



Entangled Relativity

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- Black holes

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4. Discussion & conclusion

- Singularities

- Quantum gravity

- The doom of space-time



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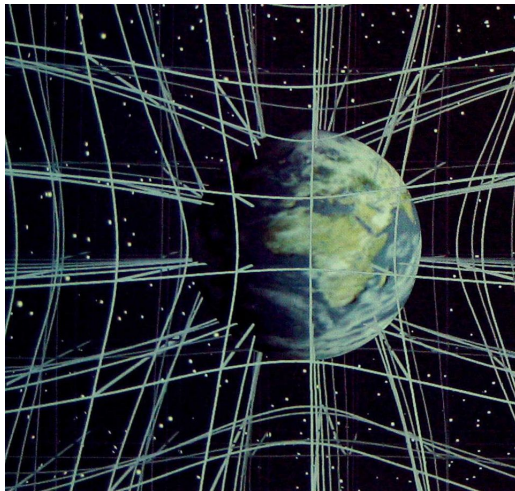
4. Discussion & conclusion

- Singularities
- Quantum gravity
- The doom of space-time



General relativity

matter tells spacetime how to bend, and spacetime tells matter how to move





General relativity may have the following issues :

- Singularities (black holes, big-bang)
- Spacetime might not have an operational meaning beyond the Planck scale (likely related to the quantum gravity conundrum)
- Inertia can be defined from nothing, i.e. GR does not satisfy *Mach's principle* of Einstein



General relativity's issues: singularities

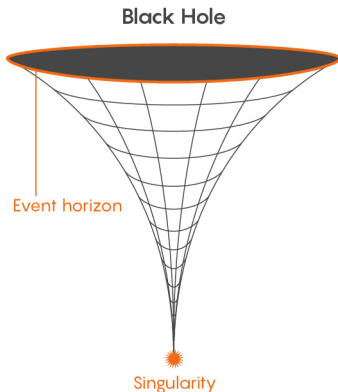


Figure: Penrose's theorem on the inevitability of singularities inside black holes.



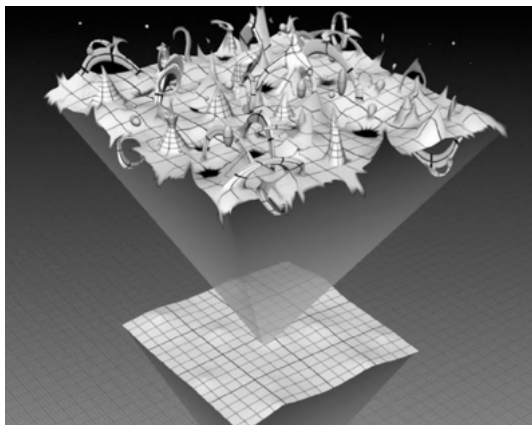
General relativity may have the following issues :

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General relativity's issues

no operational meaning beyond the Planck scale?



Main reason: size of black hole r_S related to G (as a fundamental constant) via $r_S \sim 2G \text{ Energy}/c^4$.



General relativity may have the following issues :

- Singularities (black holes, big-bang)
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General relativity's issues

inertia can be defined *ex nihilo* in general relativity: violation of Mach's principle

Einstein believed in the *relativity of inertia*

“c. Mach's Principle. [Spacetime] is completely determined by [matter] [...]. With (c), according to the field equations of gravitation, **there can be no [spacetime] without matter.** ” Einstein [1918a]

To the press during his first visit in the US in 1921:

“It was formerly believed that if all material things disappeared out of the universe, time and space would be left. According to relativity theory, however, time and space disappear together with the things.”
Robinson [2018]



The actual reason for the cosmological constant

The cosmological constant was meant (but failed) to satisfy Mach's Principle of Einstein

Response to the paper of de Sitter. Einstein [1918b]

"If the de Sitter solution were valid everywhere, it would show that the introduction of the λ -term does not fulfill the purpose I intended. Because, in my opinion, the general theory of relativity is a satisfying system only if it shows that the physical qualities of space are *completely* determined by matter alone. Therefore no $g_{\mu\nu}$ -field must exist (that is, no space-time continuum is possible) without matter that generates it."



