AXION COUPLINGS IN GRAND UNIFIED THEORIES*

*: together with P. Agrawal, M. Nee

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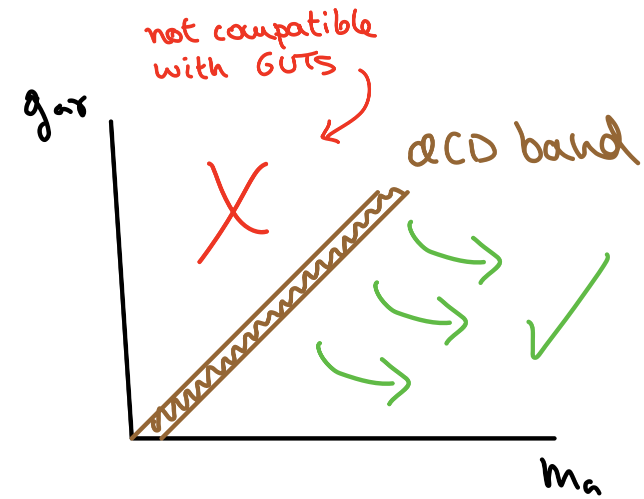
WHAT DO WE LEARN?

i) Quantization of axion-photon coupling allows knowledge about UV: charge quantization, GUTs, ...

ii) In (all??) unified theories the only axion* coupled to photons is the QCD axion.



* Any other axion necessarily has $\dfrac{g_{a\gamma}^{ALP}}{m_{ALP}} < \dfrac{g_a^{QCD}}{m_{QCD}^a}$

(SOME) STANDARD MODEL PUZZLES

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

* Unexplained phenomena: DM, baryogenesis, CC problem, hierarchy (?) problem ...

* 3 independent (?) interactions @ low E

  Quarks, leptons, LH, RH fields, 3 families .... !?

* Plethora of independent charges & quantum numbers

  but still:

  CHIRALITY + ANOMALY FREEDOM

  $g_{strong}, g_{weak}, g_Y \sim \mathcal{O}(1)$ @ TeV

  very different from Yukawa couplings

  $$Y_{1st} \ll Y_{2nd} \ll Y_{3rd}$$

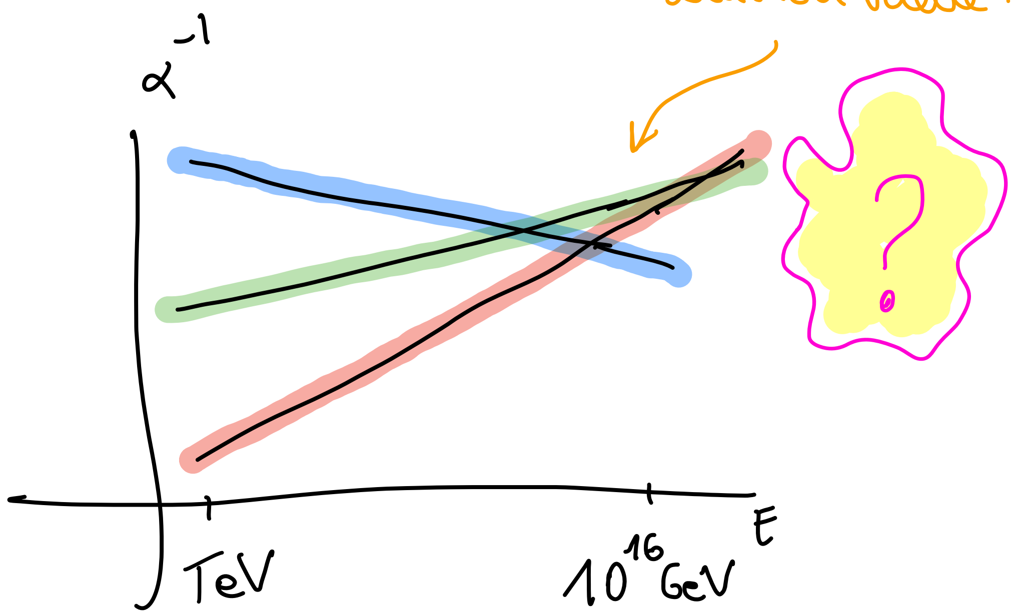
THE STANDARD MODEL

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$g_s, g_{weak}, g_Y \sim \mathcal{O}(1)$ at low energies $\rightarrow$

$\hookrightarrow$ looks even better at high E!

approach a common value?!



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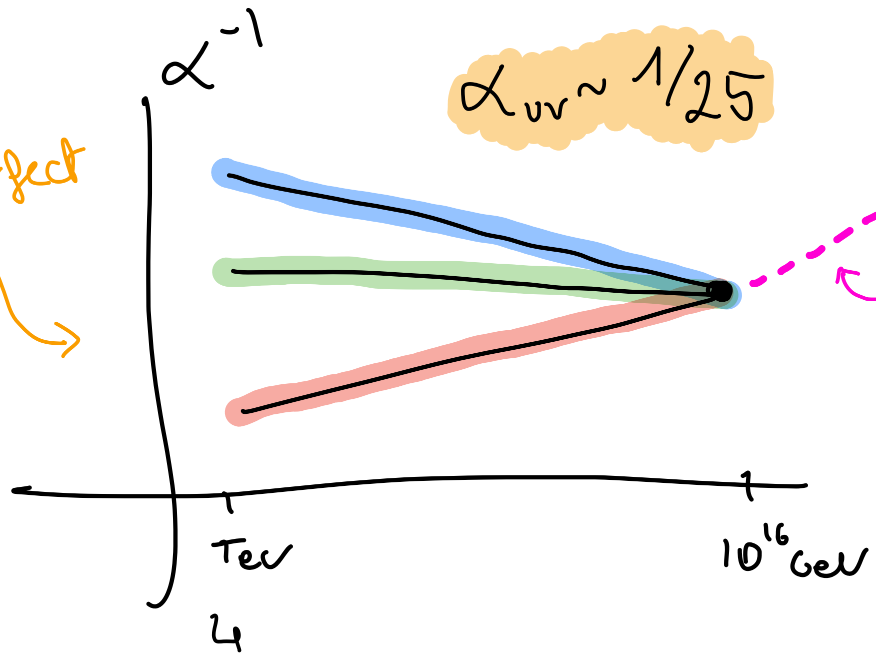
THE STANDARD MODEL

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

* To keep couplings together we need a **SIMPLE GROUP!** G_{GUT}

$\alpha_{UV} \sim 1/25$

Unification almost perfect in MSSM!



unified theory!

GEORGI-GLASHOW SU(5)

* minimal GUT : $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

$$\underbrace{5 + \overline{10}}_{\text{Chiral + anomaly free}} \longrightarrow \underbrace{q\,,\ \ell\,,\ u^c\,,\ d^c\,,\ e^c}_{\text{SM family}}$$

* $\sin^2\theta_W = \dfrac{g'^2}{g^2 + g'^2} = \dfrac{3}{8}$ @ M_{GUT} $\longrightarrow$ @ low E $\sim$ $\sin^2\theta_W \simeq 0.23$

$g' = \sqrt{\frac{3}{5}}\, g_1$

* Other GUTs : $\overbrace{SO(10),\ SO(18),\ E_6,\ E_8 \ldots}^{\text{contain } SU(5) \text{ as subgroup}}$

spinor unification

particularly appealing : CHIRAL + ANOMALY FREE

SO(10) spinor : $16 \rightarrow q + u^c + d^c + \ell + e^c + \nu^c$

DISCLAIMER

I will be talking about general features of GUTs but will be using all the time SU(5) as particular example.

