ell m}\right\}$$

Properties of the state $|\mathrm{B}\rangle$

- Equilibrium state

$$\mathcal{K}_{\mathrm{B}} = 0 \qquad \mathcal{L}_{\mathrm{B}} = 0$$

- Empty as possible at infinity

[Bernar, Balakumar & EW]

"Boulware"-like state on RN

$$\langle \mathrm{B}|\hat{\mathrm{O}}|\mathrm{B}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{\int_{-\infty}^{\infty} d\omega\,\left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m}\right] - 2\int_{\min\{0,\frac{qQ}{r_+}\}}^{\max\{0,\frac{qQ}{r_+}\}} d\omega\, o^{\mathrm{up}}_{\omega\ell m}\right\}$$

Properties of the state $|\mathrm{B}\rangle$

- Equilibrium state

$$\mathcal{K}_{\mathrm{B}} = 0 \qquad \mathcal{L}_{\mathrm{B}} = 0$$

- Empty as possible at infinity
- Diverges on the event horizon

[Bernar, Balakumar & EW]

“Hartle-Hawking”-like state on RN

"Hartle-Hawking"-like state on RN

$|\mathrm{CCH}\rangle$

"Hartle-Hawking"-like state on RN

$|\mathrm{CCH}\rangle$

$$\langle \mathrm{CCH}|\hat{O}|\mathrm{CCH}\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\omega \coth\left|\frac{\omega}{2T_H}\right| o^{\mathrm{in}}_{\omega\ell m} + \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\widetilde{\omega} \coth\left|\frac{\widetilde{\omega}}{2T_H}\right| o^{\mathrm{up}}_{\omega\ell m}$$

"Hartle-Hawking"-like state on RN

$|\mathrm{CCH}\rangle$

$$\langle \mathrm{CCH}|\hat{O}|\mathrm{CCH}\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\omega \coth\left|\frac{\omega}{2T_H}\right| o^{\mathrm{in}}_{\omega\ell m} + \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\widetilde{\omega} \coth\left|\frac{\widetilde{\omega}}{2T_H}\right| o^{\mathrm{up}}_{\omega\ell m}$$

$\mathcal{K}_{\mathrm{CCH}} = \mathcal{K}_{\mathrm{U}}$

$\mathcal{L}_{\mathrm{CCH}} = \mathcal{L}_{\mathrm{U}}$

"Hartle-Hawking"-like state on RN

$|\mathrm{CCH}\rangle$

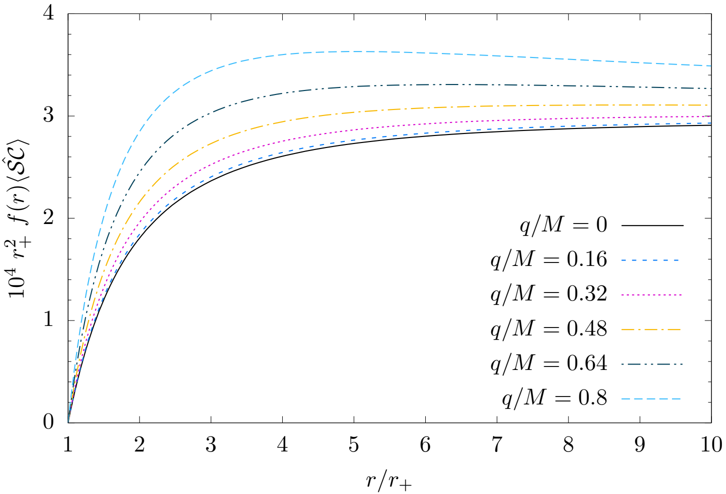
$$\langle \mathrm{CCH}|\hat{O}|\mathrm{CCH}\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\omega \coth\left|\frac{\omega}{2T_H}\right| o^{\mathrm{in}}_{\omega\ell m} + \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\widetilde{\omega} \coth\left|\frac{\widetilde{\omega}}{2T_H}\right| o^{\mathrm{up}}_{\omega\ell m}$$