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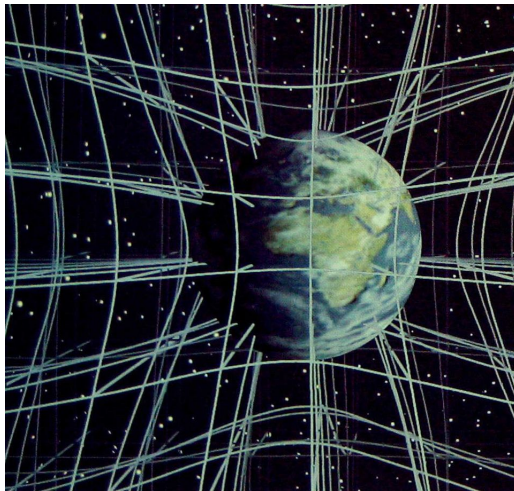
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- The doom of space-time



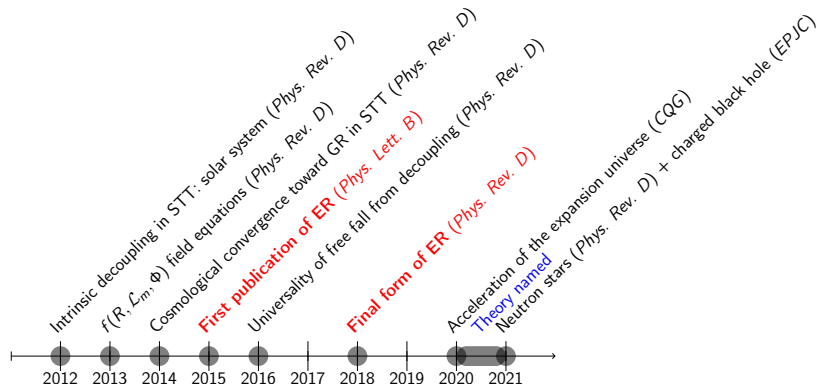
Entangled relativity

matter tells spacetime how to bend, and spacetime tells matter how to move





Chronology of entangled relativity



Arruga and Minazzoli [2021], Arruga et al. [2021], Harko et al. [2013], Ludwig et al. [2015], Minazzoli [2014, 2018, 2021], Minazzoli and Hees [2013, 2014, 2016], Minazzoli and Santos [2021]



Action of entangled relativity and its field equations

Trade the **additive coupling** of GR for a **pure multiplicative coupling** . Ludwig et al. [2015]

$$\frac{R}{2\kappa} + \mathcal{L}_m \longrightarrow -\frac{\xi}{2} \frac{\mathcal{L}_m^2}{R} \quad (1)$$

$$\kappa := \frac{8\pi G}{c^4}$$

It has nothing to do, a priori, with quantum entanglement.

ξ : new constant that does not impact the classical dynamics.

$[\xi] = [\kappa]$ but $\xi \neq \kappa$ a priori.



Action of entangled relativity and its field equations

general relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \quad (2)$$

entangled relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{R}{\mathcal{L}_m} T_{\mu\nu} + \frac{R^2}{\mathcal{L}_m^2} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \frac{\mathcal{L}_m^2}{R^2} \quad (3)$$

(special case valid $\forall \mathcal{L}_m \neq 0$ on-shell)



Action of entangled relativity and its field equations

Matter field equation ($\forall$ tensorial material field χ)

$$\frac{\partial \mathcal{L}_m}{\partial \chi} - \frac{1}{\sqrt{-g}} \partial_\sigma \left(\frac{\partial \sqrt{-g} \mathcal{L}_m}{\partial (\partial_\sigma \chi)} \right) = \frac{\partial \mathcal{L}_m}{\partial (\partial_\sigma \chi)} \frac{R}{\mathcal{L}_m} \partial_\sigma \left(\frac{\mathcal{L}_m}{R} \right) \quad (4)$$

Conservation equation

$$\nabla_\sigma \left(\frac{\mathcal{L}_m}{R} T^{\alpha\sigma} \right) = \mathcal{L}_m \nabla^\alpha \left(\frac{\mathcal{L}_m}{R} \right) \quad (5)$$

$$T_{\mu\nu} := - \frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}} \quad (6)$$



Satisfaction of Einstein's 1918 version of Mach's Principle

$$S \propto \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad \text{hence} \quad \mathcal{L}_m = 0 \Rightarrow S = 0 \quad (7)$$

It does not make sense to consider space-time without considering matter fields in this framework $\Rightarrow$ **Inertia cannot be defined *ex nihilo*!**

Matter and curvature are *entangled* (in the etymological sense) at the level of the action density.