THE GOOD: HINTS FOR UNIFICATION

* **Charge quantisation**: ALL isolated states have integer electric charge.

$\frac{|q_p + q_e|}{e} \lesssim 10^{-21}$ [PDG]

* **Anomaly freedom**: SM quantum numbers "conspire" to cancel gauge anomalies.

* **Unification of couplings**: $\sin^2\theta_W$ & $\frac{m_b}{m_\tau}$

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{3}{8} \quad , \quad \frac{m_b}{m_\tau} \simeq 3 \text{ at low } E$$

$g' = \sqrt{\frac{3}{5}}\, g_1$

THE BAD: PROTON DECAY

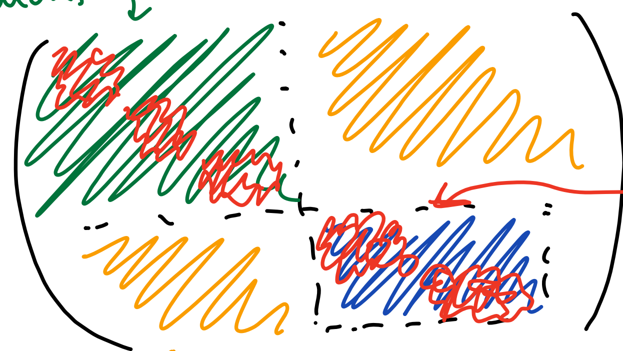
* Generic GUT prediction

SU(5) ADJOINT MATRIX: 24

* GUT bosons:

gluons

mediate proton decay



hyperchare boson $\propto \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right)$

$X \sim (3, 2, -5/6)$

5×5

Weak bosons

$$\boxed{\Gamma_{p \to \pi e^+} \sim \frac{m_p^5}{M_{GUT}^4}} \rightarrow$$

$M_{GUT} \gtrsim 10^{16}$ GeV

$\tau_{proton} > 1.6 \times 10^{34}$ yr.

Hyper-K expected to improve by factor of 10!

(data-taking ~ 2027)

Current limits from Super-K

THE UGLY: GUT DRAWBACKS

* Doublet-triplet splitting:

$$\bar{5} \sim \begin{pmatrix} H_3 \\ H_{SM} \end{pmatrix}$$

lead to proton decay unless $m_{H_3} \sim M_{GUT}$

* Mass ratios: $\dfrac{m_b}{m_\tau}$ is OK... but $\dfrac{m_s}{m_\mu}$ or $\dfrac{m_d}{m_e}$?!

NOT EVERYTHING IS LOST...

↳ requires model building effort: orbifold GUTs, flavor sym...

! Our results will be independent of GUT model building details...

AXION REVIEW

* Axion: periodic (compact) scalar with discrete shift-symmetry.

AKA axion-like particle (ALP)

$$\boxed{a \to a + 2\pi f_a}$$

* (periodic) Interactions shaped by shift-symmetry

$$\frac{\partial_\mu a}{f_a} \bar{f} \gamma^\mu \gamma^5 f \; ; \; \frac{a}{f_a} F\tilde{F} \; ; \; V(a) = \Lambda^4 \cos(a/f_a)$$

* Field theory language: pNGB of (anomalous) symmetries

↳ $U(1)_{PQ}$ for QCD axion

$$[SU(3)_c]^2 \times U(1)_{PQ} = \mathcal{A}_{QCD}$$

anomaly coefficient

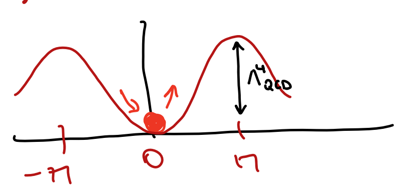
WHY AXIONS?

* Appear BSM models & String Theory (i.e. AXIVERSE)
* solve strong CP problem: QCD axion
* Dark matter candidates
* Dark energy, or even inflation (?)

Ex: QCD AXION

$$\theta_{QCD}\, G\tilde{G} \longrightarrow \frac{a}{F_a}\, G\tilde{G}$$

* solves strong CP: $\frac{\langle a \rangle}{F_a} = 0$
* $V(a) = \Lambda_{QCD}^4 \left(1 - \cos\left(\frac{a}{F_a}\right)\right) \Rightarrow m_a \sim \frac{\Lambda_{QCD}^2}{F_a}$



WHY AXIONS ?

* Appear in many BSM constructions
* solve strong CP problem: QCD axion
* Dark matter candidates
* Dark energy, or even inflation (?)
* Topological, quantized couplings to gauge bosons

$$\mathcal{L}_a = \frac{(\partial_\mu a)^2}{2} + \mathcal{A} \frac{a}{F_a} \frac{\alpha_{GUT}}{8\pi} G_{GUT} \tilde{G}_{GUT}$$

Chern-Simons-like coupling

↳ QUANTISATION:

Anomaly coefficient $\mathcal{A} \in \mathbb{Z}$, an integer!

TOPOLOGICAL COUPLINGS TO GAUGE BOSONS

* Anomaly coeff. unaffected by renormalization [see anomaly matching]

$$\boxed{\mathcal{A}_{UV} = \mathcal{A}_{IR}}$$ → directly probing the far UV !

↳ Together with gauge invariance offers info about gauge group.

WAIT !!
WHAT ABOUT MIXING ?

* AXION-PHOTON COUPLING (QCD axion case)

topological nature

axion-pion mixing

$$\frac{\alpha_{em}}{F_a}\left(E/N - 1.92\right) a F\tilde{F}$$

↳ We obtain solid information in the massless axion limit:

MIXING EFFECTS VANISH IN THIS LIMIT → $m^2 \ll m^2_{QCD} \sim \Lambda^4_{QCD}/f_a^2$

WILL BE SHOWN LATER...

MINIMAL 4dim GUT

* Starting point: $G_{GUT} \times \prod_i U(1)_{PQ\,i}$

simple gauge group — e.g. SU(5)

Set of commuting, global unbroken symmetries

↳ Analogy with SM: $\underbrace{U(1)_B \text{ and } U(1)_L}_{\text{weak interaction } SU(2)} \rightarrow \begin{cases} U(1)_{B-L} \text{ anomaly-free} \\ U(1)_{B+L} \text{ ANOMALOUS!} \end{cases}$

applications for baryogenesis etc.

* After symmetry redefinition:

$$G_{GUT} \times U(1)_{PQ} \times \prod_i U(1)_i$$

axion

non anom.

exact or decoupled Goldstone bosons

Important !!

$$[G_{GUT}]^2 \times U(1)_{PQ} = \mathcal{A}$$

$$[G_{GUT}]^2 \times U(1)_i = 0$$

MINIMAL 4dim GUT

$$\prod_i U(1)_{PQ\,i} \xrightarrow{} \underbrace{U(1)_{PQ}}\times \prod_i^{\text{non-anom}} \tilde{U}(1)_i$$

field redef.

↳ only this linear combination gives an axion coupled to gauge bosons.

$\mathcal{A}_{PQ} \neq 0$

$\mathcal{A}_i = 0$

and due to quantisation $\underline{\underline{\mathcal{A}^{UV} = \mathcal{A}^{IR}}}$

↳ CURRENTS:

Above PQ & GUT SSB scales

$$\begin{cases} U(1)_{PQ}: & \partial^\mu J_\mu^{PQ} = \mathcal{A}_{PQ} \frac{\alpha_{GUT}}{8\pi} \underbrace{G \tilde{G}_{GUT}} \\ \tilde{U}(1)_i: & \partial^\mu J_\mu^{\tilde{U}(1)_i} = 0 \end{cases}$$

↳ This axion couples to both, photons and gluons!!

decoupled Goldstones! (from gauge bosons)

DEPENDENCE ON PQ SCALE ?

