From the black hole, through the wormhole to ultra-compact objects

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Outline

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- The Einstein-Scalar-Gauss-Bonnet Theories
- Black-Hole Solutions in EsGB
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G. Antoniou (Nottingham U), A. Bakopoulos (Ioannina U.), B. Kleihaus & J. Kunz (Oldenburg U.) and N. Pappas (Athens U.)

Introduction

Einstein's General Theory of Relativity based on his field equations

$$\boxed{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}}$$

- describes extremely accurately gravity around ordinary astrophysical bodies, such as our Sun,
- predicts the existence of "exotic" objects such as black holes or wormholes

Although a beautiful mathematical theory, Einstein's General Relativity is not considered as the final theory of Gravity due to a number of problems (dark matter, dark energy, coincidence problem, initial singularity problem, non-unification with the other forces etc)...

Gravitational Solutions in General Relativity

In addition, General Relativity is a rather *restrictive* theory from the point of view of gravitational solutions:

- it admits only three families of Black-Hole solutions: Schwarzschild (1916), Reissner-Nordstrom (1921) and Kerr(-Newman) (1963)

  According to the "no-hair" theorems of GR (Birkhoff; Israel; Carter; Price; Hartle; Teitelboim; Bekenstein), these may be characterized *only* by their mass M, electromagnetic charge Q and angular momentum J

  A BH has no colour, baryon and lepton number, or scalar charges...

- it does admit Wormholes hidden in the interior of all black-hole solutions but these are not *traversable* wormholes

Generalised Theories of Gravity

In the absence of the fundamental Quantum Theory of Gravity, we may consider modified *effective theories* of gravity by introducing extra fields and/or higher gravitational terms

A generalised theory of gravity could have the form

$$S = \int d^4x \sqrt{-g} \left[f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}, \Phi_i) + \mathcal{L}_X(\Phi_i) \right]$$

There is a plethora of modified theories of gravity in the literature:

Scalar-tensor theories, $f(R)$ theories, Higher-derivative theories, Chern-Simons gravity, Einstein-aether theory, Massive gravity, Gravitational aether, $f(T)$ theories, TeVes, ...

The hope: such a modified theory may describe more accurately the stronger gravity regime which we are slowly starting to explore (neutron-star properties, gravitational waves by LIGO/Virgo, bounds from EHT ...)

Scalar No-Hair Theorems

Can we find new black-hole solutions in the context of such a theory? And, what happens to the old GR solutions already observed in nature?

In the context of theories with scalar fields, non-existence theorems readily emerged: the ‘Scalar No-Hair Theorem’ (Bekenstein, 1972; Teitelboim, 1972) excluded novel BH solutions in minimally-coupled and non-minimally coupled (Bekenstein, 1994) scalar-tensor theories - new formulations (Sotiriou & Faraoni, 2012; Hui & Nicolis, 2013) covered the case of more general scalar-tensor theories

However, ... as the conditions of the non-existence theorems are violated, new black-hole solutions easily emerge: solutions with a Skyrme field (Luckock & Moss, 1986; Droz et al, 1994), a conformally-coupled scalar field (Bekenstein, 1974), a dilaton field (Kanti et al, 1996) and a gauge field (Torii et al, 1997; Kanti et al, 1997), rotating BHs (Kleihaus et al, 2011; Pani et al, 2011) and shift-symmetric Galileon BHs (Babichev & Charmousis, 2014; Sotiriou & Zhou, 2014), ...

The Einstein-Scalar-Gauss-Bonnet Theory

We therefore need to consider a gravity theory that evades the no-hair requirements and remains legitimate, such as

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + f(\phi) \, R^2_{GB} \right],$$

with $f(\phi)$ a coupling function between a scalar field ϕ and the GB term

$$R^2_{GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

Such a theory arises

- as part of the string effective action at low energies
- as part of a Lovelock effective theory in four dimensions
- as part of a modified scalar-tensor (Horndeski or DHOST) theory

It contains a quadratic gravitational term (next important term in strong curvature regimes) but leads to field equations with up to 2nd-order derivatives, and with no Ostrogradski instabilities

Black-Hole Solutions in Einstein-Scalar-GB

• The Basic Question: For what forms of the coupling function $f(\phi)$ can one get a static, spherically-symmetric black-hole solution?