$$\mathcal{K}_{\mathrm{CCH}} = \mathcal{K}_{\mathrm{U}} - \frac{q}{64\pi^3}\sum_{\ell=0}^{\ell}\int_0^{\infty} d\omega \frac{(2\ell+1)\,\widetilde{\omega}^2}{\omega\left(\exp\left[\frac{\omega}{T_H}\right]-1\right)}\left[\frac{1}{\widetilde{\omega}}\left|B^{\mathrm{in}}_{\omega\ell}\right|^2 - \frac{1}{\omega}\left|B^{\mathrm{in}}_{-\omega\ell}\right|^2\right]$$

$$\mathcal{L}_{\mathrm{CCH}} = \mathcal{L}_{\mathrm{U}} - \frac{1}{16\pi^2}\sum_{\ell=0}^{\ell}\int_0^{\infty} d\omega \frac{(2\ell+1)\,\widetilde{\omega}^2}{\left(\exp\left[\frac{\omega}{T_H}\right]-1\right)}\left[\frac{1}{\widetilde{\omega}}\left|B^{\mathrm{in}}_{\omega\ell}\right|^2 + \frac{1}{\omega}\left|B^{\mathrm{in}}_{-\omega\ell}\right|^2\right]$$

"Hartle-Hawking"-like state on RN

$\langle \mathrm{CCH}|\widehat{SC}|\mathrm{CCH}\rangle - \langle \mathrm{U}|\widehat{SC}|\mathrm{U}\rangle$



[Bernar, Balakumar & EW]

“Hartle-Hawking”-like state on RN

"Hartle-Hawking"-like state on RN

$|\mathrm{FT}\rangle$

"Hartle-Hawking"-like state on RN

$|\mathrm{FT}\rangle$

$$\langle \mathrm{FT}|\hat{O}|\mathrm{FT}\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\widetilde{\omega}\,\coth\left|\frac{\widetilde{\omega}}{2T_H}\right| o^{\mathrm{in}}_{\omega\ell m} + \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\widetilde{\omega}\,\coth\left|\frac{\widetilde{\omega}}{2T_H}\right| o^{\mathrm{up}}_{\omega\ell m}$$

"Hartle-Hawking"-like state on RN

$|\mathrm{FT}\rangle$

$$\langle \mathrm{FT}|\hat{O}|\mathrm{FT}\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\tilde{\omega}\coth\left|\frac{\tilde{\omega}}{2T_H}\right| o^{\mathrm{in}}_{\omega\ell m} + \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\tilde{\omega}\coth\left|\frac{\tilde{\omega}}{2T_H}\right| o^{\mathrm{up}}_{\omega\ell m}$$

$$\mathcal{K}_{\mathrm{FT}} = \frac{q}{64\pi^3}\sum_{\ell=0}^{\infty}\int_{\min\left\{\frac{qQ}{r_+},0\right\}}^{\max\left\{\frac{qQ}{r_+},0\right\}} d\omega\,\frac{|\tilde{\omega}|}{\omega}\,(2\ell+1)\coth\left|\frac{\tilde{\omega}}{2T_H}\right|\left|B^{\mathrm{in}}_{\omega\ell}\right|^2$$

$$\mathcal{L}_{\mathrm{FT}} = \frac{1}{16\pi^2}\sum_{\ell=0}^{\infty}\int_{\min\left\{\frac{qQ}{r_+},0\right\}}^{\max\left\{\frac{qQ}{r_+},0\right\}} d\omega\,|\tilde{\omega}|\,(2\ell+1)\coth\left|\frac{\tilde{\omega}}{2T_H}\right|\left|B^{\mathrm{in}}_{\omega\ell}\right|^2$$

