Second-order stochastic theory for self-interacting scalar fields in de Sitter spacetime

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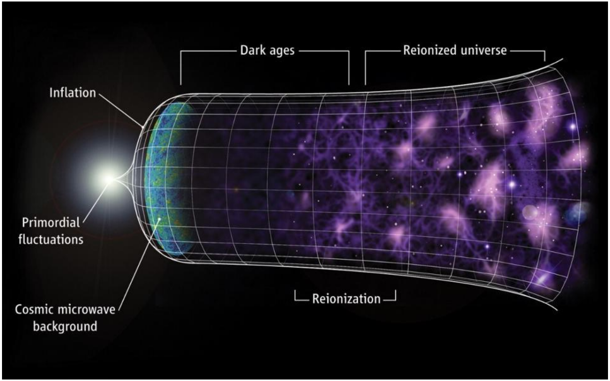
arXiv:2209.02545

Inflation

The period of accelerated expansion in the early Universe before structure was formed.



Why study inflation?

- To better understand the early Universe
- To constrain physical parameters

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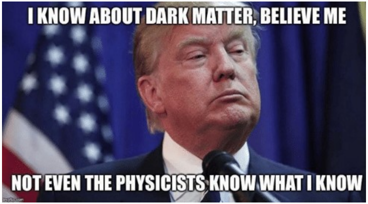
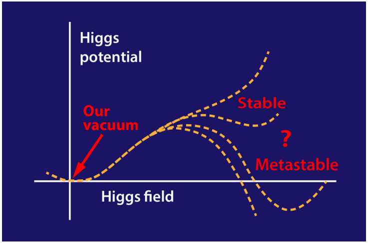
- To constrain physical parameters
- Examples:
  - dark matter
  - curvature/isocurvature perturbations
  - primordial black holes
  - EW vacuum decay



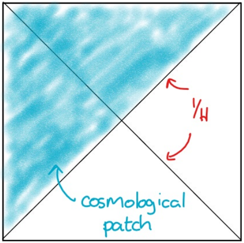


Cosmological de Sitter spacetime

Metric:

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2 d\Omega_2^2) \qquad ; \qquad a(t) = e^{Ht}$$

- Horizon at $R_H = 1/H$.
- Subhorizon: scales $< 1/H$
  Superhorizon: scales $> 1/H$



Spectator scalar field in de Sitter

- Action:

$$S[\phi] = \int d^4x a(t)^3 \left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2a(t)^2}(\nabla\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 \right]$$

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- Equation of motion:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\pi} \end{pmatrix} = \begin{pmatrix} \pi \\ -3H\pi + \frac{1}{a(t)^2}\nabla^2\phi - m^2\phi - \lambda\phi^3 \end{pmatrix}$$

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Long distance behaviour of scalar fields during inflation

Why?

- Modes are amplified by the spacetime expansion, causing them to exit the de Sitter horizon
- These are "frozen"
- Later (today!), they re-enter the de Sitter horizon

The Feynman propagator

The object of interest is

$$i\Delta_F(t,\mathbf{x};t',\mathbf{x}') = \langle 0_{BD}|\,\hat{T}\hat{\phi}(t,\mathbf{x})\hat{\phi}(t',\mathbf{x}')\,|0_{BD}\rangle$$

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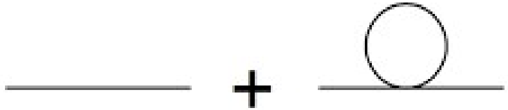
Perturbative QFT can be used to compute this...

... but results contain infrared (IR) divergences that cannot be renormalised with current techniques

Feynman propagator to one-loop order

Feynman propagator to one-loop order in ϕ^4 theory is given by



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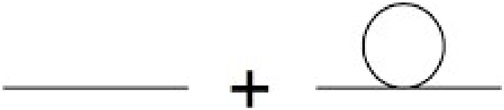

The tadpole diagram contains both infrared and ultraviolet (UV) divergences.

To deal with the UV divergences, we renormalise the mass $m \longrightarrow m_R$.