Satisfaction of Einstein's 1918 version of Mach's Principle

General relativity may have the following issues:

- Singularities (black holes, big-bang)
- Spacetime might not have an operational meaning beyond the Planck scale (likely related to the quantum gravity conundrum)
- ~~Inertia can be defined from nothing~~ ✓



Dilaton equivalent form

As in $f(R)$ theories, the theory can be re-written as a scalar-tensor theory.

$$-\frac{\xi}{2} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad \Leftrightarrow \quad \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (8)$$

$$\forall \mathcal{L}_m \neq \emptyset.$$

κ is a field that settles to a constant during the evolution of the universe. (Discussed later on, p. 31)



Dilaton equivalent form: field equations

$$\int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)$$

gives

$$G_{\alpha\beta} = \kappa T_{\alpha\beta} + \kappa^2 [\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square] \kappa^{-2} \quad (9)$$

with

$$3\kappa^2 \square \kappa^{-2} = \kappa (T - \mathcal{L}_m) \quad (10)$$

and

$$\nabla_\sigma \left(\frac{T^{\alpha\sigma}}{\kappa} \right) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \quad (11)$$



Check that one gets the same field equations

$$-\frac{\xi}{2} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad \& \quad \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)$$

$$\frac{\delta}{\delta \kappa} \left[\int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \right] = 0 \Rightarrow \kappa = -\frac{R}{\mathcal{L}_m}$$

Hence (e.g.)

$$G_{\alpha\beta} = \kappa T_{\alpha\beta} + \kappa^2 [\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square] \kappa^{-2} \quad (12)$$

$\Leftrightarrow$

$$G_{\mu\nu} = -\frac{R}{\mathcal{L}_m} T_{\mu\nu} + \frac{R^2}{\mathcal{L}_m^2} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \frac{\mathcal{L}_m^2}{R^2}$$



An even more usual form

$$S = \frac{\xi}{\tilde{\kappa}} \int d^4x \sqrt{-g} \left(\frac{\phi^2 R}{2\tilde{\kappa}} + \phi \mathcal{L}_m \right) \quad (13)$$

$\tilde{\kappa}$: normalization constant, such that $\phi := \tilde{\kappa}/\kappa$.

$$G_{\alpha\beta} = \tilde{\kappa} \frac{T_{\alpha\beta}}{\phi} + \phi^{-2} [\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square] \phi^2 \quad (14)$$

with

$$3 \frac{\square \phi^2}{\phi^2} = \frac{\tilde{\kappa}}{\phi} (T - \mathcal{L}_m) \quad (15)$$

and

$$\nabla_\sigma (\phi T^{\alpha\sigma}) = \mathcal{L}_m \nabla^\alpha \phi \quad (16)$$



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General relativity limit of entangled relativity

Field equations

$$G_{\alpha\beta} = \kappa T_{\alpha\beta} + \kappa^2 [\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square] \kappa^{-2} \quad (17)$$

with

$$3\kappa^2 \square \kappa^{-2} = \kappa (T - \mathcal{L}_m) \quad (18)$$

and

$$\nabla_\sigma \left(\frac{T^{\alpha\sigma}}{\kappa} \right) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \quad (19)$$



General relativity limit of entangled relativity

Field equations

$$G_{\alpha\beta} = \kappa T_{\alpha\beta} + \kappa^2 [\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square] \kappa^{-2} \quad (20)$$

with

$$3\kappa^2 \square \kappa^{-2} = \kappa (T - \cancel{\mathcal{L}_m}) \quad (21)$$

and

$$\nabla_\sigma \left(\frac{T^{\alpha\sigma}}{\kappa} \right) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \quad (22)$$

$\mathcal{L}_m = T \Rightarrow \kappa = \text{cste}$ can be sol^o $\Rightarrow$ equations of general relativity



What is the ratio κ ?

Field equations

$$G_{\alpha\beta} = \kappa T_{\alpha\beta} + \kappa^2 [\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square] \kappa^{-2} \quad (23)$$

with

$$3\kappa^2 \square \kappa^{-2} = \kappa (T - \mathcal{L}_m) \quad (24)$$

$$\kappa = -\frac{R}{\mathcal{L}_m} \quad (25)$$

When $\mathcal{L}_m = T$, one simply recovers the trace of Einstein's equation, that is $R = -\kappa T$. And κ can be a constant $\Rightarrow$ GR.

The ratio $\kappa = -R/\mathcal{L}_m$ corresponds to a field that gives the amplitude with which spacetime is bent by matter fields.



Dust cases → general relativity

$$\mathcal{L}_m = -\rho = T \quad (26)$$

Action for a dust field

$$S \propto \int d^4x \sqrt{-g} \frac{\phi^2 R}{2\tilde{\kappa}} - c^2 \sum_A \int_A d\tau \phi \bar{m}_A \quad (27)$$

Equivalent to general relativity. Minazzoli and Hees [2015]

Conformal transformation $\tilde{g}_{\alpha\beta} = e^{-2\varphi/\sqrt{3}} g_{\alpha\beta}$, and $\phi = e^{-\varphi/\sqrt{3}}$

$$S \propto \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\tilde{\kappa}} \left(\tilde{R} - 2\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) \right] - c^2 \sum_A \int_A d\tilde{\tau} \bar{m}_A \quad (28)$$