"Hartle-Hawking"-like state on RN

$|\mathrm{FT}\rangle$

$$\langle \mathrm{FT}|\hat{O}|\mathrm{FT}\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\tilde{\omega}\, \coth\left|\frac{\tilde{\omega}}{2T_H}\right| o^{\mathrm{in}}_{\omega\ell m} + \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\tilde{\omega}\, \coth\left|\frac{\tilde{\omega}}{2T_H}\right| o^{\mathrm{up}}_{\omega\ell m}$$

$$\mathcal{K}_{\mathrm{FT}} = \frac{q}{64\pi^3}\sum_{\ell=0}^{\infty}\int_{\min\left\{\frac{qQ}{r_+},0\right\}}^{\max\left\{\frac{qQ}{r_+},0\right\}} d\omega\, \frac{|\tilde{\omega}|}{\omega}\,(2\ell+1)\coth\left|\frac{\tilde{\omega}}{2T_H}\right| \left|B^{\mathrm{in}}_{\omega\ell}\right|^2$$

$$\mathcal{L}_{\mathrm{FT}} = \frac{1}{16\pi^2}\sum_{\ell=0}^{\infty}\int_{\min\left\{\frac{qQ}{r_+},0\right\}}^{\max\left\{\frac{qQ}{r_+},0\right\}} d\omega\, |\tilde{\omega}|\,(2\ell+1)\coth\left|\frac{\tilde{\omega}}{2T_H}\right| \left|B^{\mathrm{in}}_{\omega\ell}\right|^2$$

$$\langle \mathrm{FT}|\widehat{SC}|\mathrm{FT}\rangle - \langle \mathrm{U}|\widehat{SC}|\mathrm{U}\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_{-\infty}^{\infty} d\omega\, \frac{1}{\exp\left|\frac{\tilde{\omega}}{T_H}\right| - 1}\left|\phi^{\mathrm{in}}_{\omega\ell m}\right|^2$$

“Hartle-Hawking”-like state on RN

"Hartle-Hawking"-like state on RN

$|H\rangle$

"Hartle-Hawking"-like state on RN

$|\mathrm{H}\rangle$

$$\langle \mathrm{H}|\hat{\mathrm{O}}|\mathrm{H}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{\int_{-\infty}^{\infty} d\widetilde{\omega}\ \left[o_{\omega\ell m}^{\mathrm{in}} + o_{\omega\ell m}^{\mathrm{up}}\right]\coth\left|\frac{\widetilde{\omega}}{2T_H}\right|\right.$$

"Hartle-Hawking"-like state on RN

$|\mathrm{H}\rangle$

$$\langle \mathrm{H}|\hat{\mathrm{O}}|\mathrm{H}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{ \int_{-\infty}^{\infty} d\widetilde{\omega}\ \left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m}\right] \coth\left| \frac{\widetilde{\omega}}{2T_H}\right| \right.$$
$$\left. -2\int_{\min\{0,\frac{qQ}{r_+}\}}^{\max\{0,\frac{qQ}{r_+}\}} d\omega\, o^{\mathrm{in}}_{\omega\ell m} \coth\left|\frac{\widetilde{\omega}}{2T_H}\right| \right\}$$

"Hartle-Hawking"-like state on RN

$|\mathrm{H}\rangle$

$$\langle \mathrm{H}|\hat{O}|\mathrm{H}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{ \int_{-\infty}^{\infty} d\widetilde{\omega}\ \left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m}\right] \coth\left|\frac{\widetilde{\omega}}{2T_H}\right| \right.$$
$$\left. -2\int_{\min\{0,\frac{qQ}{r_+}\}}^{\max\{0,\frac{qQ}{r_+}\}} d\omega\, o^{\mathrm{in}}_{\omega\ell m} \coth\left|\frac{\widetilde{\omega}}{2T_H}\right| \right\}$$