Feynman propagator to $\mathcal{O}(\lambda H^4/m^4)$

The UV-finite part of the Feynman propagator to one-loop is

$$i\Delta_F(t,\mathbf{0};t,\mathbf{x}) = \left(\frac{H^2}{16\pi^2} \frac{\Gamma(\frac{3}{2}-\nu_R)\Gamma(2\nu_R)4^{\frac{3}{2}-\nu_R}}{\Gamma(\frac{1}{2}+\nu_R)} - \frac{27\lambda H^8}{64\pi^4 m_R^6} + \mathcal{O}\left(\frac{\lambda H^6}{m_R^4}\right)\right) \times |Ha(t)\mathbf{x}|^{-2\left(\frac{3}{2}-\nu_R+\frac{3\lambda H^2}{8\pi^2 m_R^2}+\mathcal{O}(\lambda)\right)}$$

$$\nu_R = \sqrt{\frac{9}{4}-\frac{m_R^2}{H^2}}.$$

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IR divergent unless $\lambda \ll m^4/H^4$

UV renormalised (scale-dependent)

The stochastic approach

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Quantum modes can be approximated by a statistical noise contribution to the classical equations of motion.

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Quantum modes can be approximated by a statistical noise contribution to the classical equations of motion.

We expect it to work if fields are sufficiently light $m \lesssim H$ such that long wavelength modes are stretched by spacetime expansion.

The overdamped (OD) stochastic approach

- One can derive stochastic equations by introducing a strict cut-off between sub and superhorizon modes, $\hat{\phi} = \phi_L + \hat{\phi}_S$ where

$$\hat{\phi}_S = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\hat{\phi}_k(t)\theta(k - \epsilon a(t)H).$$

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- In the limits $m \ll H$ and $\lambda \ll m^2/H^2$, the stochastic equations are

$$0 = 3H\dot{\phi_L} + m^2\phi_L + 3\lambda\phi_L^2 - \xi_{OD}(t, \mathbf{x})$$

where $\langle \xi(t, \mathbf{x})\xi(t', \mathbf{x})\rangle = \dfrac{9H^5}{4\pi^2}\delta(t - t')$.

OD field correlator

- To one-loop order,

$$\langle\phi(t,\mathbf{0})\phi(t,\mathbf{x})\rangle = \left(\frac{3H^4}{8\pi^2 m^2} - \frac{27\lambda H^8}{64\pi^4 m^6} + \mathcal{O}(\lambda^2)\right)|Ha(t)\mathbf{x}|^{-2\left(\frac{m^2}{3H^2}+\frac{3\lambda H^2}{8\pi^2 m^2}+\mathcal{O}(\lambda^2)\right)}$$

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same as free term in QFT if $m^2 \ll H^2$

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same as $\mathcal{O}\left(\lambda H^4/m^4\right)$ term in QFT

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  (b) Never includes the renormalisation scale-dependent terms.

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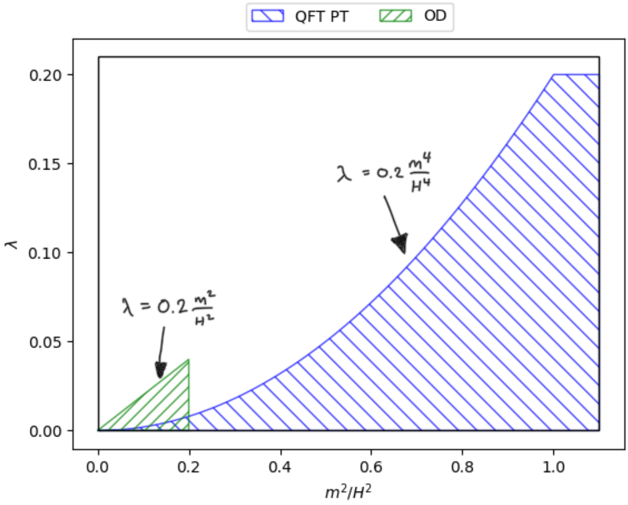
- N.B. Non-perturbative methods are available to compute the OD field correlator [*arXiv:1904.11917*]

The state of play



A second-order stochastic effective theory

Make the ansatz

$$\begin{pmatrix} \dot{\phi} \\ \dot{\pi} \end{pmatrix} = \begin{pmatrix} \pi \\ -3H\pi - m^2\phi - \lambda\phi^3 \end{pmatrix} + \begin{pmatrix} \xi_\phi(t, \mathbf{x}) \\ \xi_\pi(t, \mathbf{x}) \end{pmatrix}$$

with a stochastic white noise contribution

$$\left\langle \xi_i(t, \mathbf{x})\xi_j(t', \mathbf{x}) \right\rangle = \sigma_{ij}^2 \delta(t - t').$$

where $i, j \in \{\phi, \pi\}$.