Dust cases → general relativity

$$\mathcal{L}_m = -\rho = T \quad (29)$$

Action for a dust field

$$S \propto \int d^4x \sqrt{-g} \frac{\phi^2 R}{2\tilde{\kappa}} - c^2 \sum_A \int_A d\tau \phi \bar{m}_A \quad (30)$$

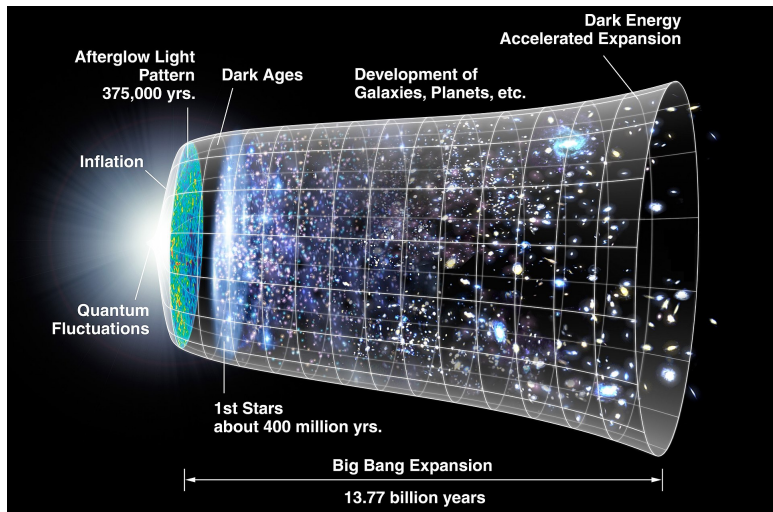
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$$S \propto \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\tilde{\kappa}} \left(\tilde{R} - 2\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) \right] - c^2 \sum_A \int_A d\tilde{\tau} \bar{m}_A \quad (31)$$



Cosmological evolution of the universe





Cosmological matter era and the Equivalence Principle

Matter era $\approx$ dust case

Assuming a flat Friedman-Lemaître-Robertson-Walker metric, **the phenomenology of entangled relativity converges toward the one of general relativity without a cosmological constant during the matter era.** Minazzoli [2014], Minazzoli and Hees [2014]

$$\text{That is } \boxed{\kappa(t) \rightarrow \kappa_0}.$$

It means Newton's constant G is an asymptotic value of an actual field.

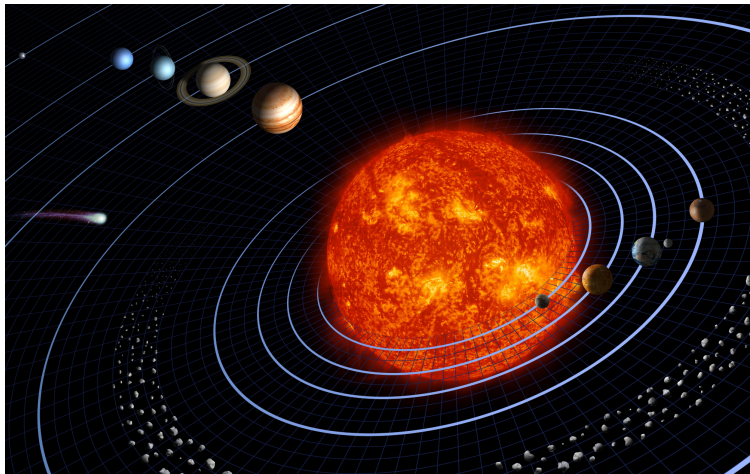
Because dust + EM radiation $\Rightarrow \mathcal{L}_m = T$.

$(\mathcal{L}_m^{\text{EM}} \propto E^2 - B^2 = 0 = T^{\text{EM}} \text{ for radiation}).$

NLO remains to be studied.



Post-Newtonian solution: e.g. Solar System





An ambiguity in the field equations

One needs to know the value of the on-shell matter Lagrangian

$$3\kappa^2 \square \kappa^{-2} = \kappa (T - \mathcal{L}_m) \quad (32)$$

- if $\mathcal{L}_m = T$, as argued in e.g. Avelino and Azevedo [2018], then the scalar degree of freedom has no source at all $\Rightarrow$ GR.
- if $\mathcal{L}_m = -\rho$, as argued in e.g. Minazzoli and Harko [2012], then the scalar degree of freedom is sourced by pressure only $\rightarrow$ *Pressuron*. Minazzoli and Hees [2014].
- $\mathcal{L}_m = P$, one **does not** have general relativity at leading post-Newtonian order. Should only be valid for exotic objects such as boson stars Arruga et al. [2021]. (Because $\mathcal{L}_m = K - V = P$ for scalar fields).



