Rotating black hole analogues in quantum fluids

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Outline

- Analogue models
- Quantum fluids
- Draining vortex flow
- Experimental status
- Theoretical progress: vortex decay
- Moving forward

Quantum Simulators for Fundamental Physics

QSimFP

Black hole analogues in **superfluid ^4He & fibre-optics**

False vacuum decay using 2-component **Bose-Einstein condensates**

University of Nottingham UK | CHINA | MALAYSIA

Royal Holloway University of London

University of St Andrews

University of Cambridge

UCL

Perimeter Institute

University of Waterloo

King's College London

The University of British Columbia

TU Wien Technische Universität Wien

HZDR Helmholtz Zentrum Dresden Rossendorf

Lancaster University

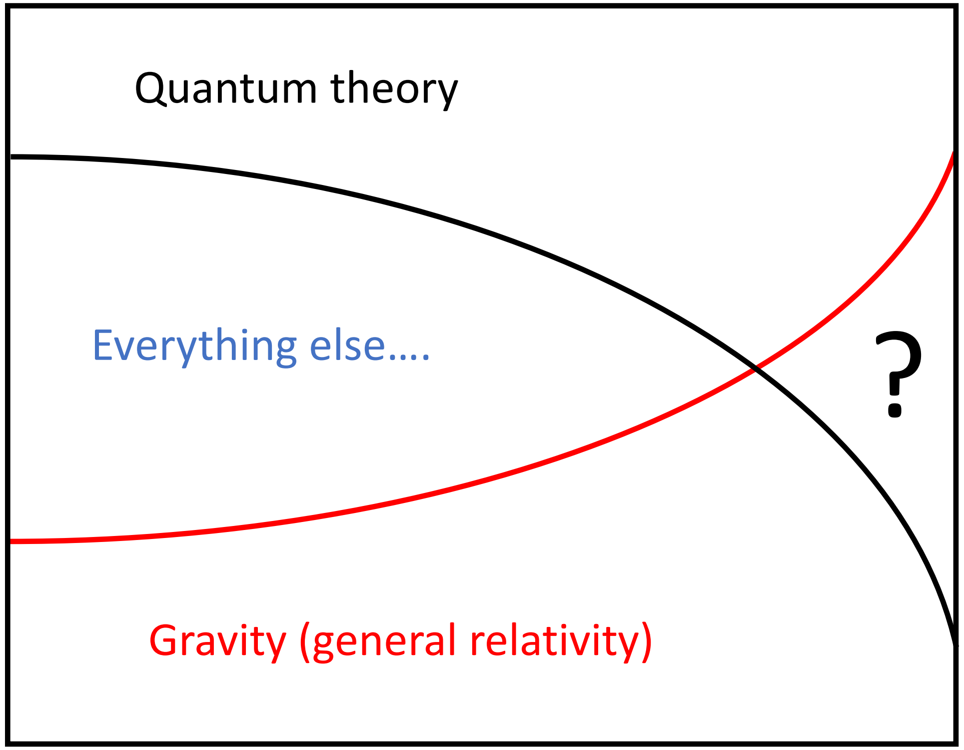
CITA ICAT

Newcastle University

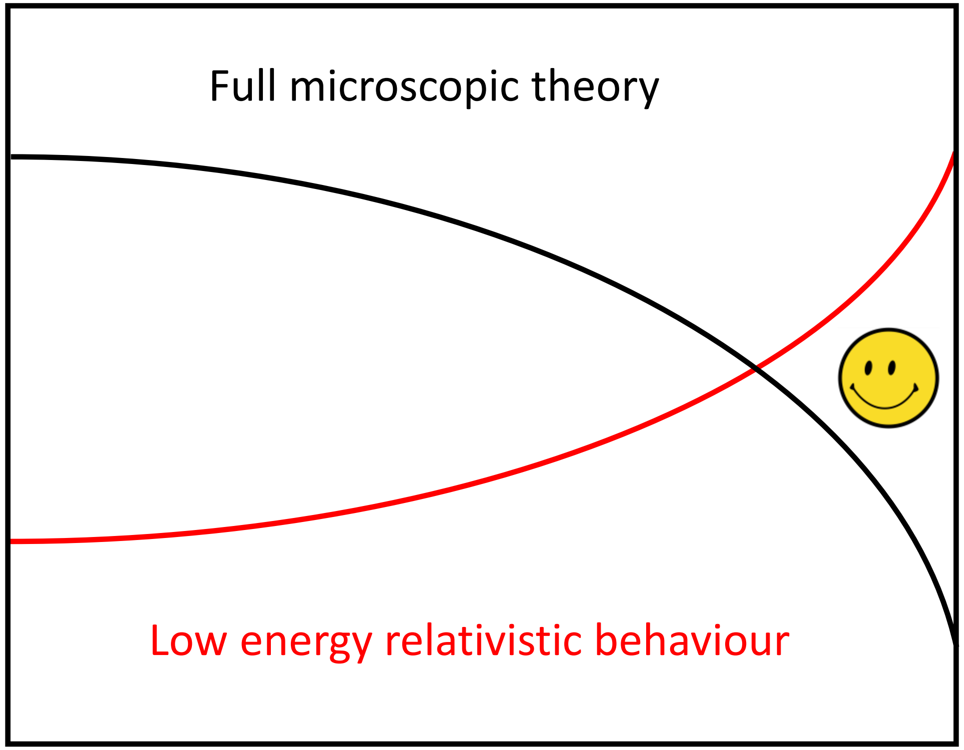
Analogue gravity

- Use non-gravitational systems to mimic features of gravitational ones

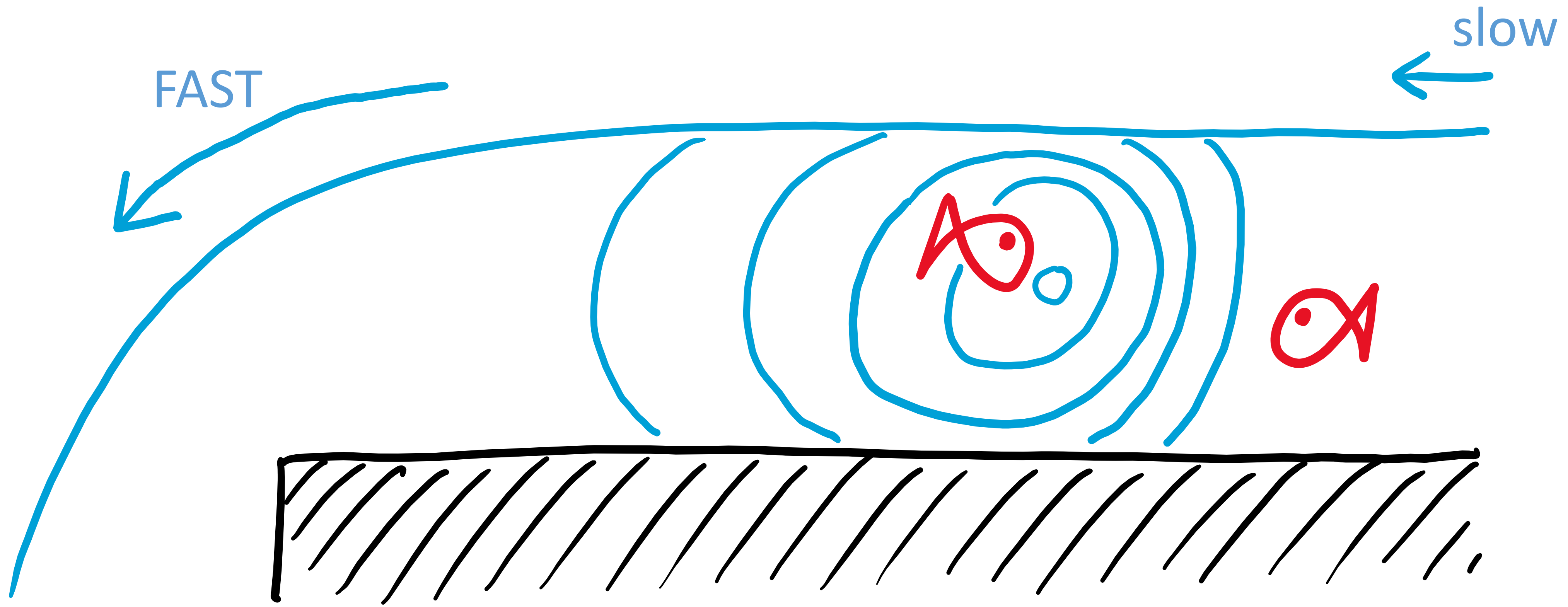
All of nature…



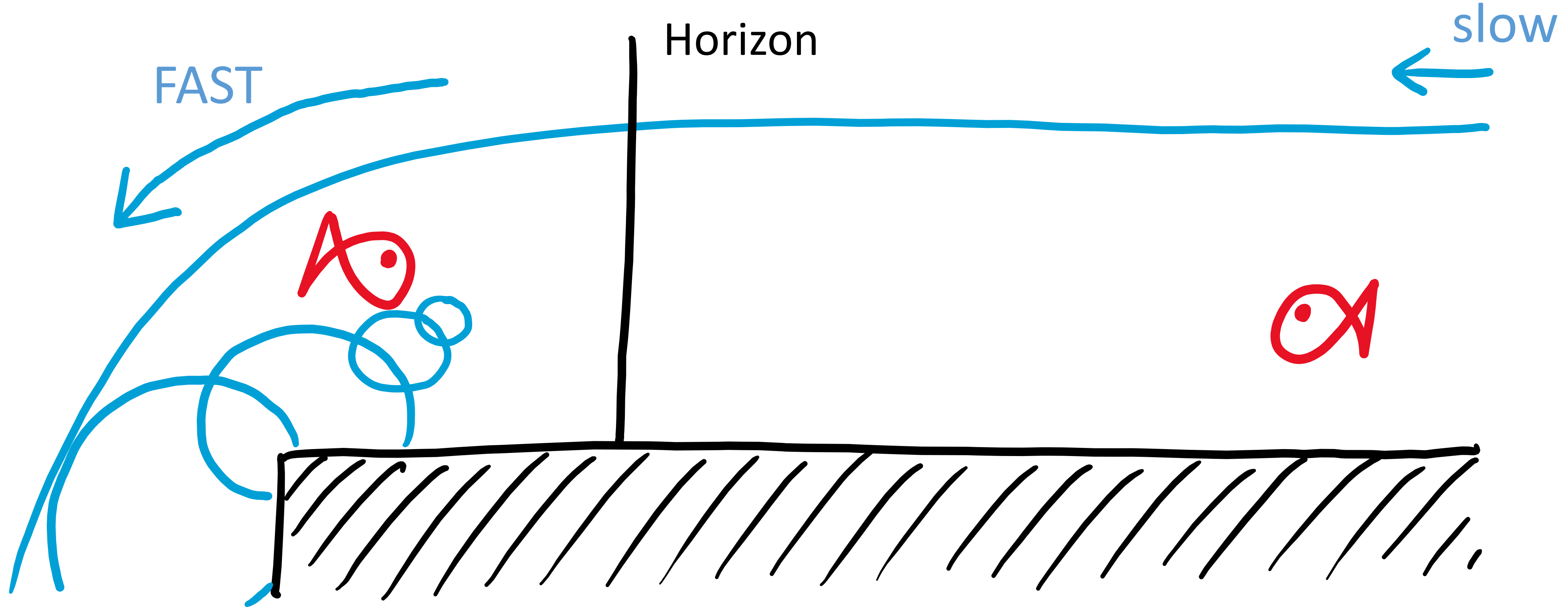
Analogue system



Classical example



Classical example



Other systems

- Surface waves/sound waves in a flowing fluid
- Electromagnetic waves in fibre-optics
- Quantum fluids (Bose-Einstein condensates, superfluid ^4He)
- Photon fluids in non-linear optics
- Exciton-polariton condensates
- Superconducting, microwave circuits with Josephson junctions