$$\mathcal{K}_{\mathrm{H}} = 0 \qquad \mathcal{L}_{\mathrm{H}} = 0$$

"Hartle-Hawking"-like state on RN

$|\mathrm{H}\rangle$

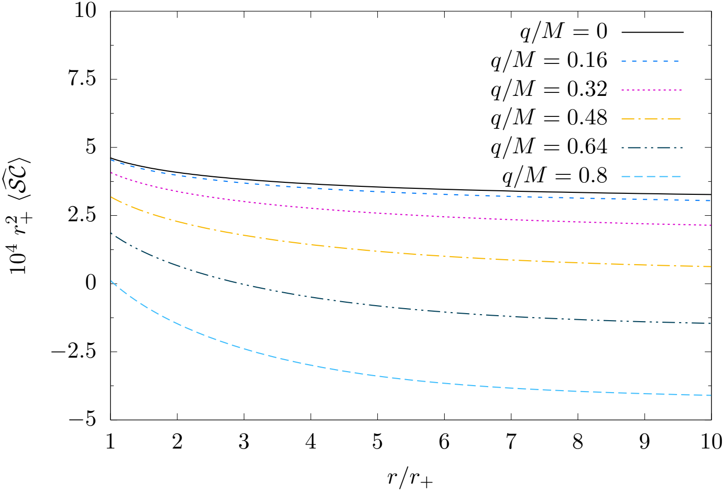
$$\langle \mathrm{H}|\hat{O}|\mathrm{H}\rangle = \frac{1}{2}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\left\{\int_{-\infty}^{\infty} d\widetilde{\omega}\ \left[o^{\mathrm{in}}_{\omega\ell m} + o^{\mathrm{up}}_{\omega\ell m}\right] \coth\left|\frac{\widetilde{\omega}}{2T_H}\right| \right.$$
$$\left. -2\int_{\min\{0,\frac{qQ}{r_+}\}}^{\max\{0,\frac{qQ}{r_+}\}} d\omega\, o^{\mathrm{in}}_{\omega\ell m} \coth\left|\frac{\widetilde{\omega}}{2T_H}\right|\right\}$$

$$\mathcal{K}_{\mathrm{H}} = 0 \qquad \mathcal{L}_{\mathrm{H}} = 0$$

$$\langle \mathrm{H}|\widehat{SC}|\mathrm{H}\rangle - \langle \mathrm{U}|\widehat{SC}|\mathrm{U}\rangle$$
$$= \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_0^{\infty} d\omega \left\{\frac{\left|\phi^{\mathrm{in}}_{\omega\ell m}\right|^2}{\exp\left(\frac{\widetilde{\omega}}{T_H}\right)-1} + \frac{\left|\phi^{\mathrm{in}}_{-\omega\ell m}\right|^2}{\exp\left(\frac{\overline{\omega}}{T_H}\right)-1}\right\}$$

"Hartle-Hawking"-like state on RN

$\langle \mathrm{H}|\widehat{SC}|\mathrm{H}\rangle - \langle \mathrm{U}|\widehat{SC}|\mathrm{U}\rangle$



[Bernar, Balakumar & EW]

Quantum states for a neutral scalar field

Quantum states for a neutral scalar field

Schwarzschild

Quantum states for a neutral scalar field

Schwarzschild

- Unruh $|\mathrm{U}\rangle$ - Hawking flux at $\mathscr{I}^+$

Quantum states for a neutral scalar field

Schwarzschild

- Unruh $|\mathrm{U}\rangle$ - Hawking flux at $\mathscr{I}^+$
- Boulware $|\mathrm{B}\rangle$ - empty as possible at infinity

Quantum states for a neutral scalar field

Schwarzschild

- Unruh $|\mathrm{U}\rangle$ - Hawking flux at $\mathscr{I}^+$
- Boulware $|\mathrm{B}\rangle$ - empty as possible at infinity
- Hartle-Hawking $|\mathrm{H}\rangle$ - equilibrium, regular at horizon

Quantum states for a neutral scalar field

Schwarzschild

- Unruh $|\mathrm{U}\rangle$ - Hawking flux at $\mathscr{I}^+$
- Boulware $|\mathrm{B}\rangle$ - empty as possible at infinity
- Hartle-Hawking $|\mathrm{H}\rangle$ - equilibrium, regular at horizon

Kerr

Quantum states for a neutral scalar field

Schwarzschild

- Unruh $|U\rangle$ - Hawking flux at $\mathscr{I}^+$
- Boulware $|B\rangle$ - empty as possible at infinity
- Hartle-Hawking $|H\rangle$ - equilibrium, regular at horizon