Post-Newtonian solutions assuming $\nabla_\sigma(\rho_0 u^\sigma) = 0$

No source $\Rightarrow$ GR post-Newtonian phenomenology

$$\mathcal{L}_m = T \quad \Rightarrow \quad \square\phi^2 = 0. \quad (33)$$

Or “Pressuron” $\rightarrow$ name given in Minazzoli and Hees [2014]

$$\mathcal{L}_m = -\rho \quad \Rightarrow \quad \frac{1}{\phi^2}\square\phi^2 = -\frac{\tilde{\kappa}}{\phi}P, \quad (34)$$

Solution for *pressuron* Minazzoli and Hees [2013]

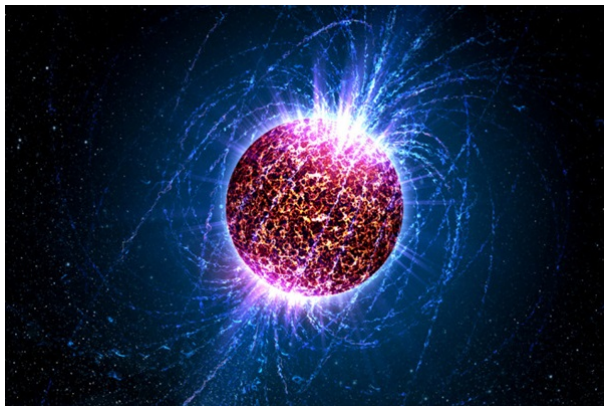
$$\boxed{g_{\alpha\beta}^{ER} = g_{\alpha\beta}^{GR} + \mathcal{O}(P/(\rho c^2), 1/c^4)} \quad (35)$$

$$P/(\rho c^2) = \mathcal{O}(10^{-10}) \text{ for the Earth}$$

However $P/(\rho c^2)$ not negligible for neutron stars



Neutron stars in entangled relativity

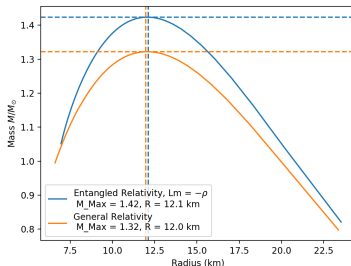
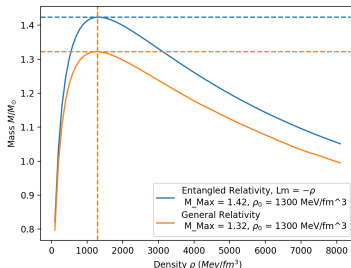




Neutron stars

Always heavier in entangled relativity w.r.t. general relativity

Assuming $P = K\rho^\gamma$, with $\gamma = 5/3$, $K = 1.5 \times 10^{-3} (fm^3/MeV)^{2/3}$.
 And $\mathcal{L}_m = -\rho$. (Otherwise, same result as GR for $\mathcal{L}_m = T$).



Max: 8% more massive Arruga et al. [2021]

No free theoretical parameter!



Neutron stars

Birkoff's theorem no longer valid

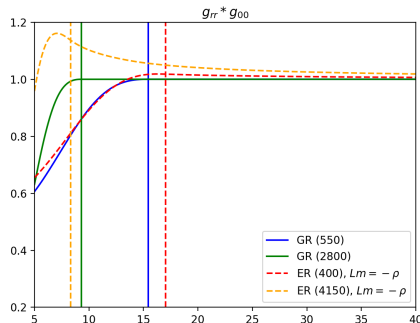


Figure: $g_{00}g_{rr}$ of the two solutions of an object of mass $M = 1.25M_{\odot}$ in general relativity and entangled relativity. The vertical: radius of the solutions. (Central density for each solution, in MeV/fm^3). Arruga et al. [2021]



Neutron stars

Toward experimental tests

Comparison with X-ray pulse profiles

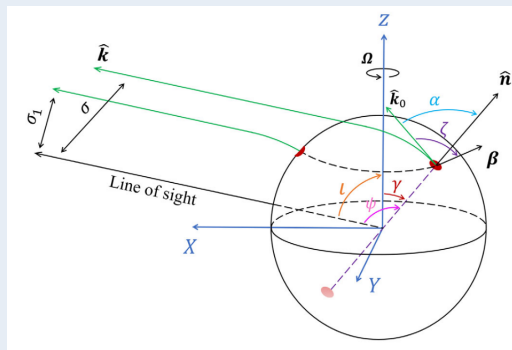
Observation of the x-ray pulse profile emitted by hot spots on the surface of neutron stars offers a way to probe the mass, the radius as well as the spacetime generated by these stars.