PQ current above F_a, M_{GUT}: $\partial^\mu J_\mu^{PQ} = \mathcal{A}_{PQ} \frac{\alpha_{GUT}}{8\pi} G\tilde{G}_{GUT}$ → What if $F_a < M_{GUT}$!?

A) $F_a > M_{GUT}$: effects of anomaly captured by dim-5 op.

$$\mathcal{A}_{PQ} \frac{a}{F_a} \frac{\alpha_{GUT}}{8\pi} G\tilde{G}_{GUT}$$

axion couples to both photons and gluons!

K_3, K_2, K_1 levels of embedding of $SU(3), SU(2), U(1)$ in G_{GUT}

B) $F_a < M_{GUT}$:

$$\partial^\mu J_\mu^{PQ} = \mathcal{A}_{PQ} \left\{ K_3 \frac{\alpha_3}{8\pi} G\tilde{G}_{QCD} + K_2 \frac{\alpha_2}{8\pi} W\tilde{W} + K_1 \frac{\alpha_1}{8\pi} B\tilde{B} \right\}$$

↓ After PQ breaking...

$$\mathcal{A}_{PQ} \frac{a}{F_a} \left\{ K_3 \frac{\alpha_3}{8\pi} G\tilde{G}_{QCD} + K_2 \frac{\alpha_2}{8\pi} W\tilde{W} + K_1 \frac{\alpha_1}{8\pi} B\tilde{B} \right\}$$

↳ Again, axion couples to both photons & gluons!

RESULT:

MINIMAL 4dim GUT

TOPOLOGY + GAUGE INVARIANCE $\rightarrow$ $\frac{a}{f_a}\left[\alpha_{em} E F_{em}\tilde{F}_{em} + \alpha_s N G\tilde{G}_{QCD}\right]$

generates QCD potential!

$V(a) \simeq \Lambda_{QCD}^4 (1 - \cos(a/f_a))$

* Generic GUT prediction:

$\hookrightarrow$ $\frac{E}{N} = \frac{k_1 + k_2}{k_3}$

Standard GUT embedding $\begin{cases} k_3 = k_2 = 1 \\ k_1 = 5/3 \end{cases}$

$E/N = 8/3$

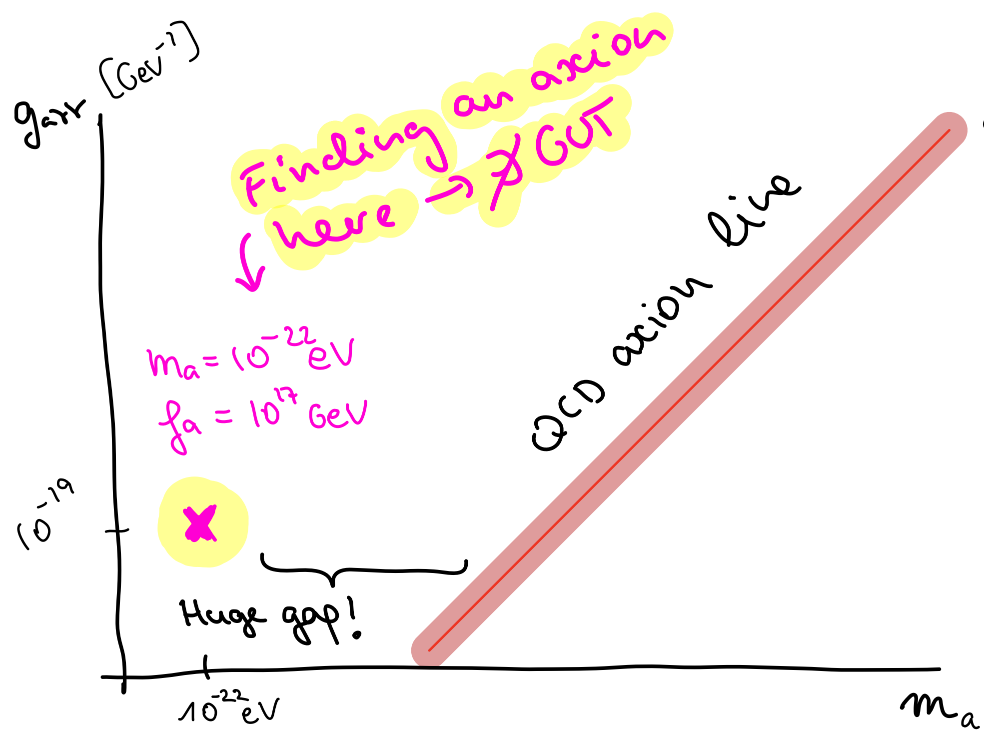
$\hookrightarrow$ Only one axion coupled to photons: the QCD axion!

$\boxed{\text{CAN WE TEST GUTS WITH AXIONS?}}$

TESTING GUTs WITH AXIONS ?!

ZERO ORDER: CLAIM

In the presence of a unified gauge group, any axion coupled to photons must lie on the QCD line.



(minimal) GUT PREDICTION

- $m_{QCD} = \frac{\Lambda_{QCD}^2}{f_a}$
- $g_{a\gamma\gamma} = \frac{\alpha_{em}}{f_a}\left(\frac{E}{N} - 1.92\right)$

ANOTHER EXAMPLE:

- Rotation of CMB polarisation by light axion coupled to photons
- Requires: $\begin{cases} H_{CMB} > m_{alp} > H_0 \\ F_a \sim M_P \end{cases}$

INCOMPATIBLE WITH GUTs

ASSUMPTIONS MADE SO FAR...

* Axion is a GUT singlet. See $U(1)_{PQ}$ SSB

* Axions have no mass/kinetic mixing

* GUT group is simple (e.g. no $U(1)$ factors)

(If time allows...)

* 4D GUTs

Does the main GUT result change if we relax these assumptions??

What if the axion is not a GUT singlet?

What if the axion is not a GUT singlet?

* We've assumed a is pNGB from $U(1)_{PQ}$

  e.g. $\Phi = \rho(x) e^{ia(x)/f_a}$

* Photon coupling from: $[G_{GUT}]^2 \times U(1)_{PQ}$ anomaly

  Anomaly matching: GUT anomaly $\rightarrow$ EM anomaly

* Example: $U(1)_{PQ}$ does not commute with G_{GUT}

  ↳ axion coupled to $U(1)_{EM}$ without QCD coupling.

  Avoides anomaly matching argument ?!

WARM UP: THE NEUTRAL PION CASE

* QCD provides an example of an "alp" coupled to photons!

Flavor symmetry → $SU(2)_L \times SU(2)_R \xrightarrow[\langle \bar{q}q \rangle]{} SU(2)_V$ + pions as NGb
$\pi^a(x) \sim 3$ of $SU(2)_V$

GAUGE EM!

* Conseq. 1: gauging $U(1)_{EM} \subset SU(2)_V$ induces an anomaly for neutral pion current!

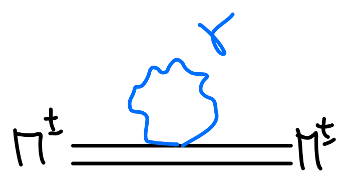
Triangle anomaly!

$$\partial^\mu J_\mu^{\pi^0} = \frac{N_c}{16\pi^2} F\tilde{F}\, \mathrm{Tr}\left[\frac{\sigma^3}{2} Q_{EM}^2\right]$$

$Q_{EM} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}$
$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\pi^0 \longrightarrow \gamma\gamma$$

* Conseq. 2: $\pi^\pm$ receive a mass from EM effects



$$\boxed{\Delta m_{\pi^\pm}^2 \simeq \alpha_{EM} f_\pi^2}$$

generators of $\pi^\pm$ do not commute with Q_{EM}!