The Fokker-Planck equation

The time evolution of the probability distribution function (PDF) $P(\phi, \pi; t)$ is given by the Fokker-Planck equation associated with the Langevin equation

$$\begin{aligned}\partial_t P(\phi, \pi; t) =& \Big[3H - \pi\partial_\phi + \left(3H\pi + m^2\phi + \lambda\phi^3\right)\partial_\pi \\ & + \frac{1}{2}\sigma^2_{\phi\phi}\partial^2_\phi + \sigma^2_{\phi\pi}\partial_\phi\partial_\pi + \frac{1}{2}\sigma^2_{\pi\pi}\partial^2_\pi \Big] P(\phi, \pi; t) \\ & = \mathcal{L}_{FP} P(\phi, \pi; t).\end{aligned}$$

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6. Solve the Fokker-Planck equation numerically to obtain non-perturbative results

1. The spectral expansion: 1-PDF

Write the 1-PDF in terms of eigenfunctions $\Psi_N(\phi, \pi)$ and eigenvalues Λ_N

$$P(\phi, \pi; t) = \Psi_0^*(\phi, \pi) \sum_N \Psi_N(\phi, \pi) e^{-\Lambda_N t}$$

where the eigenproblem is

$$\mathcal{L}_{FP} \Psi_N(\phi, \pi) = -\Lambda_N \Psi_N(\phi, \pi),$$
$$\mathcal{L}_{FP}^* \Psi_N^*(\phi, \pi) = -\Lambda_N \Psi_N^*(\phi, \pi).$$

$\{\Psi_N(\phi, \pi)\}$ obey biorthogonality and completeness relations.

1. The spectral expansion: correlators

From this basis, one can find an expression for the spacelike correlation function of two functions $f(\phi, \pi)$ and $g(\phi, \pi)$ composed purely of the eigenfunctions and eigenvalues:

$$\begin{aligned}\langle f(\phi, \pi; t, \mathbf{0}) g(\phi, \pi; t, \mathbf{x})\rangle = &\int d\phi_r \int d\pi_r \frac{\Psi_0(\phi_r, \pi_r)}{\Psi_0^*(\phi_r, \pi_r)} \sum_{N' N} \Psi_N^*(\phi_r, \pi_r) \Psi_{N'}^*(\phi_r, \pi_r) \\ &\times \int d\phi_1 \int d\pi_1 \Psi_0^*(\phi_1, \pi_1) \Psi_N(\phi_1, \pi_1) f(\phi_1, \pi_1) \\ &\times \int d\phi_2 \int d\pi_2 \Psi_0^*(\phi_2, \pi_2) \Psi_{N'}(\phi_2, \pi_2) g(\phi_2, \pi_2) \\ &\times |H(a(t)\mathbf{x}|^{-\frac{\Lambda_N + \Lambda_{N'}}{H}}.\end{aligned}$$

2. Free field solutions: field correlator

Using these solutions, we evaluate the free field spacelike stochastic correlator to be

$$\langle\phi(t,\mathbf{0})\phi(t,\mathbf{x})\rangle^{(0)} = \frac{1}{4\nu^2H^3}\left[\frac{1}{2\alpha}\left(\sigma_{\pi\pi}^{2(0)} + 2\beta H\sigma_{\phi\pi}^{2(0)} + \beta^2H^2\sigma_{\phi\phi}^{2(0)}\right)|Ha(t)\mathbf{x}|^{-3+2\nu}\right.$$
$$+ \frac{1}{2\beta}\left(\sigma_{\pi\pi}^{2(0)} + 2\alpha H\sigma_{\phi\pi}^{2(0)} + \alpha^2H^2\sigma_{\phi\phi}^{2(0)}\right)|Ha(t)\mathbf{x}|^{-3-2\nu}$$
$$\left. - \frac{2}{3}\left(\sigma_{\pi\pi}^{2(0)} + 3H\sigma_{\phi\pi}^{2(0)} + m^2\sigma_{\phi\phi}^{2(0)}\right)|Ha(t)\mathbf{x}|^{-3}\right]$$

$\alpha/\beta = \frac{3}{2} +/- \nu$ with $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$.