- Vibrational modes in ion traps

[Living Rev. Relativ. 14, 1 (2011), Phil. Trans. R. Soc. A **378**: 20190239 (2020)]

Analogue spacetimes in Bose-Einstein condensates

- System of bosons will condense into the ground state at $T = 0$K

Gross-Pitaevskii equation (GPE): $$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2M}\nabla^2\Psi + U(\mathbf{x})\Psi + g|\Psi|^2\Psi$$

Madelung representation

$$\Psi = \sqrt{n(\mathbf{x},t)}\, e^{\frac{i}{\hbar}(M\Phi(\mathbf{x},t)-\mu t)}$$

$\mathbf{v} = \boldsymbol{\nabla}\Phi$ Fluid velocity

Continuity of number density

$$\partial_t n + \boldsymbol{\nabla}\cdot(n\mathbf{v}) = 0$$

Healing length $\xi = \frac{\hbar}{\sqrt{M\mu}}$

Bernoulli equation ~~+ "quantum pressure"~~

$$\partial_t\Phi + \frac{1}{2}\mathbf{v}^2 + \frac{gn+U-\mu}{M} - \cancel{\frac{\hbar^2}{2M^2}\frac{\nabla^2\sqrt{n}}{\sqrt{n}}} = 0$$

if $\frac{|\boldsymbol{\nabla}n|}{n} \sim L^{-1} \ll \xi^{-1}$

Perturb the fluid variables

$$n \to n + \delta n, \qquad \Phi \to \Phi + \delta\phi$$

$$(\partial_t + \boldsymbol{\nabla} \cdot \mathbf{v})\delta n + \boldsymbol{\nabla} \cdot (n \boldsymbol{\nabla} \delta\phi) = 0$$

$$(\partial_t + \mathbf{v} \cdot \boldsymbol{\nabla})\delta\phi + \frac{g}{m}\delta n = 0$$

Wave equation

$$(\partial_t + \boldsymbol{\nabla} \cdot \mathbf{v})(\partial_t + \mathbf{v} \cdot \boldsymbol{\nabla})\delta\phi - \boldsymbol{\nabla} \cdot (c^2 \boldsymbol{\nabla} \delta\phi) = 0$$