Pion-like GUT-charged Axions ?!

$SU(5)_{GUT} \times SU(N)_{HC}$ → confining interaction: $\Lambda_{HC} \simeq f_a \gg EW$

* Fermions: $SM + \Psi \sim (5, N) + \psi \sim (1, N)$ $\langle \bar{\Psi}\Psi \rangle = \Lambda^3_{HC}$

* Flavor symmetry: $SU(6)_L \times SU(6)_R \longrightarrow \underbrace{SU(6)_{L+R}}$

As for π^0 in QCD we weakly gauge a subgroup of the flavor symmetry

pion-like field: $\Pi^a \sim 35$

Is there an ALP here?

Example:

* SU(5) is gauged!

$35 \rightarrow 24 + 5 + \bar{5} + 1$ → QCD-like axion: $m_a \sim \frac{\Lambda^2_{QCD}}{f_a}$

contains SM singlet!

$24 \rightarrow (8,1,0) + (1,3) + (3,2,5/6) + (\bar{3},2,-5/6)$
$+ (1,1,0)$

↳ COMPOSITE ALP

Large perturbative mass! $M \sim \alpha_{SM} \Lambda_{HC}$

COUPLED TO PHOTONS WITHOUT QCD ?!

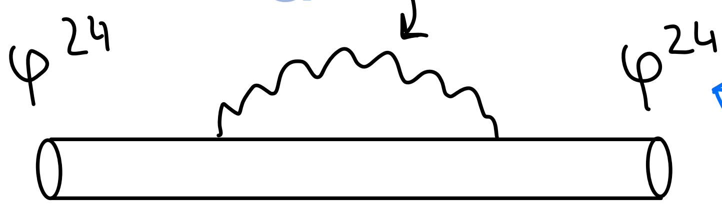
Pion-like GUT-charged AXIONS ?!

* What happens to $(1,1,0)$? $\rightarrow$ φ^{24}: aligned in "hyper-charge direction"

coupled to photons

$$\alpha_{em} \frac{\varphi^{24}}{\Lambda_{HC}} F\tilde{F}$$

$X_{GUT} \sim (\bar{3},2,5/6)$



COMPOSITE ALP

$(\simeq$ equiv. to π^0 in QCD$)$

Integrating out GUT gauge bosons $\rightarrow$

Mass prediction:

$$m^2_{\varphi^{24}} \sim \alpha_{GUT} \frac{\Lambda^4_{HC}}{M^2_{GUT}}$$

$\langle \bar{\Psi}\Psi \rangle = \Lambda^3_{HC}$ $\longrightarrow$ $F_a \sim \Lambda_{HC}$ for pion-like axions !

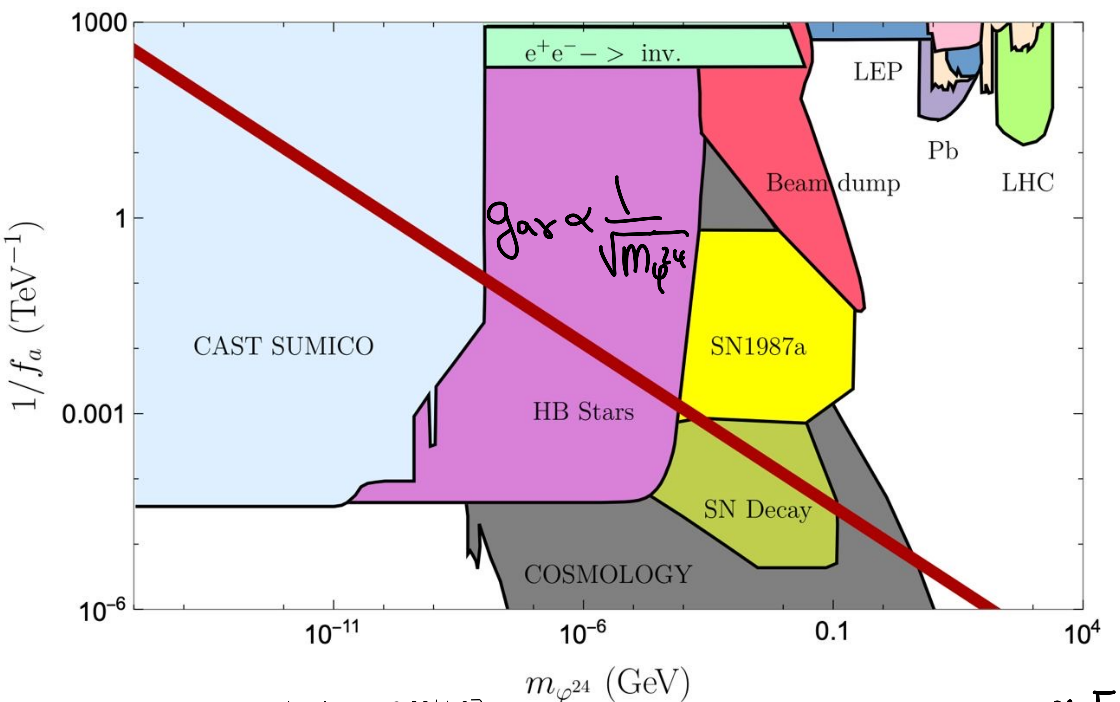
Exp. input $M_{GUT} \gtrsim 10^{16}$ GeV

Pion-like GUT-charged Axions ?!

* Emergent, charged axion does NOT invalidate previous results

No axion parametrically lighter than QCD can arise after GUT breaking.

$\boxed{m_\varphi \gtrsim 1 \text{ GeV}}$



[Adapted from Bauer et al. 1708.00443]

$m_{a\varphi} \sim \frac{F_a^2}{M_{GUT}}$

$g_{a\gamma\gamma} \sim \frac{\alpha_{em}}{F_a}$

qualitatively different to QCD axion!

* $F_a \sim \Lambda_{HC}$ for pion-like ALPs
* Assuming $M_{GUT} \simeq 10^{16}$ GeV

How does axion mixing change the result?

How does axion mixing change the result?

* In the absence of mixing: 1 anomalous $U(1)_{PQ}$
  $\updownarrow$
  1 axion coupled to photons

$$\prod_i U(1)_{PQ_i} \longrightarrow U(1)_{PQ} \times \prod_i^{\text{non-anom}} \tilde{U}(1)_i$$

only possible if unbroken!

* Small explicit breaking of shift symmetries may turn on non-quantised mixing...
  (see a-π^0 mixing)

↳ Does it change the result?

KINETICALLY MIXED AXIONS?

$$\frac{K_{ij}\,\partial a_i \partial a_j}{2} + \hat{a}\, G\tilde{G}_{GUT}$$

Axion kinetic mixing matrix

linear combination coupled to GUT

Remember about redef. of "anomalous" U(1)'s

* Massless limit: freedom to rotate away K_{ij} ✓

Canonical basis

$$\frac{\delta_{ij}}{2}\partial a_i \partial a_j + a_{QCD}\, G\tilde{G}_{GUT} + \left\{ \text{bunch of massless decoupled axions} \right\}$$

$[E F\tilde{F} + N G\tilde{G} \ldots]$

SINGLE AXION COUPLED TO PHOTONS: QCD AXION!

Mass Mixed Axions?

$$\delta_{ij}\frac{\partial a_i \partial a_j}{2} + a_{QCD}\, G\tilde{G} + m_{ij}\, a_i a_j$$

$\det(m_{ij}) \stackrel{!}{=} 0$
to solve strong CP

<u>Axion mixing!</u>

as we turn on m_{ij}...