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$$+\frac{1}{2\beta}\left(\sigma_{\pi\pi}^{2(0)} + 2\alpha H \sigma_{\phi\pi}^{2(0)} + \alpha^2 H^2 \sigma_{\phi\phi}^{2(0)}\right)|Ha(t)\mathbf{x}|^{-3-2\nu}$$
$$-\frac{2}{3}\left(\sigma_{\pi\pi}^{2(0)} + 3H \sigma_{\phi\pi}^{2(0)} + m^2 \sigma_{\phi\phi}^{2(0)}\right)|Ha(t)\mathbf{x}|^{-3}\Bigg]$$

$\alpha/\beta = \frac{3}{2} + / - \nu$ with $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$.

3. Free field noise

To reproduce the free Feynman propagator,

$$\sigma^{2(0)}_{\phi\phi} = \frac{H^3\Gamma(2\nu)\Gamma\left(\frac{5}{2}-\nu\right)}{2\pi^{5/2}},$$

$$\sigma^{2(0)}_{\phi\pi} = -\frac{H^4\alpha\Gamma(2\nu)\Gamma\left(\frac{5}{2}-\nu\right)}{2\pi^{5/2}},$$

$$\sigma^{2(0)}_{\pi\pi} = \frac{H^5\alpha^2\Gamma(2\nu)\Gamma\left(\frac{5}{2}-\nu\right)}{2\pi^{5/2}}.$$

4. The perturbative calculation

We now perform a perturbative expansion around the free eigenspectrum i.e.

$$\begin{aligned}\Lambda_N =&\Lambda_N^{(0)} + \lambda\Lambda_N^{(1)}\\ \Psi_N^{(*)}(\phi,\pi) =&\Psi_N^{(0)(*)}(\phi,\pi) + \lambda\Psi_N^{(1)(*)}(\phi,\pi)\end{aligned}$$

Using standard perturbative techniques, these are written as

$$\begin{aligned}\Lambda_N^{(1)} &= -\int d\phi \int d\pi \Psi_N^{(0)*}(\phi,\pi)\mathcal{L}_{FP}^{(1)}\Psi_N^{(0)}(\phi,\pi)\\ \Psi_N^{(1)}(\phi,\pi) &= \sum_{N'} \Psi_{N'}^{(0)}(\phi,\pi)\frac{\int d\phi' \int d\pi' \Psi_{N'}^{(0)*}(\phi',\pi')\mathcal{L}_{FP}^{(1)}\Psi_N^{(0)}(\phi',\pi')}{\Lambda_{N'}^{(0)} - \Lambda_N^{(0)}}.\end{aligned}$$

4. The perturbative calculation: $\mathcal{O}(\lambda)$ field correlators

To $\mathcal{O}(\lambda)$,

$$\begin{aligned}
&\langle\phi(t,\mathbf{0})\phi(t,\mathbf{x})\rangle \\
&\quad=\left[\frac{H^2}{16\pi^2}\frac{\Gamma\left(\frac{3}{2}-\nu\right)\Gamma(2\nu)4^{\frac{3}{2}-\nu}}{\Gamma\left(\frac{1}{2}+\nu\right)}+\frac{3\lambda(3-4\nu)H^4\Gamma(\nu)^2\Gamma\left(\frac{3}{2}-\nu\right)^2}{32\pi^5\nu m^2}+\mathcal{O}\left(\lambda^2\right)\right] \\
&\qquad\times|Ha(t)\mathbf{x}|^{-3-2\nu-\frac{3\lambda\Gamma(\nu)\Gamma\left(\frac{3}{2}-\nu\right)}{8\pi^{5/2}\nu}+\mathcal{O}\left(\lambda^2\right)} \\
&\qquad-\left(\frac{\lambda H^4\Gamma(\nu)^2\Gamma\left(\frac{5}{2}-\nu\right)^2}{8\pi^5\nu m^2}+\mathcal{O}(\lambda)\right)|Ha(t)\mathbf{x}|^{-3+\mathcal{O}(\lambda^2)}.
\end{aligned}$$

4. The perturbative calculation: $\mathcal{O}\big(\lambda H^4/m^4\big)$ field correlators

To $\mathcal{O}\left(\dfrac{\lambda H^4}{m^4}\right)$,

$$\langle\phi(t,\mathbf{0})\phi(t,\mathbf{x})\rangle = \left[\frac{H^2}{16\pi^2}\frac{\Gamma\big(\frac{3}{2}-\nu\big)\Gamma(2\nu)4^{\frac{3}{2}-\nu}}{\Gamma\big(\frac{1}{2}+\nu\big)} - \frac{27\lambda H^8}{64\pi^4 m^6} + \mathcal{O}\left(\frac{\lambda H^6}{m^4}\right)\right] \times |Ha(t)\mathbf{x}|^{-3-2\nu+\frac{3\lambda H^2}{8\pi^2 m^2}+\mathcal{O}(\lambda)}.$$

This has the same form as the Feynman propagator to $\mathcal{O}\left(\dfrac{\lambda H^4}{m^4}\right)$!