Sound speed

$$c^2 = \frac{gn}{m}$$

Klein-Gordon equation

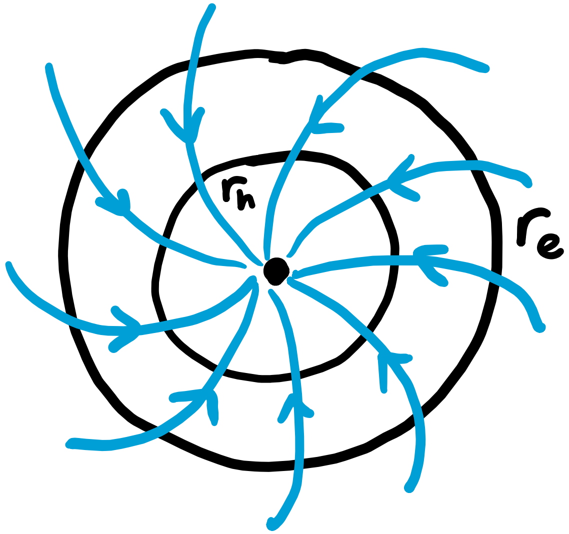
$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\delta\phi) = 0$$

Effective metric

$$g_{\mu\nu} = \frac{n}{mc}\begin{pmatrix} -c^2 + \mathbf{v}^2 & -v_j \\ -v_i & \delta_{ij} \end{pmatrix}$$

	Gravity	Analogue
Linear	$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\delta\phi) = 0$	$(\partial_t + \boldsymbol{\nabla}\cdot\mathbf{v})(\partial_t + \mathbf{v}\cdot\boldsymbol{\nabla})\delta\phi - \boldsymbol{\nabla}\cdot(c^2\boldsymbol{\nabla}\delta\phi) = 0$
Classical	$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$	$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2M}\nabla^2\Psi + U(\mathbf{x})\Psi + g\lvert\Psi\rvert^2\Psi$
T.O.E	Quantum gravity?	$i\hbar\partial_t\lvert\psi_{\mathrm{MB}}\rangle = \hat{H}\lvert\psi_{\mathrm{MB}}\rangle$

Draining vortex flow



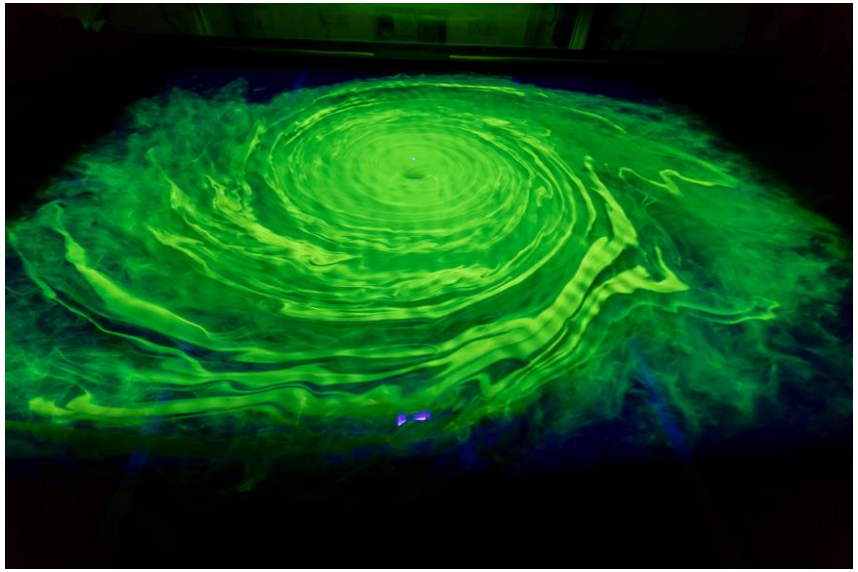
Ergosphere: $||\mathbf{v}(r_e)|| = c$

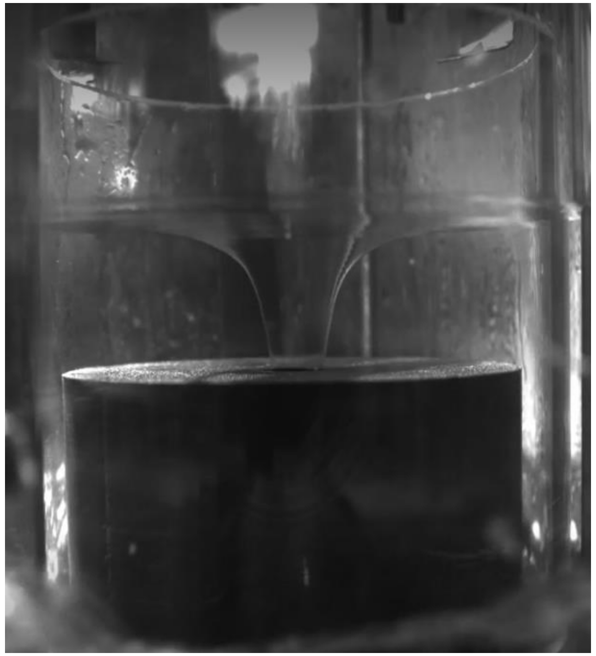
Horizon: $||v_r(r_h)|| = c$

$$\xi \ll r_h \qquad n \simeq \text{const}$$

$$\mathbf{v} = \frac{\hbar}{M}\left(-\frac{d}{r}\hat{\mathbf{e}}_r + \frac{\ell}{r}\hat{\mathbf{e}}_\theta\right)$$

$$r_h = \xi d, \qquad r_e = \xi\sqrt{\ell^2 + d^2}$$





$$\Psi \sim e^{i\ell\theta} \quad \therefore \quad \ell \in \mathbb{Z}$$

Vortex quantisation

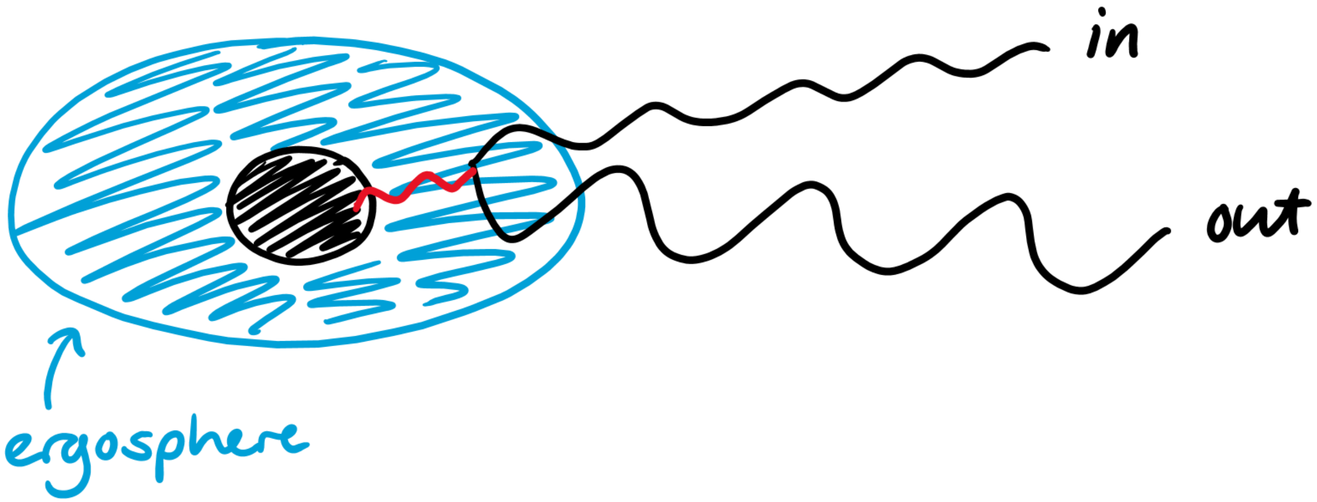
Angular momentum:

$$L = \hbar\ell N$$

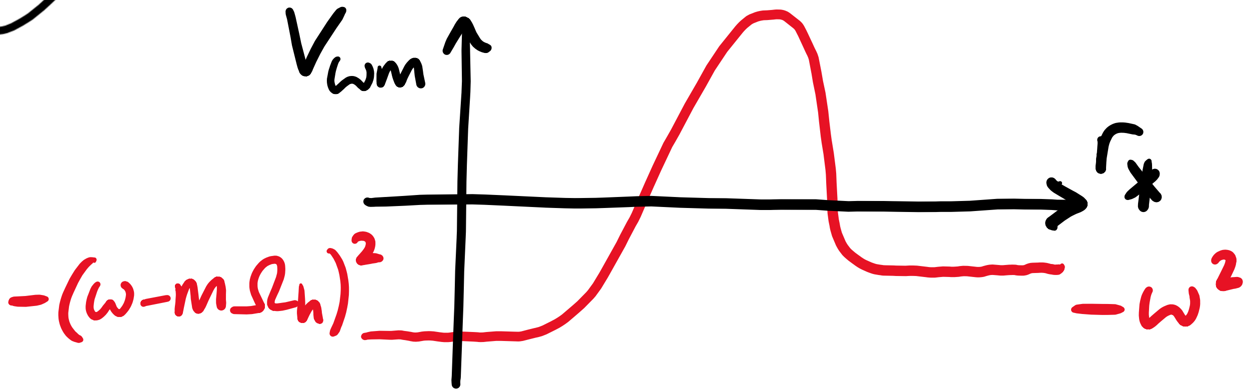
Superradiant scattering

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\delta\phi) = 0$$

$$\delta\phi \sim \phi_r(r)e^{im\theta-i\omega t}$$



$$-\partial_{r_*}^2\phi_r + V_{\omega m}(r)\phi_r = 0$$

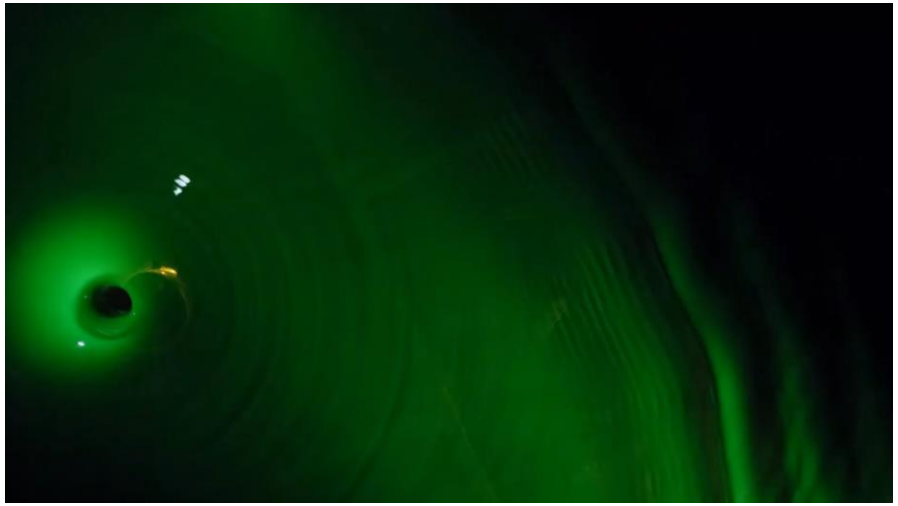


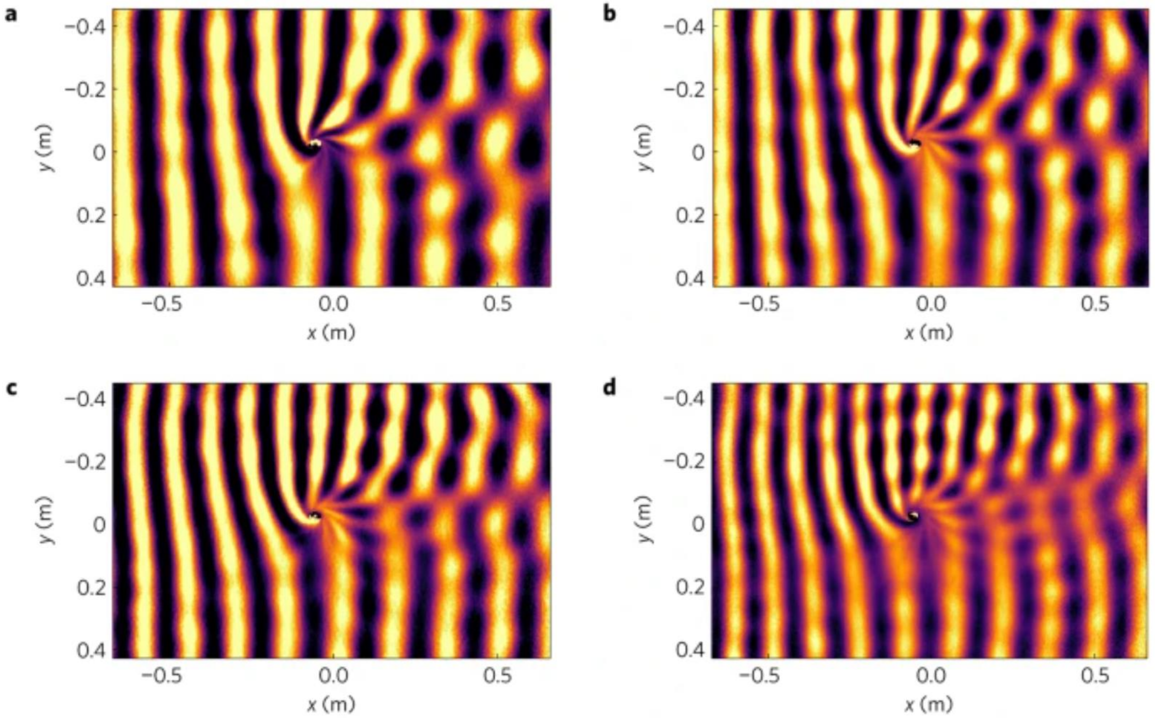
Scattering