↳ No longer freedom to rotate away axions!

<u>Spectrum of Axions</u>

- Heavy (decoupled) axions $m_i^2 \gg m_{QCD}^2$
- Light axions with $m_i^2 \ll m_{QCD}^2$

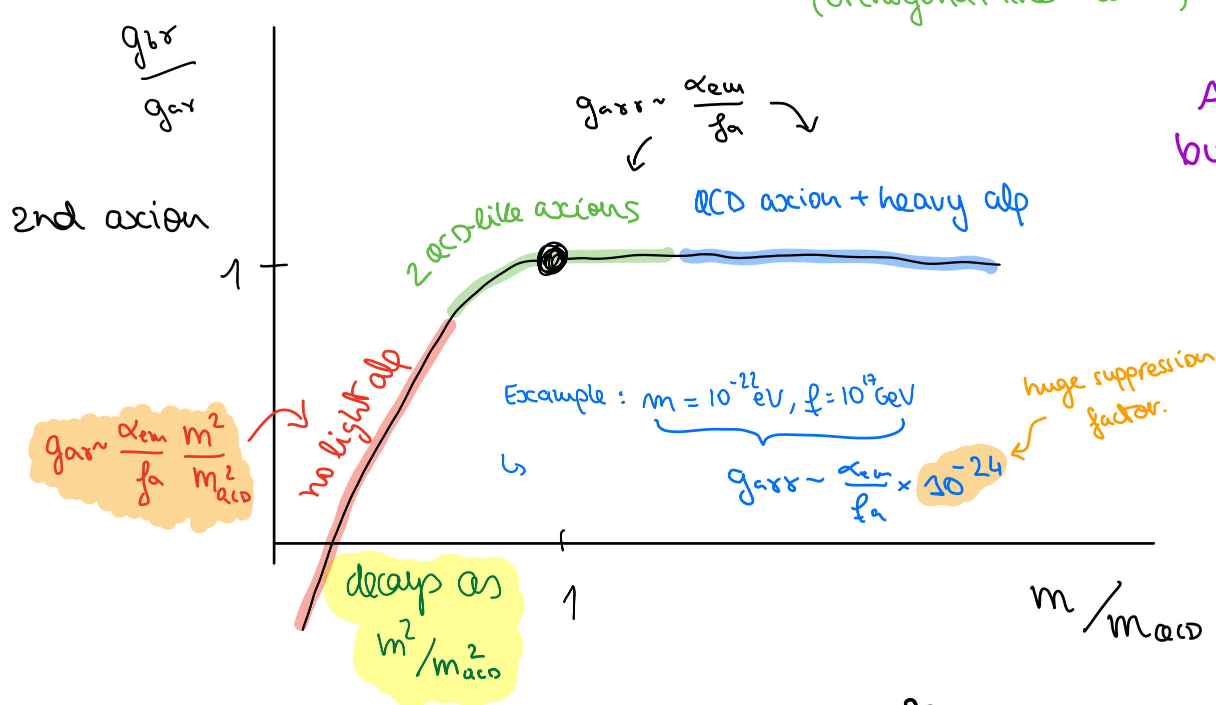
TOY MODEL WITH 2 AXIONS

$$\mathcal{L} = \left(\frac{a}{f_a}+\frac{b}{f_b}\right) G\tilde{G} + \frac{1}{2} m_b^2 b^2$$

May be generalised to arbitrary number of axions

↳ a) $m_b \gg \Lambda_{QCD}^2/f_a$ →
- QCD axion: $a_{QCD} \simeq a$
- heavy ALP b: mass m_b, coupling $g_{a\gamma} \sim \frac{\alpha}{f_b}$

↳ b) $m_b \ll \Lambda_{QCD}^2/f_a$ →
- QCD axion: $\frac{a_{QCD}}{F} = \frac{a}{f_a}+\frac{b}{f_b}$
- decoupled light ALP: (orthogonal linear comb.) $g_{a\gamma} \sim \frac{m^2}{m_{QCD}^2} \times \frac{\alpha_{em}}{f}$



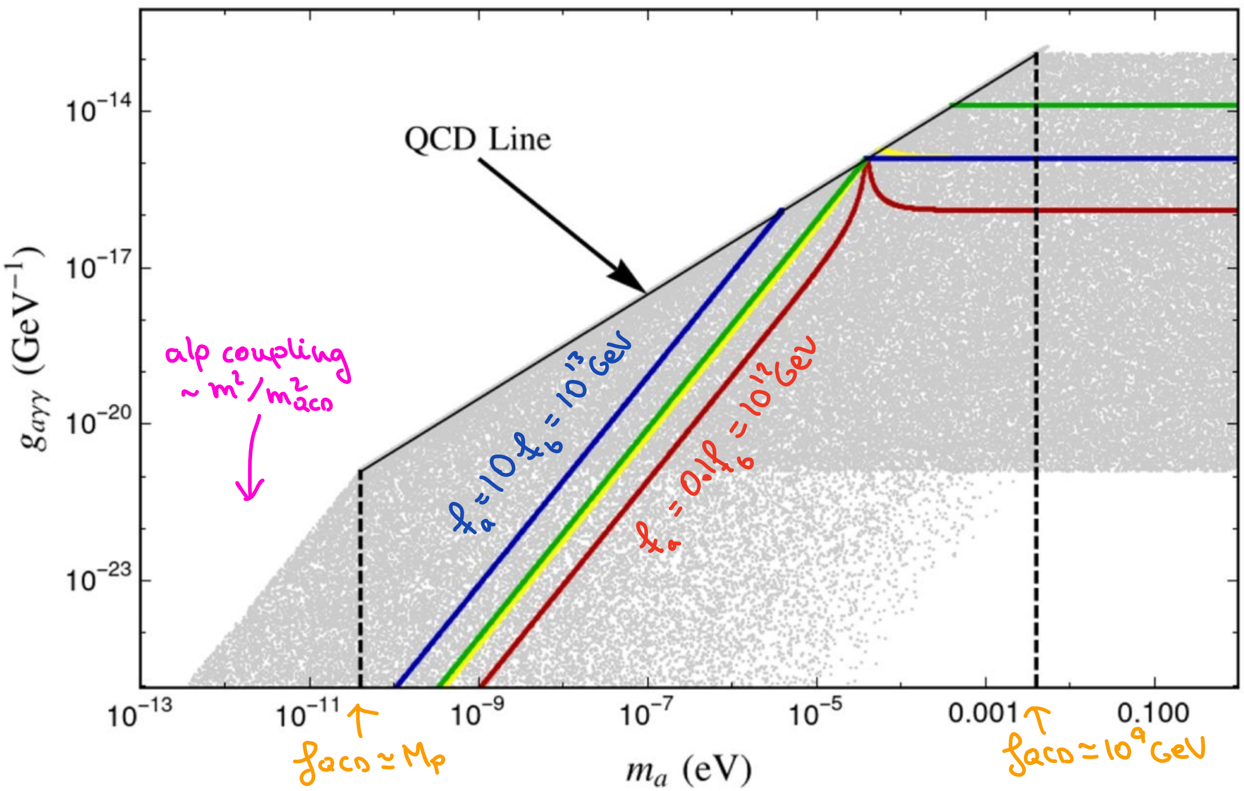
ALP-photon coupling induced by mixing effects vanishes in the massless limit!