Silva and Yunes [2019]

NASA's NICER (Neutron star Interior Composition Explorer) mission should allow such studies. Gendreau and Arzoumanian [2017]

Toward experimental tests

Schematic illustration for the x-rays emitted from a hot spot on a rotating NS and reaching the observer at infinity

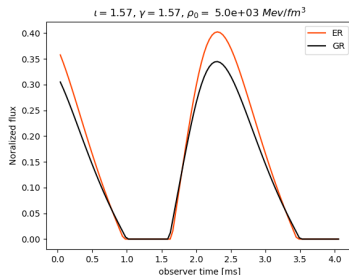
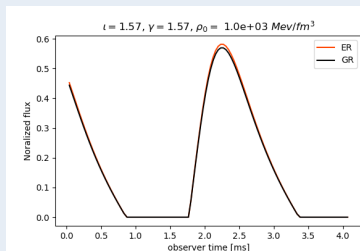


Xu et al. [2020]



Neutron stars: X-ray pulse profiles

Preview: unpublished results of Denis Arruga

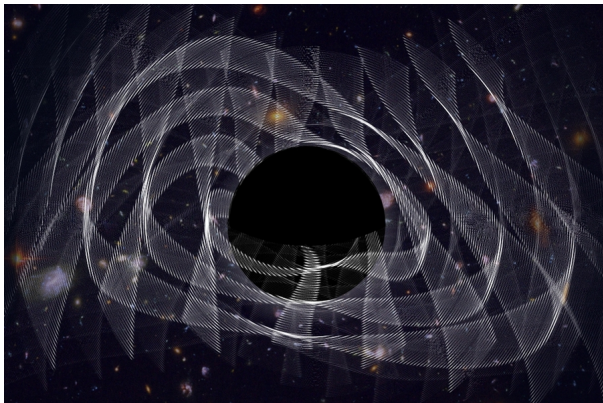


Takes into account the non-conservation of the *photon number* close to the neutron star, which follows from $\nabla_\nu (\phi F^{\mu\nu}) = 0$. Same assumptions as in Silva and Yunes [2019] otherwise.



Black holes in entangled relativity

Unlike in general relativity: cannot be vacuum solutions





Black holes

Charged black hole: special case of string dilaton charged black hole

With conformal transformation $\tilde{g}_{\alpha\beta} = e^{-2\varphi/\sqrt{3}} g_{\alpha\beta}$, and $\phi = e^{-\varphi/\sqrt{3}}$, one recovers an usual dilaton theory for which charged black hole solutions have been found during the first superstring revolution. Garfinkle et al. [1991], Gibbons and Maeda [1988]

$$S \propto \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\tilde{\kappa}} \left(\tilde{R} - 2\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) - e^{-\varphi/\sqrt{3}} \tilde{F}^2 \right] \quad (36)$$

One only needs to make a conformal transformation back to the original frame.



Charged black hole: Minazzoli and Santos [2021]

$$ds^2 = -\lambda_0^2 dt^2 + \lambda_r^{-2} dr^2 + \rho^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (37)$$

$$\begin{aligned} \rho^2 &= r^2 \left(1 - \frac{r_-}{r}\right)^{6/13} \\ \lambda_0^2 &= \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)^{15/13} \\ \lambda_r^2 &= \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)^{7/13} \end{aligned} \quad (38)$$

Tends to usual Schwarzschild metric for $r_- \rightarrow 0$ ($F^2 \rightarrow 0$)

$\Rightarrow$ Usual Schwarzschild metric is a good approximation of spherical black holes in entangled relativity for $F^2 \rightarrow 0$



Black holes/Generalization: conjectures

First conjecture

Usual Schwarzschild metric is a good approximation of spherical black holes in entangled relativity for $T_{\mu\nu}(\text{ext}) \rightarrow 0, \forall \mathcal{L}_m \neq 0$.

Second conjecture

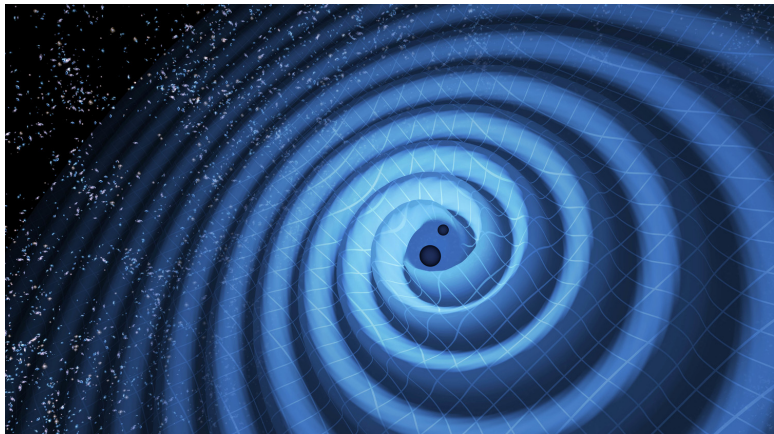
Usual Kerr metric is a good approximation of rotating black holes in entangled relativity for $T_{\mu\nu}(\text{ext}) \rightarrow 0, \forall \mathcal{L}_m \neq 0$.