Keeping therefore the form of $f(\phi)$ arbitrary, we assume the line-element

$$ds^2 = -e^{A(r)} dt^2 + e^{B(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

while the equations of motion read

$$\nabla^2\phi + \dot{f}(\phi) R^2_{GB} = 0 \,, \qquad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$$

where

$$T_{\mu\nu} = -\frac{1}{4} g_{\mu\nu} (\partial\phi)^2 + \frac{1}{2} \partial_\mu\phi \partial_\nu\phi - \frac{1}{2} (g_{\rho\mu} g_{\lambda\nu} + g_{\lambda\mu} g_{\rho\nu}) \eta^{\kappa\lambda\alpha\beta} \tilde{R}^{\rho\gamma}{}_{\alpha\beta} \nabla_\gamma \partial_\kappa f \,,$$

$$\Downarrow$$

$$\boxed{A'' = \frac{P}{S} \,, \qquad \phi'' = \frac{Q}{S} \,, \qquad P, Q, S = g(r, \phi, \phi', A')}$$

Black-Hole Solutions in Einstein-Scalar-GB

- For the existence of a regular black-hole horizon, we demand that

$$e^{A(r)} \to 0, \quad e^{-B(r)} \to 0, \quad \phi(r) \to \phi_h$$

Demanding that ϕ'' is also finite at the horizon r_h, we find the constraint

$$\phi'_h = \frac{r_h}{4\dot{f}_h}\left(-1 \pm \sqrt{1 - \frac{96\dot{f}_h^2}{r_h^4}}\right), \qquad \dot{f}_h^2 < \frac{r_h^4}{96}$$

- At large distances from the horizon, we obtain a power series expansion in $1/r$ of the form

$$e^A = 1 - \frac{2M}{r} + \frac{MD^2}{12r^3} + \ldots, \qquad e^B = 1 + \frac{2M}{r} + \frac{16M^2 - D^2}{4r^2} + \ldots$$

$$\phi = \phi_\infty + \frac{D}{r} + \frac{MD}{r^2} + \frac{32M^2D - D^3}{24r^3} + \frac{12M^3D - 24M^2\dot{f} - MD^3}{6r^4} + \ldots$$

A general coupling function f does not interfere with the existence of a regular event horizon or an asymptotically-flat limit

Black-Hole Solutions in Einstein-Scalar-GB

The construction of a *complete* black-hole solution demands the smooth matching of these two asymptotic solutions

This is where the no-hair theorems enter... Bekenstein's *No-Hair* theorem (1994) dictated that this cannot be done if "T_r^r is negative near the BH horizon"

However, in the Einstein-scalar-Gauss-Bonnet theories with arbitrary f, this is not true since it holds that

$$sign(T_r^r)_h = -sign(\dot{f}_h \phi'_h)) > 0$$

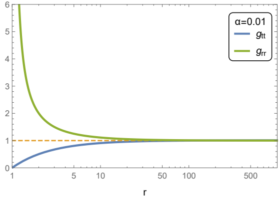
The presence of the Gauss-Bonnet term in the theory guarantees the evasion of the scalar no-hair theorem

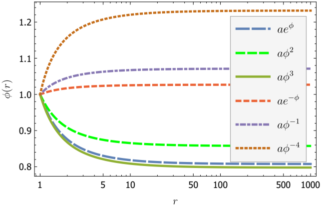
The aforementioned analysis was followed in 1996 to prove that the *dilatonic* theory with $f(\phi) = \alpha e^{\phi}$ evades Bekenstein's theorem and numerically demonstrate the existence of the dilatonic black holes (Kanti, Mavromatos, Rizos, Tamvakis, Winstanley, PRD 1996)

Black-Hole Solutions in Einstein-Scalar-GB

Choosing $f(\phi)$ and then (ϕ_h, ϕ'_h), we found numerous BH solutions:

(Antoniou, Bakopoulos & Kanti, PRL 2018, PRD 2018)

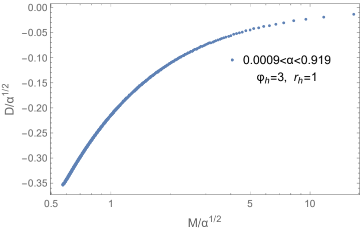


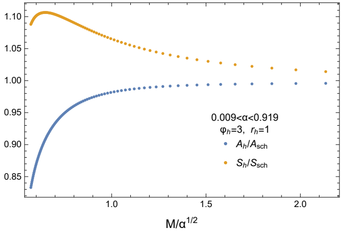


There is a huge literature on different types of black holes with a non-trivial scalar field...