(QCD axion "portal")

ALP-photon coupling via mixing

$$\mathcal{L} = \left(\frac{a}{f_a} + \frac{b}{f_b}\right) G\tilde{G} + \frac{1}{2} m_b^2 b^2$$

Generate sets of "points"
$(a, g_{a\gamma}) + (b, g_{b\gamma})$



Ranges:

- $m_b = [10^{-11}, 1]$ eV
- $f_a, f_b = [10^9, 10^{18}]$ GeV

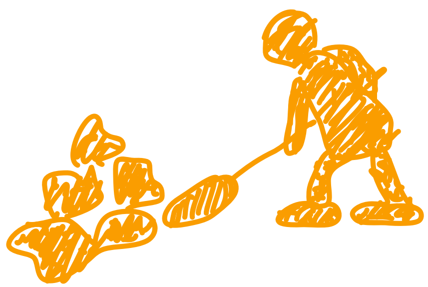
* $\dfrac{g_{alp}^{\gamma}}{m_{alp}}$ is always smaller than QCD axion $\dfrac{g_{a\gamma}}{m_{QCD}}$

[Does not depend on number of axions]

Do axions in extra dimensions change the story?

WORK IN PROGRESS!

23XX-YYYYY



Do axions in extra dimensions change the story?

WORK IN PROGRESS!

- Arise from extra dim. gauge fields:

↳ Ex: 5D model $\qquad A_M = (A_\mu ; A_5)$

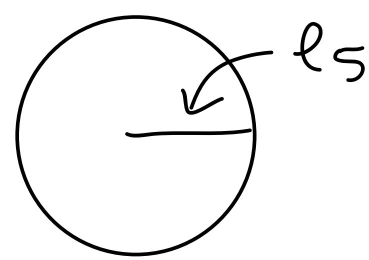
- Shift symmetry ⟷ Higher dimensional gauge invariance

- Charged particles in the bulk → (non-local) axion potential!

$$V(a) = \ell_5^{-4} e^{-S} \cos(a/F_a)$$

"Instanton" action:

$S \sim 2\pi \ell_5 M_{UV}$



STRING AXIONS & GUTs

* SM is embedded in higher dim. simple gauge group: **GUT symmetry is exact everywhere in extra dimension**

CS-like coupling (e.g. 5D) — 4-dimensions

$$S^{(5)}_{CS} = \frac{K}{16\pi^2}\int d^5x\, \epsilon^{MNPQR} B_M \,\mathrm{Tr}[G_{NP} G_{QR}] \supseteq \frac{a}{F_a} G_{GUT}\tilde{G}_{GUT} \rightarrow$$

4d result trivially extended to higher D!

K → CS level ~ anom. coeff.

B_M → $B_5 \equiv$ axion "a"

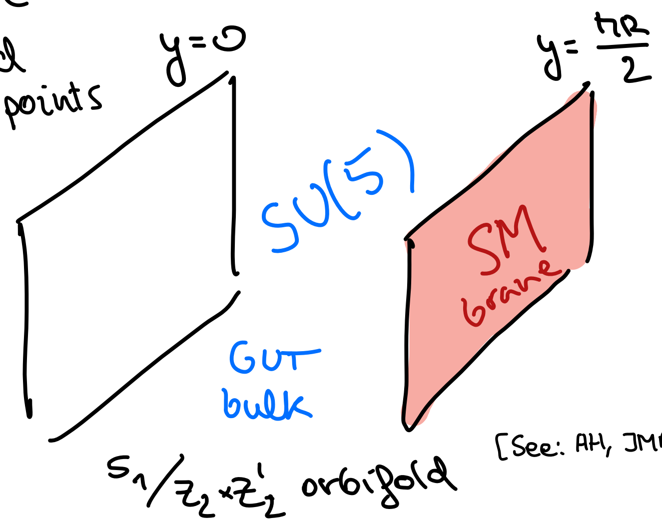
* String theory offers richer possibilities: **orbifold GUTs**

"position dependent" gauge symmetry

↓

leading to: **APPARENT UNIFICATION**

branes @ orbifold fixed points



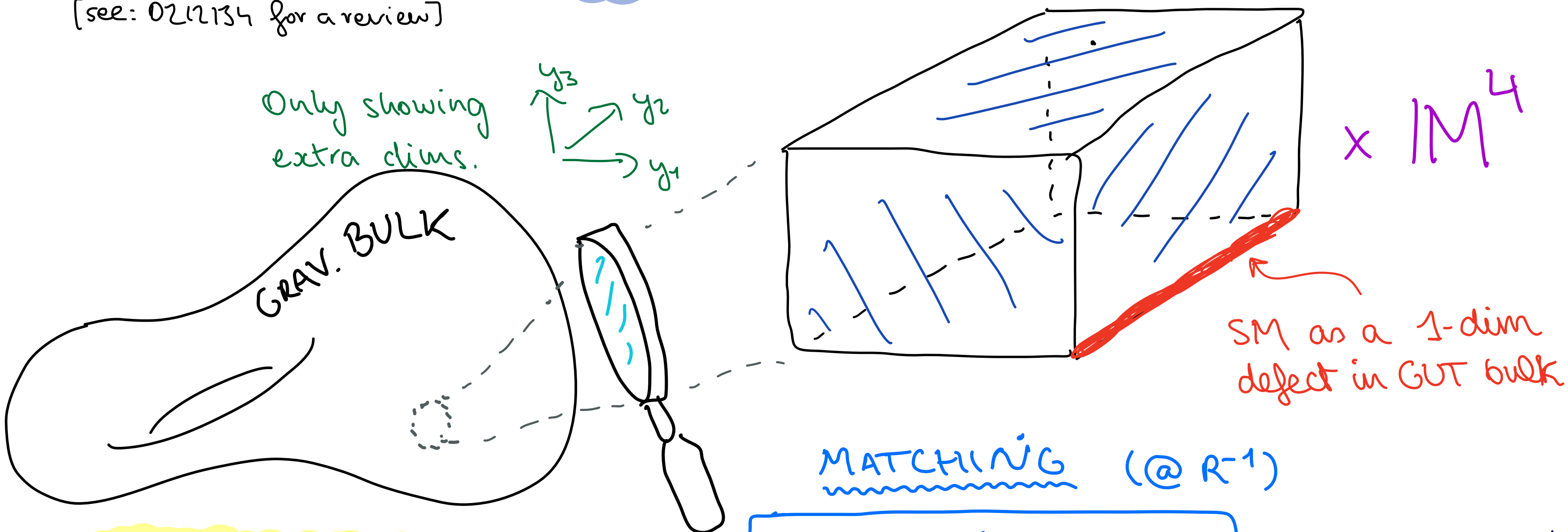
[See: AH, JMR 0106166]

ALP without QCD coupling ?!

TOY MODEL OF "APPARENT UNIFICATION"

- General **GUT-symmetric bulk** with **SM-like defect**

[see: 0212134 for a review]



APPARENT UNIFICATION

$$\frac{1}{\alpha^{bulk}_{GUT}} \gg \frac{1}{\alpha^{loc}_{i}}$$

MATCHING (@ R^{-1})

$$\boxed{\frac{1}{\alpha^{4D}_{i}} = \frac{1}{\alpha^{bulk}_{GUT}} + \frac{1}{\alpha^{loc.}_{i}}}$$

$$\frac{1}{\alpha} = \frac{1}{g_s}\frac{Vol(\Sigma_p)}{\ell_s^p}$$

For long distance physics, boundary effects become important:

↳ **localized axion:** coupled to photons $\quad a_{loc.} F_{em} \tilde{F}_{em}$

AXION-PHOTON COUPLING IN HIGHER-DIMENSIONAL GUTS

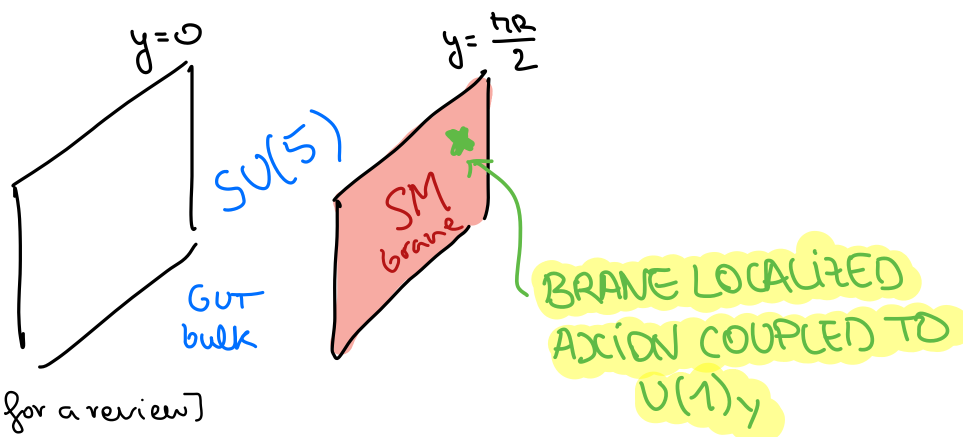
* We can get a (brane-localized) ALP: $a_{loc.} F_{em} \tilde{F}_{em}$

* **Is there any effect generating a mass?**

"Instantons are as abundant as axions in ST" [see non-exist. of global sym.]

↳ Effects breaking axionic shift-symmetries are expected.

branes @ orbifold fixed points



What's the mass of this guy?

[see: 0212134 for a review]

(STRINGY) LIST OF INGREDIENTS*

* **Gauge sectors** arise from D(p+3)-branes wrapping p-cycles in X (compact manifold).

$$\boxed{\frac{1}{\alpha_{D(p+3)}} = \frac{1}{g_s}\frac{\mathrm{Vol}(\Sigma_p)}{\ell_s^p}}$$

[see: 0605206 for review]

* **Axions** arise from p-form fields integrated over p-cycles.

$$\boxed{\theta_i = \frac{1}{2\pi}\int_{W_p} C_p}$$

[see: 0902.3251 for review]

↙ point-like in Minkowski!

* **Instantons** given by D(p-1)-branes wrapping p-cycles (D-instantons)

$$\boxed{S_{D(p-1)} = \frac{2\pi}{g_s}\frac{\mathrm{Vol}(\gamma_p)}{\ell_s^p}}$$

Ex: type I: D1, D5-instantons
type II { A: D2-instantons
B: D(-1), D3-instantons

* Important!
In general $\Sigma_p \neq W_p \neq \gamma_p$

STRINGY AXION COUPLED TO PHOTONS ?

* Axion-gauge boson coupling from S_{CS}

$$2\pi \int_{M \times \mathcal{C}} C_p \wedge \frac{1}{8\pi^2} \mathrm{Tr}\, F^2$$

$M \times \mathcal{C}$ ↳ Compact space where gauge cycle γ_p "lives".

* Let γ_p be the gauge cycle hosting $U(1)_Y$ or $SU(2)_L$.

Associated D-instanton action

$$\boxed{S_{D(p-1)} = \frac{2\pi}{g_s} \frac{\mathrm{Vol}(\gamma_p)}{\ell_s^p} = \frac{2\pi}{\alpha_i(R^{-1})}}$$

$i = SU(2)_L ; U(1)_Y$

(p-1)-brane is charged wrt p-form fields! (giving axions)

We use DBI action to relate $\mathrm{Vol}(\gamma_p)$ to gauge coupling!

This corresponds to the particular case : $\Sigma_p = W_p = \gamma_p$

MINIMAL ALP MASS...

In some sense is the usual "PQ quality problem" applied to ALPs.

MINIMAL ALP POTENTIAL $V(a) \sim \text{Tr} \times R^{-4} e^{-S_{D(p-1)}} \cos(\theta^i)$

CHIRAL SUPPRESSION?

- Zero modes saturated by m_{SUSY} insertions
- Charged chiral matter highly constrained! (e.g. Z decays, Higgs properties...)

D-instanton action

$$\boxed{S_{D(p-1)} \sim 2\pi/\alpha_i}$$

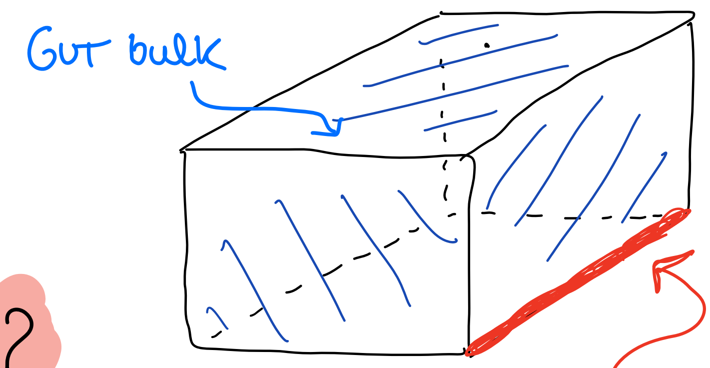
- Dominates $V(a)$
- axion massless ($m_a \ll H_0$) if $\alpha_i \lesssim 1/45$

Although $\exists$ minimum mass, it may well be orders of magnitude below H_0.

AN APPLICATION: APPARENT UNIFICATION

MATCHING (@ R^{-1})

$$\boxed{\frac{1}{\alpha_i^{4D}} = \frac{1}{\alpha_{GUT}^{bulk}} + \frac{1}{\alpha_i^{loc.}}}$$



localized axion coupled to photons $\dfrac{a_i}{F_{a_i}} F_i \tilde{F}_i$?

Example: SM as a 1-dim defect in GUT bulk

KEEP PREDICTIONS

* Gauge coupling unification $\alpha^{bulk} \sim 1/25$
* $\sin^2\theta_w$ prediction.

needs

$$\boxed{\frac{1}{\alpha^{bulk}} \gg \frac{1}{\alpha_i^{loc}}}$$

justified when bulk vol. is parametrically larger

implies

LARGE MASS FOR LOCALIZED AXIONS !!

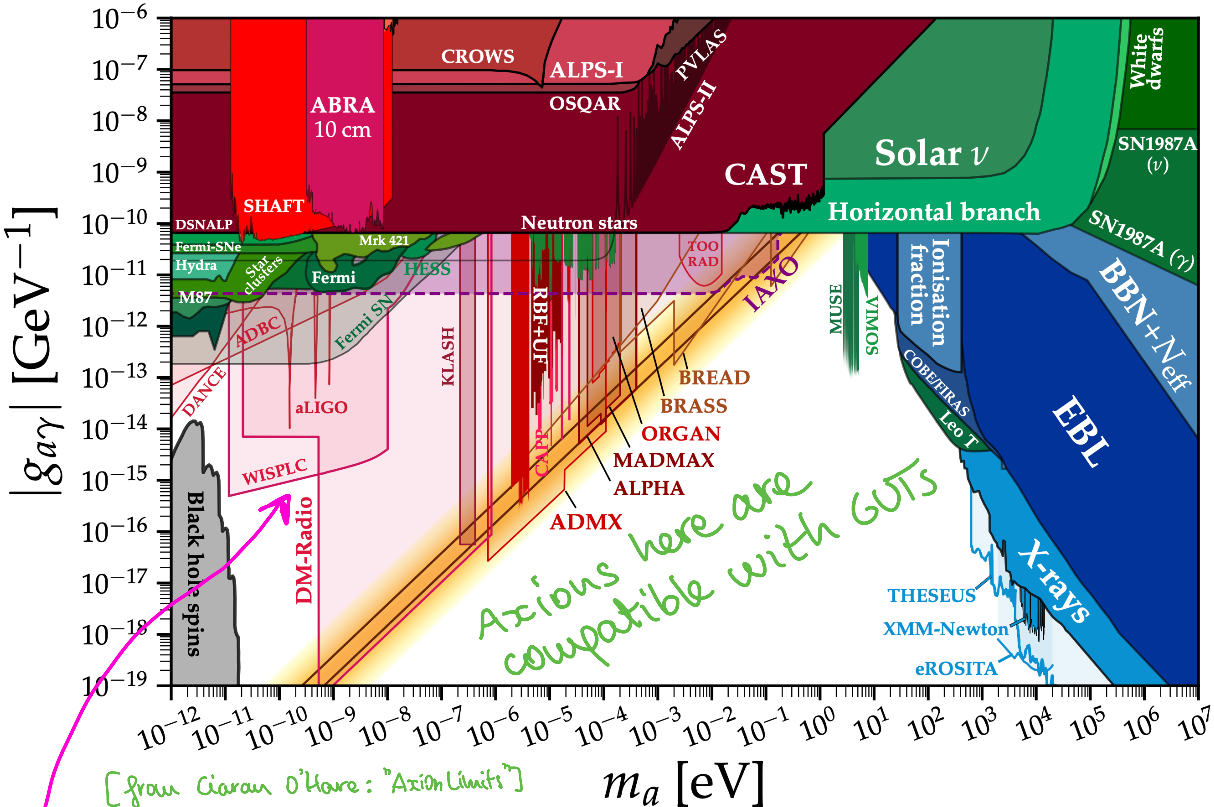
Localized coupling: $\mathcal{O}(1)$ to preserve unification!