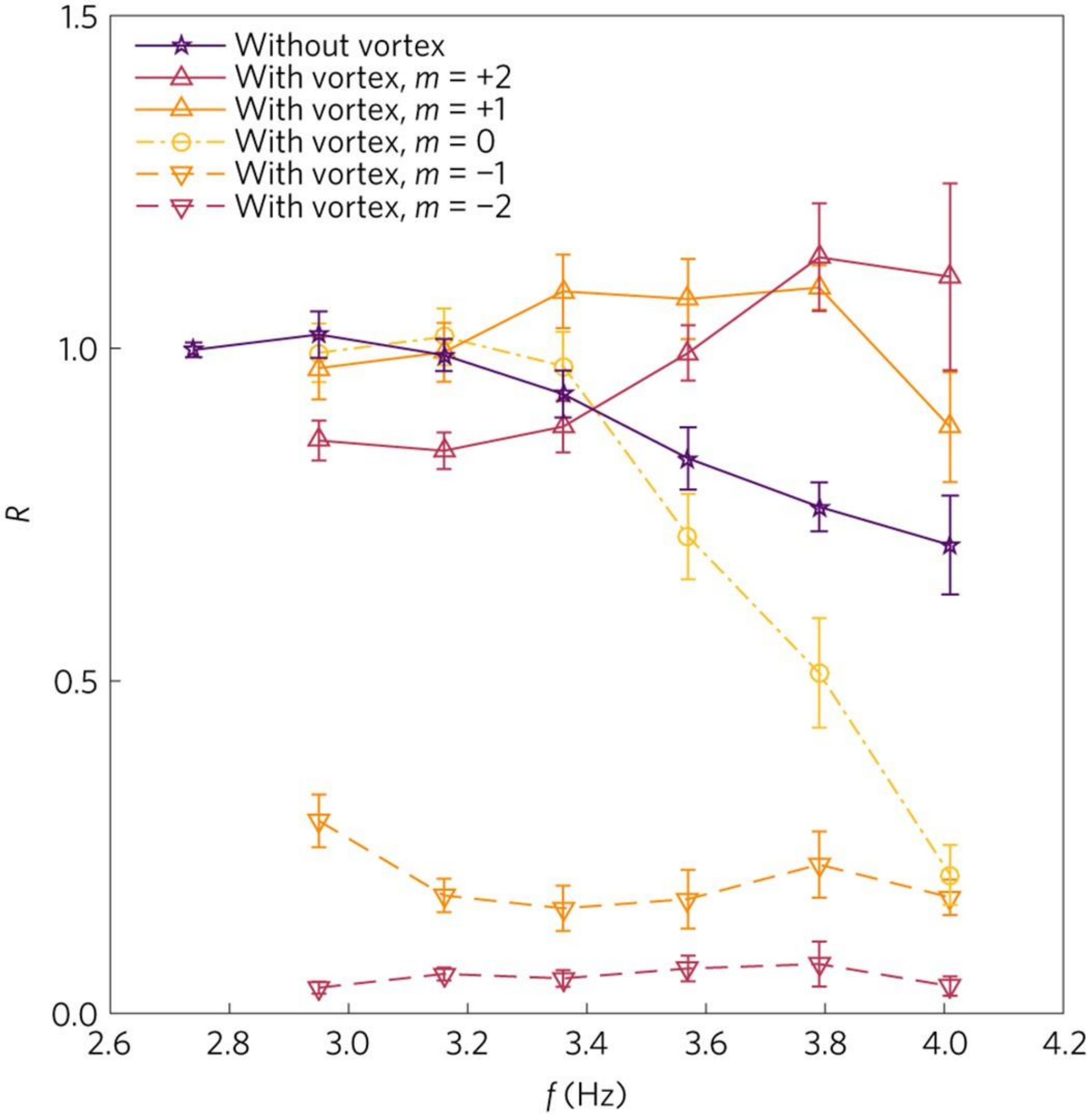
$$\phi_r \sim \begin{cases} e^{-i\omega r_*} + \mathcal{R}e^{i\omega r_*}, & r_* \to +\infty \\ \mathcal{T}e^{-i(\omega-m\Omega_h)r_*}, & r_* \to -\infty \end{cases}$$

Conserved current: $|\mathcal{R}|^2 + \dfrac{\omega - m\Omega_h}{\omega}|\mathcal{T}|^2 = 1$

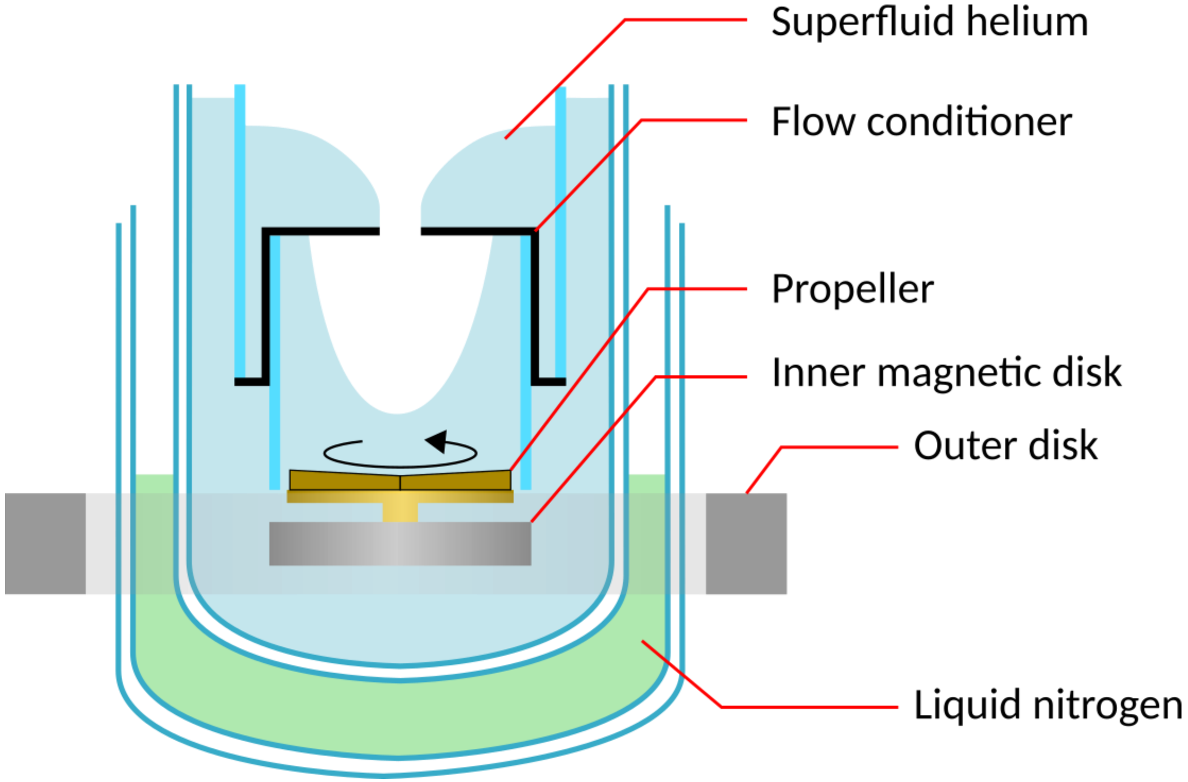
Superradiant amplification for $\omega < m\Omega_h$