Almost simultaneously two additional works appeared for $f \sim 1 - e^{-\phi^2}$ (Doneva & Yazadjiev, 2018) and $f = a\phi^2$ (Silva et al, 2018) discussing *spontaneously scalarised* black holes, i.e. BHs with a scalar field emerging when a GR solution is destabilised (Damour & Esposito-Farese, 1993)

Black-Hole Solutions in Einstein-Scalar-GB





- Our solutions are *naturally scalarised* and always have a non-trivial ϕ – the GR solution may not exist as an independent solution
- In the limit of large mass, all GB black holes reduce to the Schwarzschild solution $\Rightarrow$ *Schwarzschild BHs are large GB BHs*
- The scalar charge D is a "secondary" conserved quantity
- The entropy of the GB black holes may exceed that of the Schwarzschild solution (shown that of $f(\phi) \sim 1/\phi$)
- All GB black holes have a minimum mass M_{min}

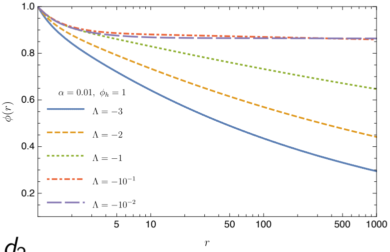
Black-Hole Solutions in Einstein-Scalar-GB

If we add a negative cosmological constant in the theory, the spacetime reduces asymptotically to an Anti-de Sitter background

A regular black-hole horizon emerges again provided that

$$\phi'_h = \frac{16\Lambda r_h \dot{f}^2 \left(\Lambda r_h^2 - 3\right) + \Lambda r_h^5 - r_h^3 \mp \sqrt{R}}{4\dot{f}[r_h^2 - \Lambda(r_h^4 - \dot{f}^2)]}$$

A plethora of solutions emerged for $f(\phi) = e^{\pm\phi}, \phi^{\pm 2n}, \phi^{\pm(2n+1)}, \ln\phi, \ldots$ as easily as the ones with asymptotically-flat behaviour (Bakopoulos, Antoniou, Kanti, PRD 2019)



At large distances, the scalar field behaves as:

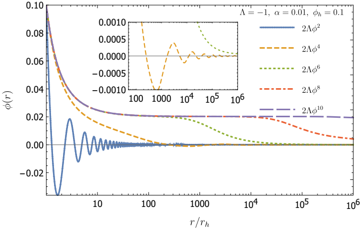
$$\phi(r) = \phi_\infty + d_1 \ln r + \frac{d_2}{r^2} + \ldots$$

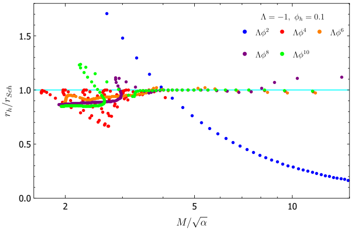
Black-Hole Solutions in Einstein-Scalar-GB

We now promote the negative cosmological constant to a dynamic potential for the scalar field (Bakopoulos, Kanti & Pappas, PRD 2020)

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2}\, \partial_\mu \phi\, \partial^\mu \phi + f(\phi)\, R^2_{GB} - 2\Lambda\, V(\phi) \right]$$

We considered $\Lambda < 0$ and different choices for $f(\phi)$ and $V(\phi)$. Again, black-hole solutions with a regular horizon and a non-trivial ϕ emerged

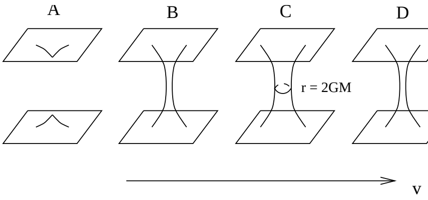




In the case where $V(\phi) = \phi^2$, the solutions are divided in two groups: large, low-mass BHs and small, large-mass (*ultracompact*) BHs

Wormholes in General Relativity

Wormholes are well motivated objects since, in GR, they hide inside the horizon of a black hole – the complete Schwarzschild geometry is:



The region inside the horizon of a Schwarzschild BH, $r < r_h = 2M$, is dynamical and a throat appears in place of the singularity as time goes by

But the throat closes so quickly that not even a light signal can pass through (Einstein-Rosen, 1935; Wheeler, 1955)

The Reissner-Nordstrom and Kerr geometries have generically internal tunnels - but the internal Cauchy horizons are unstable

Wormholes in General Relativity

Looking for a traversable wormhole, Morris & Thorne (1988) disconnected the wormhole from the black hole. Using an ansatz of the form

$$ds^2 = -e^{2\Phi(r)}\,dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2\,(d\theta^2 + \sin^2\theta\,d\varphi^2),$$

they demanded

- an asymptotically-flat regime: $\Phi \to 0$ and $b/r \to 0$, for $r \to \infty$
- the absence of a horizon or singularity: $\Phi(r)$ everywhere regular
- the presence of a throat at $r_{min} = b_0$ where $b(r_{min}) = b_0$

The above demand in turn an energy-momentum tensor

$$T_{tt} = \rho\,, \qquad T_{rr} = -\tau\,, \qquad T_{\theta\theta} = T_{\varphi\varphi} = p$$

satisfying $\tau \geq \rho \Rightarrow$ violation of Null Energy Condition $\Rightarrow$ Exotic Matter

Wormholes in EsGB Theory

There has been a large number of efforts in the literature to produce such an exotic yet realistic distribution of matter...