Minazzoli and Santos [2021]

What happens beyond the event horizon remains to be studied
(Further discussed later on, p. 49)



Gravitational waves





Gravitational waves: from the literature

From numerical studies in Einstein-Maxwell-dilaton theory

$$S \propto \int d^4x \sqrt{-g} \left[R - 2(\nabla\phi)^2 - e^{-2\alpha_0\phi} F^2 \right]. \quad (39)$$

"Finally, an immediate conclusion of our work is that for small charges differences with respect to waveforms in [general relativity] and [Einstein-Maxwell-dilaton theory] are quite small"

Hirschmann et al. [2018]

Confirmed with analytical study Khalil et al. [2018].

Reminder: entangled relativity (with $\tilde{\kappa} := 1$)

$$S \propto \int d^4x \sqrt{-\tilde{g}} \left[\tilde{R} - 2(\tilde{\nabla}\varphi)^2 - e^{-\varphi/\sqrt{3}} \tilde{F}^2 \right] \quad (40)$$



Gravitational waves: from the literature

Binary black holes mergers

From studies for general dilaton theories, it appears that **gravitational waves from the merger of binary black holes in entangled relativity are indistinguishable from the one of general relativity**—at least, provided that the matter part does not correspond to very specific types of dark matter candidates (like dark photons). Khalil et al. [2018]

Binary neutron stars mergers

Studies remain to be done. However, it might be degenerate with the various (unknown) equations of state one can consider to describe nuclear matter inside neutron stars. **Also: critically depends on on-shell matter Lagrangian!**



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Singularities

Repulsive gravity

$$G_{\alpha\beta} = \kappa T_{\alpha\beta} + \kappa^2 [\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square] \kappa^{-2} \quad (41)$$

$$\boxed{\frac{8\pi G_{eff}}{c^4} := \kappa = -\frac{R}{\mathcal{L}_m}} \quad (42)$$

In all situations that are such that $\mathcal{L}_m \approx T$, one recovers GR, and Eq. (42) is simply the trace of the metric field equation in GR

$$\frac{R}{\mathcal{L}_m} > 0 \longrightarrow \text{repulsive gravity !}$$

Conjecture: this might happen at high energy, preventing the formation of singularities (having in mind that $\mathcal{L}_m = K - V$)



Quantum gravity

A new scale (IV. A new hope)

Full action of entangled relativity

$$S = -\frac{\xi}{2c} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R}$$

$$[\xi] = [\kappa] \quad \text{but} \quad \xi \neq \kappa ! \quad \mathcal{Z} = \int [\mathcal{D}\Phi(x)] e^{i\mathcal{S}(\Phi)/\hbar}$$

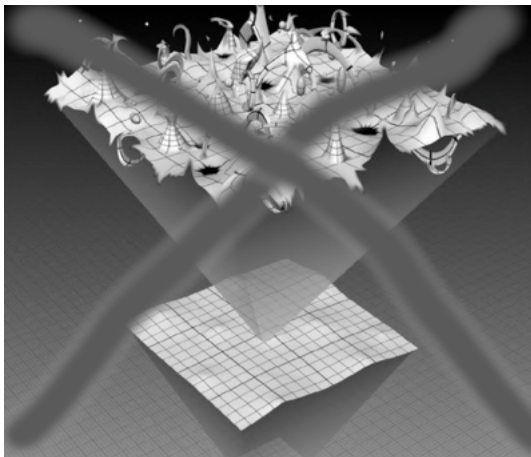
Entangled relativity has a new **purely quantum fundamental constant of nature** (which is not related to the size of black holes)

Consequences yet to be explored!



Quantum gravity

A new scale (IV. A new hope)



Spacetime might still be well-behaved in the deep UV of matter



Quantum gravity

A new scale (IV. A new hope)

$$\exp \left[-i \frac{\xi}{2c\alpha} \int \sqrt{-g} d^4x \frac{\mathcal{L}_m^2}{R} \right] = \exp \left[\frac{i}{c\alpha} \int \sqrt{-g} d^4x \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \right] \quad (43)$$

Where α is the elementary measure of a quantum action (usually thought to be $\hbar$).