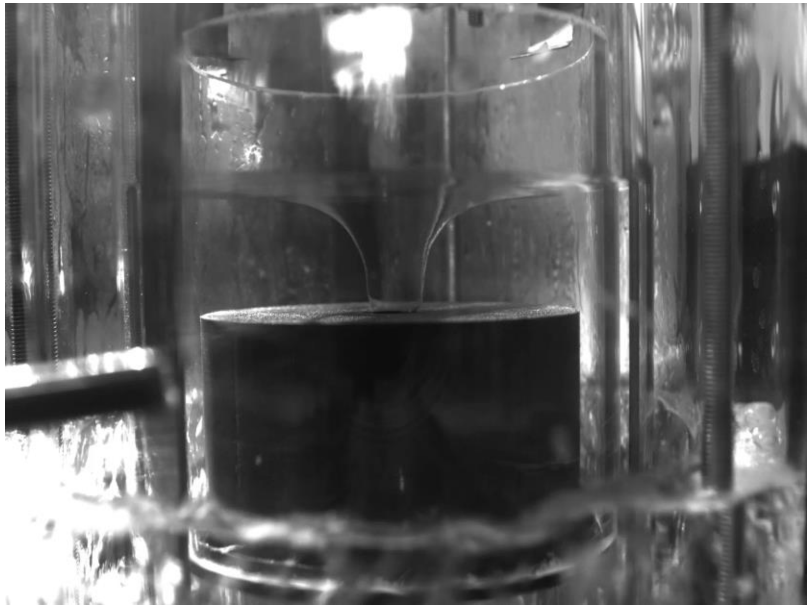


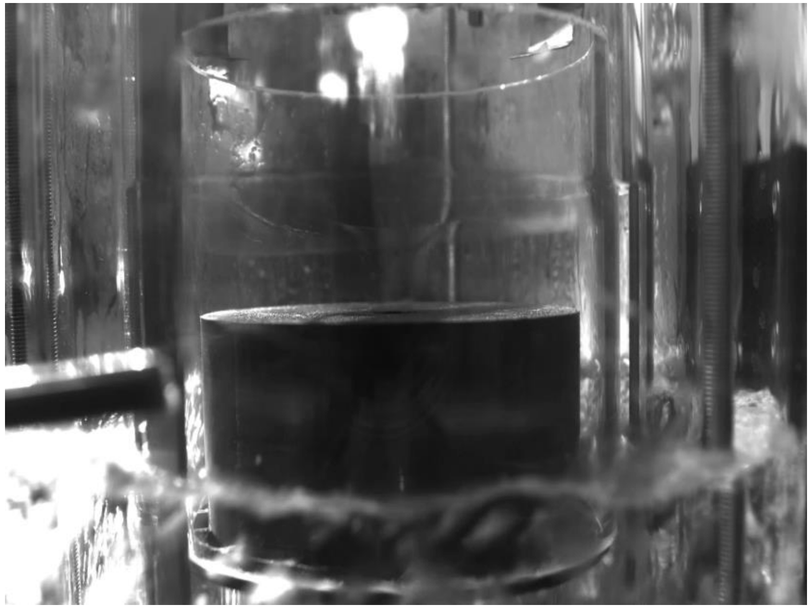

a
b
c
d
y (m)
x (m)
−0.4
−0.2
0
0.2
0.4
−0.5
0.0
0.5


Without vortex
With vortex, m = +2
With vortex, m = +1
With vortex, m = 0
With vortex, m = −1
With vortex, m = −2
R
f (Hz)
0.0
0.5
1.0
1.5
2.6
2.8
3.0
3.2
3.4
3.6
3.8
4.0
4.2

Superfluid ^{4}He draining vortex







Theoretical modelling

Surface waves on a 3D helium flow → Density waves in a 2D BEC

Superfluid ^{4}He models

Landau's 2-fluid model
Vortex filament model
Hall-Vinen-Bekarevich-Khalatnikov (HVBK) equations

Drawbacks

- No wave dynamics
- Neglects details of the vortex core
- No known derivation from first principles

BEC models

Gross-Pitaevskii equation (GPE)
Stochastic-Projected GPE (sPGPE)
Zaremba-Nikuni-Griffin (ZNG) model

Benefits

- Models wave dynamics
- Contains details of vortex core
- Derivation from first principles known
- Captures superfluidity and vortex quantisation

Drawbacks

- Assumes weak interactions
- Models a gas, not a liquid
- Model has no free surface

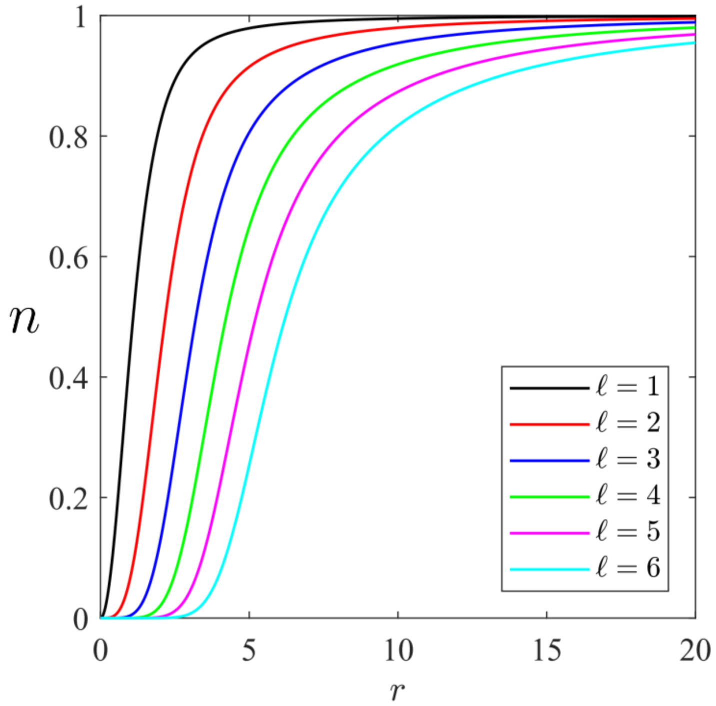
Not so bad! Surface waves on an incompressible, shallow fluid map to density waves.