In the context of the EsGB theory, one may easily observe that close to the BH horizon it holds

$$\rho \simeq -\frac{2e^{-B}}{r^2}\, B'\phi'\dot{f} < 0\,, \qquad p_r \simeq -\frac{2e^{-B}}{r^2}\, A'\phi'\dot{f} > 0$$

since as $r \to r_h$, $B' < 0$ and $A' > 0$. Thus, the GB term does create the *effective* energy-momentum tensor we need for the support of a wormhole

Indeed, our early work (Kanti et al. 1996) contained the regular solution

$$ds^2 = -e^{A(\ell)}dt^2 + e^{B(\ell)}d\ell^2 + (\ell^2 + r_0^2)\,(d\theta^2 + \sin^2\theta d\varphi^2)$$

where

$$e^A = a_0 + a_1\ell + \ldots\,, \quad e^{B(r)} = b_0 + b_1\ell + \ldots,\,, \quad \phi(\ell) = \phi_0 + \phi_1\ell + \ldots$$

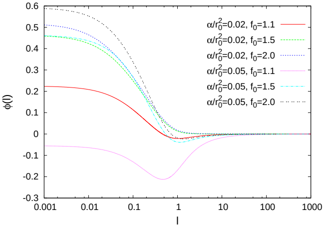
The above describes a wormhole with the throat at $r = r_0$ or $\ell = 0$

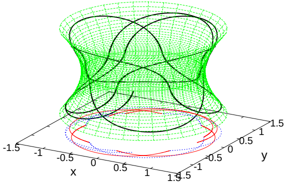
Wormholes in EsGB Theory

At large distances, we obtain (Kanti, Kleihaus & Kunz, PRL 2011, PRD 2012)

$$e^{A} \simeq 1 - \frac{2M}{\ell} + \ldots, \qquad e^{B} = 1 + \frac{2M}{\ell} + \ldots\,, \qquad \phi \simeq \phi_\infty + \frac{D}{\ell} + \ldots\,,$$

where M and D are the mass and scalar charge of the wormhole





Regular symmetric solutions arise if a perfect fluid (non-exotic!) and a gravitational source term are introduced at the throat

$$S = \int d^3x\sqrt{h}\,(\lambda_1 + \lambda_0 e^{\phi}\tilde{R})$$

Wormholes in EsGB Theory

All these were valid for $f(\phi) \sim e^{\phi}$. What happens for other forms of $f(\phi)$? In the context of the EsGB theory, we looked for solutions:

$$ds^2 = -e^{A(\eta)} dt^2 + e^{B(\eta)} \left[d\eta^2 + (\eta^2 + \eta_0^2)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

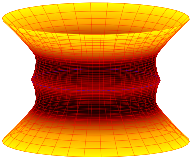
One way of studying the emergence of a throat is by studying the *circumferential radius* defined as

$$R_c(\eta) = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{g_{\varphi\varphi}}|_{\theta=\pi/2} \, d\varphi = e^{B(\eta)/2} \sqrt{\eta^2 + \eta_0^2}$$

A throat corresponds to a local minimum of R_c, thus we should have

$$\left.\frac{dR_c}{d\xi}\right|_{\eta_*} \propto B'(\eta_*) = 0\,, \quad \left.\frac{d^2R_c}{d\xi^2}\right|_{\eta_*} \propto \eta_0^2 B''(\eta_*) + 2 > 0$$

where $d\xi = e^{B/2} d\eta$ is the proper radial distance. Two extrema arise: a minimum, which stands for the throat, and a maximum, which stands for an *equator*



Wormholes in EsGB Theory

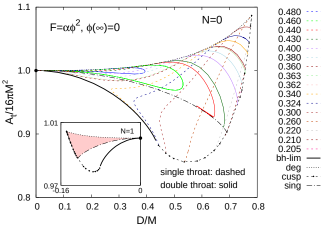
- We found regular wormhole solutions for every form of $f(\phi)$ with an asymptotically-flat behaviour and with a single or double throat
- The NEC still needs to be violated for a wormhole solution to arise. In fact, we find

$$(\rho + p_\eta)|_{\eta_*} = -2[e^{-B} R_c'']_{\eta_*}$$

- Thus the NEC is always violated at the throat but is obeyed at the equator. But, even at the throat, the violation holds for the *effective* energy-momentum tensor components

$$T_{\mu\nu} = T^{\phi}_{\mu\nu} + T^{GB}_{\mu\nu}$$

- The wormhole geometry is supported by the coupling of a non-phantom scalar field to a quadratic gravitational term