Kerr

- No Hartle-Hawking state

Quantum states for a neutral scalar field

Schwarzschild

- Unruh $|\mathrm{U}\rangle$ - Hawking flux at $\mathscr{I}^+$
- Boulware $|\mathrm{B}\rangle$ - empty as possible at infinity
- Hartle-Hawking $|\mathrm{H}\rangle$ - equilibrium, regular at horizon

Kerr

- No Hartle-Hawking state
- Classical superradiance

Quantum states for a neutral scalar field

Schwarzschild

- Unruh $|U\rangle$ - Hawking flux at $\mathscr{I}^+$
- Boulware $|B\rangle$ - empty as possible at infinity
- Hartle-Hawking $|H\rangle$ - equilibrium, regular at horizon

Kerr

- No Hartle-Hawking state
- Classical superradiance
- $|CCH\rangle$ - well defined, not equilibrium

Quantum states for a neutral scalar field

Schwarzschild

- Unruh $|U\rangle$ - Hawking flux at $\mathscr{I}^+$
- Boulware $|B\rangle$ - empty as possible at infinity
- Hartle-Hawking $|H\rangle$ - equilibrium, regular at horizon

Kerr

- No Hartle-Hawking state
- Classical superradiance
- $|CCH\rangle$ - well defined, not equilibrium
- $|FT\rangle$ - equilibrium, divergent almost everywhere

Quantum states for a charged scalar field on RN

Quantum states for a charged scalar field on RN

Unruh state $|\mathrm{U}\rangle$

Quantum states for a charged scalar field on RN

Unruh state $|\mathrm{U}\rangle$

- Hawking flux at $\mathscr{I}^+$

Quantum states for a charged scalar field on RN

Unruh state $|U\rangle$

- Hawking flux at $\mathscr{I}^+$

"Boulware"-like states

Quantum states for a charged scalar field on RN

Unruh state $|U\rangle$

- Hawking flux at $\mathscr{I}^+$

"Boulware"-like states

- $|B^-\rangle$ - superradiant flux at $\mathscr{I}^+$

Quantum states for a charged scalar field on RN

Unruh state $|U\rangle$

- Hawking flux at $\mathscr{I}^+$

"Boulware"-like states

- $|B^-\rangle$ - superradiant flux at $\mathscr{I}^+$
- Proposed state $|B\rangle$ - empty as possible at infinity

Quantum states for a charged scalar field on RN

Unruh state $|U\rangle$

- Hawking flux at $\mathscr{I}^+$

"Boulware"-like states

- $|B^-\rangle$ - superradiant flux at $\mathscr{I}^+$
- Proposed state $|B\rangle$ - empty as possible at infinity

"Hartle-Hawking"-like states

Quantum states for a charged scalar field on RN

Unruh state $|\mathrm{U}\rangle$

- Hawking flux at $\mathscr{I}^+$

"Boulware"-like states

- $|\mathrm{B}^-\rangle$ - superradiant flux at $\mathscr{I}^+$
- Proposed state $|\mathrm{B}\rangle$ - empty as possible at infinity

"Hartle-Hawking"-like states

- $|\mathrm{CCH}\rangle$ - well-defined, not equilibrium

Quantum states for a charged scalar field on RN

Unruh state $|U\rangle$

- Hawking flux at $\mathscr{I}^+$

"Boulware"-like states

- $|B^-\rangle$ - superradiant flux at $\mathscr{I}^+$
- Proposed state $|B\rangle$ - empty as possible at infinity

"Hartle-Hawking"-like states

- $|CCH\rangle$ - well-defined, not equilibrium
- $|FT\rangle$ - not equilibrium, divergent

Quantum states for a charged scalar field on RN

Unruh state $|U\rangle$

- Hawking flux at $\mathscr{I}^+$

"Boulware"-like states

- $|B^-\rangle$ - superradiant flux at $\mathscr{I}^+$
- Proposed state $|B\rangle$ - empty as possible at infinity

"Hartle-Hawking"-like states

- $|CCH\rangle$ - well-defined, not equilibrium
- $|FT\rangle$ - not equilibrium, divergent
- Proposed state $|H\rangle$ - equilibrium, regular at horizon