5. Stochastic parameters to $\mathcal{O}\left(\lambda H^4/m^4\right)$

Assuming λ is the same in both QFT and stochastic theory,

$$m^2 = m_R^2\left(1+\mathcal{O}\left(\frac{\lambda H^2}{m^2}\right)\right)$$

$$\sigma^2 = \frac{H^3\Gamma(2\nu)\Gamma\left(\frac{5}{2}-\nu\right)}{2\pi^{5/2}}\begin{pmatrix} 1 & -\frac{2m^2}{H(3+2\nu)} \\ -\frac{2m^2}{H(3+2\nu)} & \frac{4m^4}{(3+2\nu)^2H^2}\end{pmatrix}+\mathcal{O}\left(\frac{\lambda H^2}{m^2}\right).$$

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Crucially, they don't have an IR divergent part $\mathcal{O}\left(\lambda H^4/m^4\right)$

6. The numerical calculation

- To solve numerically, we expand about free eigenstates $\Psi_N(\phi,\pi) = \sum_R c_R^{(N)} \psi_R^{(0)}(\phi,\pi)$ such that the eigenequation becomes

$$\sum_R c_R^{(N)} \mathcal{L}_{FP} \psi_R^{(0)}(\phi,\pi) = \sum_{RR'} c_R^{(N)} \mathcal{M}_{RR'} \psi_{R'}^{(0)} = -\sum_R c_R^{(N)} \Lambda_N \psi_R^{(0)}(\phi,\pi)$$

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- Thus we have the matrix equation

$$\sum_{R'} \mathcal{M}_{RR'}^T c_{R'}^{(N)} = -\Lambda_N c_R^{(N)}$$

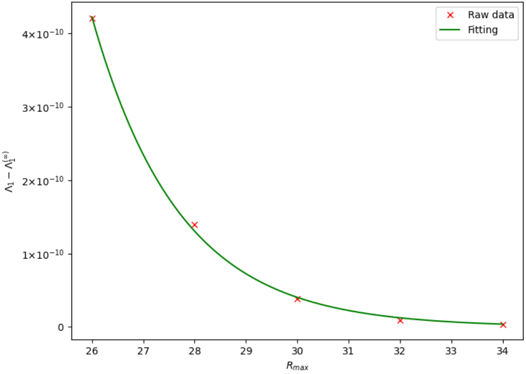
such that $\mathcal{M}^T$ can be diagonalised to find $c_R^{(N)}$ and Λ_N.

6. The numerical calculation: convergence of solutions

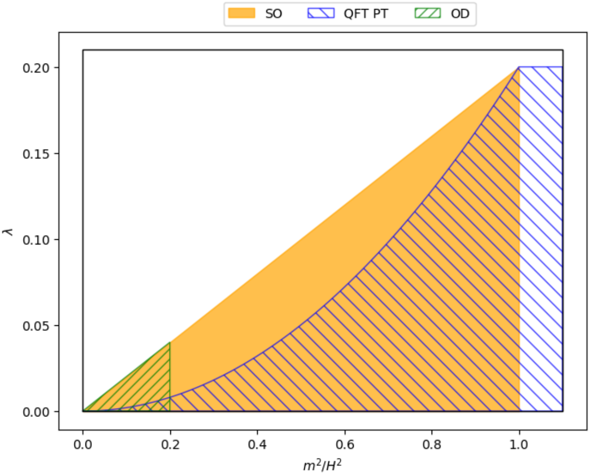
Solutions converge quickly therefore we can truncate our sum

$$\frac{m^2}{H^2} = 0.01$$
$$\lambda = 0.0005$$

$$\implies \Lambda_1^{(\infty)} = 0.00453$$



New regime of validity plot



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Thanks for listening! ☺