At late times, one has $\kappa \sim \text{constant}$, such that one can effectively take it out the integral to describe “late times low-energy” physics:

$$\sim \exp \left[\frac{i}{c\hbar} \int \sqrt{-g} d^4x \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \right], \quad (44)$$

with

$$\hbar := \frac{\kappa\alpha}{\xi} \quad (45)$$



Quantum gravity

A new scale (IV. A new hope)

$$\exp \left[-i \frac{\xi}{2c\alpha} \int \sqrt{-g} d^4x \frac{\mathcal{L}_m^2}{R} \right] = \exp \left[\frac{i}{c\alpha} \int \sqrt{-g} d^4x \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \right] \quad (46)$$

Where α is the elementary measure of a quantum action (usually thought to be $\hbar$).

At late times, one has $\kappa \sim \text{constant}$, such that one can effectively take it out the integral to describe “late times low-energy” physics:

$$\sim \exp \left[\frac{i}{c\hbar} \int \sqrt{-g} d^4x \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \right], \quad (47)$$

with

$$\frac{c\hbar}{\kappa} := \frac{c\alpha}{\xi} = E_{\text{Planck}}^2 \quad (48)$$



Conclusion

$$\boxed{\text{GR:}} \quad \mathcal{L} = \frac{R}{2\kappa} + \mathcal{L}_m \quad \longrightarrow \quad \mathcal{L} = -\frac{\xi}{2} \frac{\mathcal{L}_m^2}{R} \quad \boxed{\text{:ER}}$$

- Seems to satisfy the 3 principles demanded by Einstein in 1918.
- Possesses general relativity as limiting case whenever $\mathcal{L}_m \approx T$ (like for dust or EM-radiation (or trace anomalies?)).
- No free parameter at the classical level.
- May lead to small deviations that could be searched for (e.g. with neutron stars, and/or with magnetic fields).
- Might have the potential to solve the issue of singularities.
- May have the potential to save fate of spacetime as a fundamental structure of nature.
- Might be a description of a consistent framework for matter and gravity.



Many (difficult) things remain to be studied

- Remains widely unknown when $\mathcal{L}_m$ is very different from T .
- How to explain the acceleration of the expansion of the universe? tentative solution given in Minazzoli [2021].
- Detailed cosmological & astrophysical studies are needed.
- **On-shell Lagrangian for realistic fluids to be derived from first principles!**
- What does the theory predict inside black holes, or near the big bang?
- Stability of universal multiplicative coupling at the quantum level (necessary to satisfy the universality of free fall).
- More generally: quantum field theory behavior of the theory.
- Related: **what is $\mathcal{L}_m$ in the UV?** (Standard model of particles built upon the flat spacetime approximation $\rightarrow$ new ideas are likely required here due to pure nonlinear coupling).

Thank you!



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Appendix

Actual metric field equations $\forall \mathcal{L}_m$:

$$\frac{\mathcal{L}_m^2}{R^2} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = -\frac{\mathcal{L}_m}{R} T_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \frac{\mathcal{L}_m^2}{R^2} \quad (49)$$

If $\mathcal{L}_m \neq 0$ it reduces to:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{R}{\mathcal{L}_m} T_{\mu\nu} + \frac{R^2}{\mathcal{L}_m^2} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \frac{\mathcal{L}_m^2}{R^2} \quad (50)$$



Appendix

3 Principles of General relativity:

- Principle of relativity $\rightarrow$ covariant equations.
- Principle of equivalence $\rightarrow g_{\mu\nu}$ determines the metric property of space, the inertial behavior of bodies and the gravitational effects.
- Mach's Principle $\rightarrow$ no vacuum solution.



$$ds^2 = - \left(1 - \frac{2m}{\beta r}\right)^\beta dt^2 + \left(1 - \frac{2m}{\beta r}\right)^{-\beta} dr^2 + r^2 \left(1 - \frac{2m}{\beta r}\right)^{1-\beta} d\Omega^2 \quad (51)$$

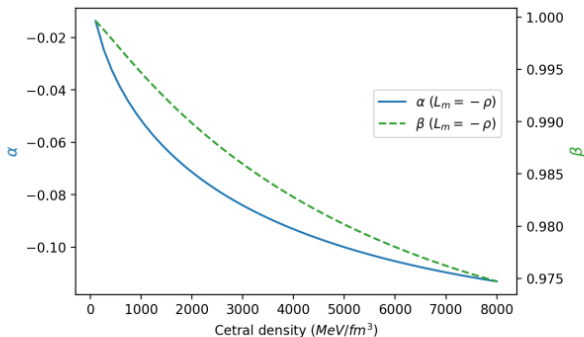


FIG. 3: Values of the parameters α and β in Eq. (23-26) with respect to the density of the compact



Neutron stars

Toward experimental tests