Non-draining vortex

$$\mathbf{v} = \frac{\hbar}{M}\frac{\ell}{r}\hat{\mathbf{e}}_\theta$$

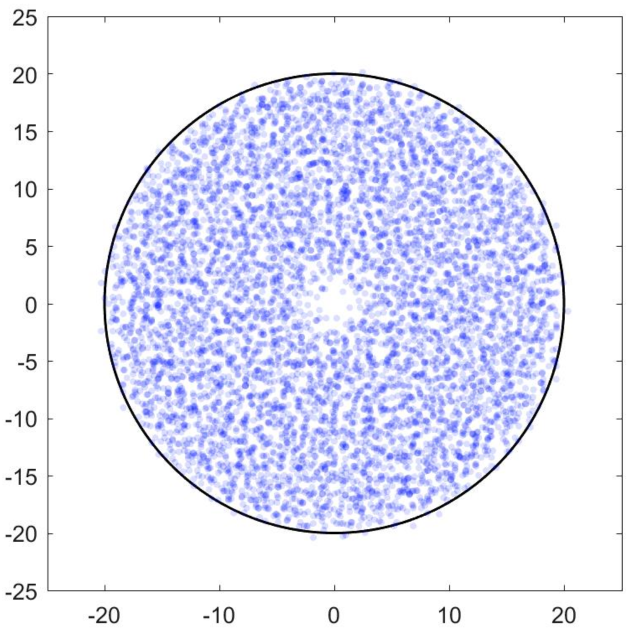
$$L = \hbar \ell N$$

- Draining flow is still difficult to model
- As a first step, consider a non-draining vortex
- Rotating horizonless geometry



$$ds^2 = \sqrt{n}\left[-\left(n - \frac{\ell^2}{r^2}\right)dt^2 - \frac{2\ell}{r}d\theta dt + dr^2 + r^2 d\theta^2\right]$$

Units: $\hbar = M = \mu = 1$

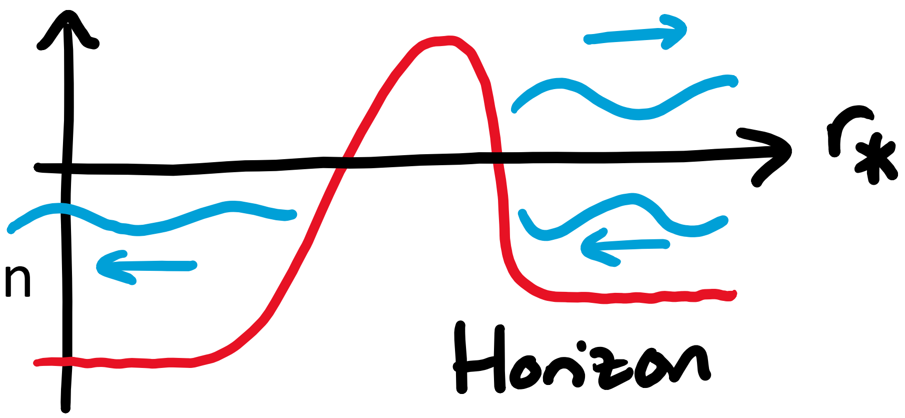


$$(\partial_t + \boldsymbol{\nabla}\cdot\mathbf{v})(\partial_t + \mathbf{v}\cdot\boldsymbol{\nabla})\delta\phi - \boldsymbol{\nabla}\cdot(c^2\boldsymbol{\nabla}\delta\phi) = 0$$

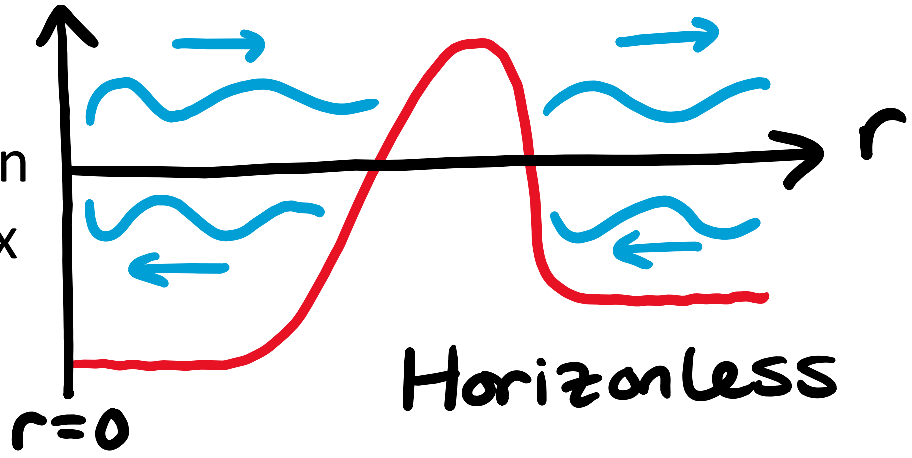
+ extra terms

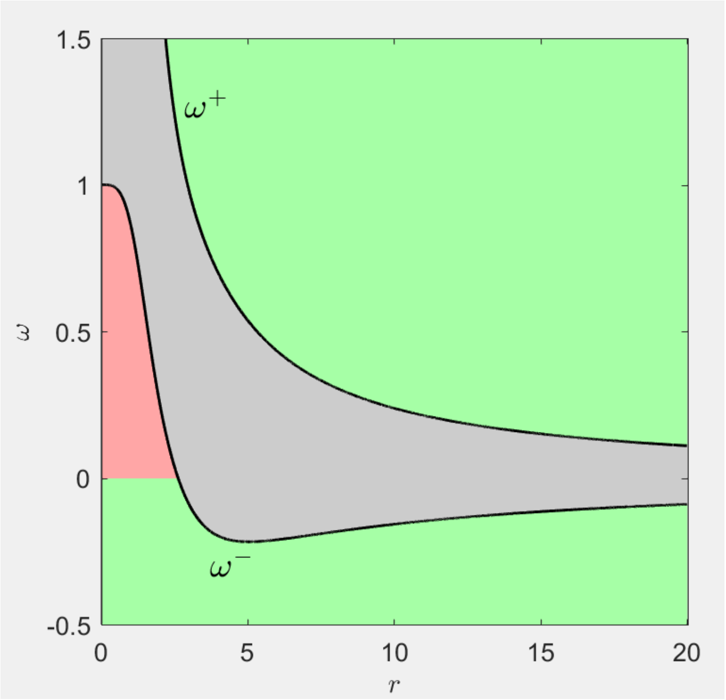
Superradiance with horizonless quantum vortex