- All EsGB WHs are bounded by the corresponding BHs and are free of any exotic matter (Antoniou, Bakopoulos, Kanti, Kleihaus, Kunz, PRD 2020)

Particle-like Solutions in EsGB Theory

Particle-like solutions are common in flat space-time, usually termed solitons. Gravity allows for similar solutions not always with attractive properties. E.g. in the context of the Einstein-scalar theory, the singular Fisher/Janis-Newman-Winicour-Wyman (1948/68) solution arises with

$$e^A \sim e^{-B} \sim \left(1 - \frac{1}{2s}\,\frac{M}{r}\right)\,, \quad \phi \sim \ln\left(1 - \frac{1}{2s}\,\frac{M}{r}\right)$$

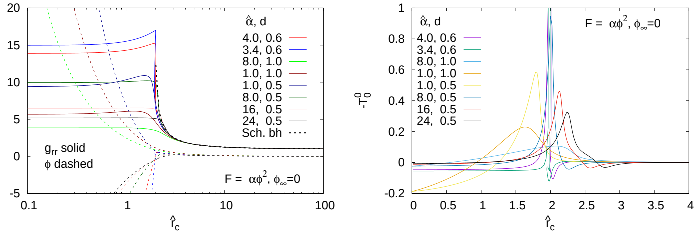
In the context of the EsGB theory, we looked also for solutions with a regular spacetime, with no singularities, no horizons and no throats

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

At large distances, the asymptotic behaviour is the same as for black holes and wormholes. At small distances, for e.g. $f(\phi) = \alpha\phi^2$, we find

$$A(r) = A_0 + A_2 r^2 + A_3 r^3 + \dots\,, \quad \phi = -\frac{c_0}{r} + \phi_0 + \phi_1 r + \phi_2 r^2 + \phi_3 r^3 + \dots$$

Particle-like Solutions in EsGB Theory



All scalar invariants are finite everywhere, as are also all components of $T_{\mu\nu}$ despite the singularity in ϕ: (Kleihaus, Kunz & Kanti, PLB 2020, PRD 2020)

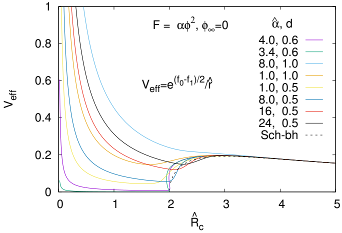
$$\rho(0) = -\frac{3}{32\alpha}\,, \qquad p(0) = \frac{2}{32\alpha}$$

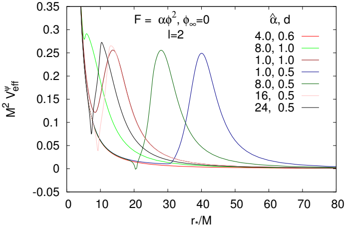
The energy-momentum tensor has a regular, shell-like behaviour and vanishes at a very small radius qualifying these solutions as ultra compact objects (UCOs) – missed by Brihaye, Hartmann and Urrestilla (2018)

Particle-like Solutions in EsGB Theory

We studied also the propagation of particles in this background - for photons this may be done through the relation $\mathcal{L} = g_{\mu\nu}\,\dot{x}^\mu\,\dot{x}^\nu = 0$

The extrema in the photon effective potential correspond to circular orbits and lead to the presence of *light rings* around these UCOs



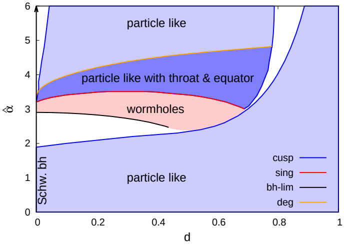


We finally considered the propagation of a scalar test particle ψ in this background. The form of the effective potential V^{ψ}_{eff} with its two potential barriers results into the creation of *echoes* in the wave signal

Conclusions

• The Generalised Theories of Gravity may be the way forward in gravitational physics

• The Einstein-scalar-Gauss-Bonnet theory is a very promising type of a quadratic theory that admits a variety of solutions



• Its solution space contains regular black holes, wormholes with no need for exotic matter and particle-like solutions all with scalar hair and for arbitrary coupling function

• What else could there be in there??? Soon (or later) to be discovered...