Into horizon



Reflection off vortex axis

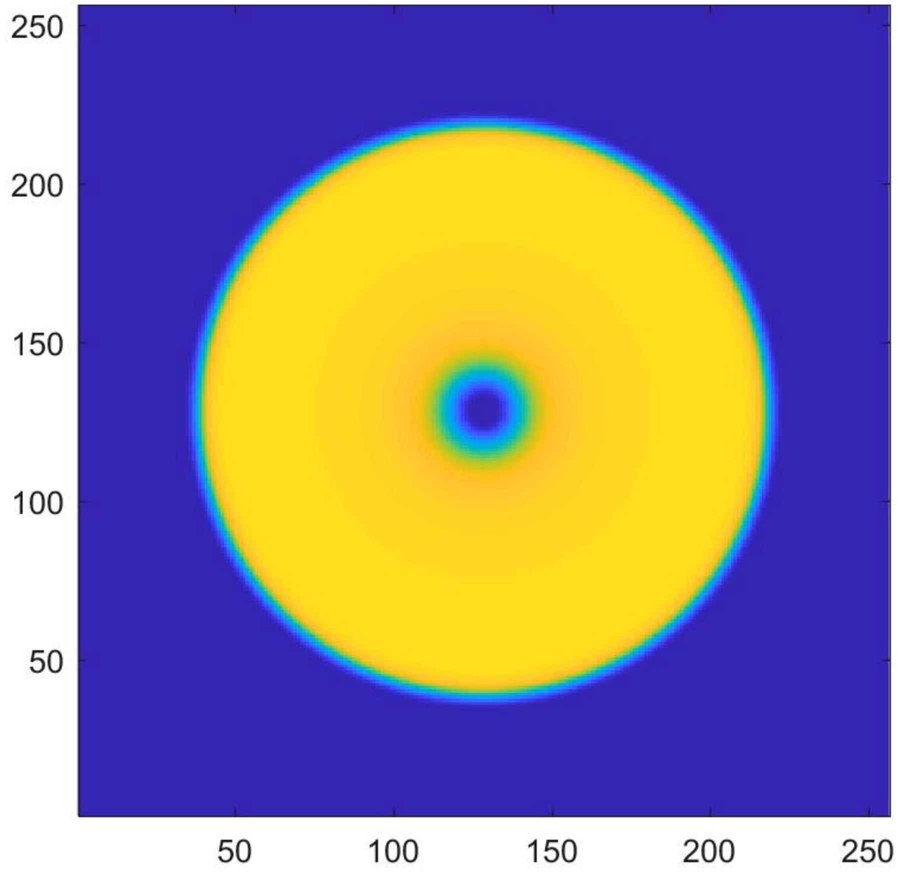




$$S(\omega) \equiv \int p(r,\omega)dr = \frac{\pi}{2} \qquad \Gamma = \frac{\log|\mathcal{R}(\omega)|}{2|\partial_\omega S(\omega)|}$$

ℓ	m	ω_{WKB}	ω_{sim}	Γ_{WKB}	Γ_{sim}
2	2	0.3954	0.4399	2.543×10^{-3}	2.431×10^{-3}
3	2	0.2475	0.2871	1.903×10^{-3}	2.189×10^{-3}
	3	0.6613	0.6648	7.839×10^{-4}	n.a.
	4	0.2161	0.2462	2.370×10^{-7}	n.a.

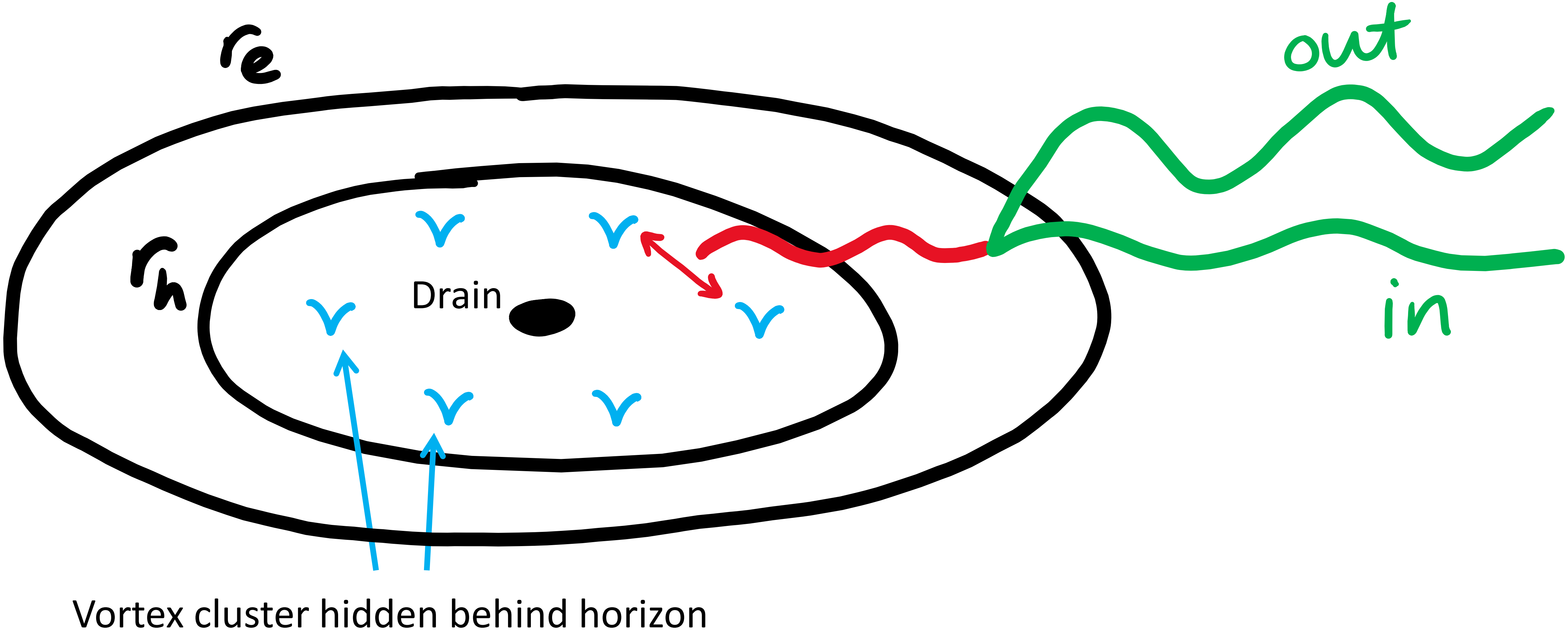
Triply quantised vortex, $m = 2$ instability



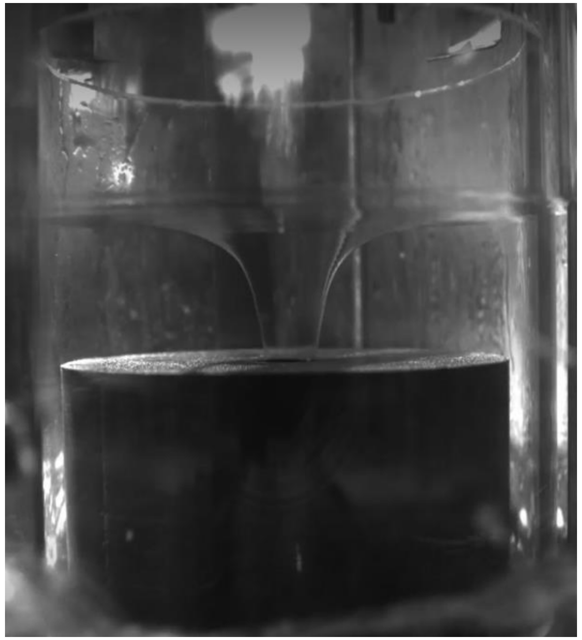
Lessons

- Quantum vortex decay arises due to a superradiant instability inside the vortex core.
- Vortex separation is a degree of freedom which facilitates superradiance.

Superradiance with draining quantum vortex?



Summary



- Draining vortex is an analogue rotating black hole
- A quantum fluid has discrete angular momentum
- Study how scattering processes are affected
- Vortex splitting with no drain
- New interpretations of fluid phenomena
- Open questions: superradiance with drain? discrete atom number?
- Working toward more realistic models of ^{4}He set-up

$$L = \hbar \ell N$$

[Class. Quant. Grav. **38** 095010 (2021), Phys. Rev. Research **4** 023099 (2022), Phys. Rev. Research **4** 033117 (2022), arXiv:2111.02567]