$$V(\theta_{loc}^i) \sim K R^{-4} e^{-2\pi/\alpha_i^{loc}} \cos(\theta^i)$$

Axion-Photon Coupling: a bright future...

... or why I think all this is exciting!

∗ lab & haloscopes ∗ astrophysics ∗ cosmology



[from Ciaran O'Hare: "Axion Limits"]

∗ Many ongoing and planned experiments looking for axions coupled to photons!

specially for: $m^2 << m^2_{QCD}$

GUT alp constraint: $\dfrac{g^{alp}_{\gamma}}{m_{alp}} < \dfrac{g^{QCD}_{a\gamma}}{m_{QCD}}$ ("to the right of QCD band")

Incompatible with GUTs

BACK-UP

N mirror sectors ?

See 1802.10093, 2102.00012

$$\mathbb{Z}_N : SM_k \longrightarrow SM_{k+1}$$
$$a \longrightarrow a + \frac{2\pi k}{N} f_a$$

N copies of SM

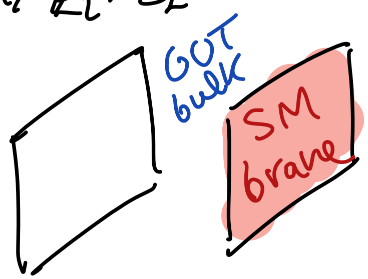
$$\boxed{m^2 \sim m_{QCD}^2 \times \frac{1}{2^N}}$$

to get $m \sim 10^{-22}\,eV$; $F_a \sim 10^{17}\,GeV$

Need: $N \sim 100$ copies of SM

TOY MODELS OF APPARENT UNIFICATION

i) $S_1/\mathbb{Z}_2\times\mathbb{Z}_2'$: 5d bulk sandwiched by 3-branes



$$\alpha_{4d}^{-1} = \alpha_{\text{bulk}}^{-1} + \alpha_{\text{brane}}^{-1}$$

* D(-1)-instanton, coupled to brane localized axions: $\boxed{S_{(-1)} \sim \frac{2\pi}{g_s} \equiv \frac{2\pi}{\alpha_{\text{brane}}} \sim \mathcal{O}(1)}$

* Bulk "instanton" is a D0-brane, equivalent to a particle going around the extra dimension, generating the classic $S_0 \sim 2\pi/\alpha_{\text{bulk}}$

that is $$S_0 \sim \frac{2\pi}{g_s}\frac{R}{\ell_s} \gg S_{(-1)}$$

$\hookrightarrow$ bulk axion, coupled to full $F_{GUT}\tilde{F}_{GUT}$ remains QCD-like (UV instanton negligible)

BEYOND THE QCD BAND: HEAVIER AXIONS?

A) Give up strong CP problem: $m_a^2 = \frac{\Lambda_{QCD}^4}{f_a^2} + \cancel{X}$

B) "Align" exotic instantons:

$SU(5)_1 \times SU(5)_2 \times \dots SU(5)_N \xrightarrow{SSB} SU(5)_{diag} \equiv GUT$

matching condition: $\alpha_{GUT}^{-1} = \sum_i \alpha_i^{-1} \approx \frac{N}{\alpha}$

$\alpha \gg \alpha_{GUT} \rightarrow$ strong SSI!

* UV instantons @ SSB scale generate large axion mass.

[Agrawal, Howe]

* $\nexists$ light axion but CP conserving minimum!

! UNCERTAIN: $\alpha(M_{SSB}) \sim \mathcal{O}(1)$? Additional matter? Why only 3 families?

EMERGENT OR "CHARGED" AXIONS

* Instead of $U(1)_{PQ} \xrightarrow{SSB}$ axion ... give it a GUT charge!

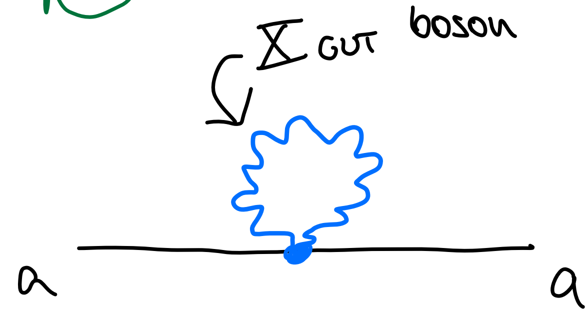
* Example: ALP from pseudoscalar in adjoint represent. (elementary)

$\Phi \sim$ GUT adjoint $\rightarrow \frac{c}{\Lambda} \mathrm{Tr}[\Phi G \tilde{G}_{GUT}]$ ← NOT A TOTAL DERIVATIVE BREAKS SHIFT SYMMETRY

↳ E.g. SU(5): $\frac{a^Y}{f_a} \propto \mathrm{diag}(-1/3, -1/3, -1/3, 1/2, 1/2)$ hyperchargedirection

Doesn't couple to gluons!

$$\frac{a^Y}{f_a} F_{U(1)_Y} \tilde{F}_{U(1)_Y}$$



↳ Mass from GUT bosons ⇝

$$m_a^2 \sim \alpha_{GUT} M_{GUT}^2$$

(if ALP is elementary) ~ analogous to hierarchy problem ⟶

What if the group is not simple?

- We've used topology + gauge invariance

- How do non-simple gauge group change the result?

  - ↳ $SU(5) \times U(1)$
  - ↳ Pati-Salam
  - ↳ Trinification

FLIPPED GUTS

what about exotic charges?!

* Theories based on $SU(5)\times U(1)_X$, or more complex groups.
* $U(1)_Y$ comes from T_{24} & X (properly normalizing)
* WEAK MIXING ANGLE (@ GUT scale)

$$\sin^2\theta_W = \frac{3/8}{1+\frac{5}{3}\left(\frac{\alpha_5}{\alpha_X}-1\right)}$$

Only if $\alpha_5 = \alpha_X \rightarrow$ { Standard GUT prediction; All couplings meet @ GUT scale; Embeddable in simple group

ONLY QCD AXION IN THIS CASE !

SO(10), E6, ...

FLIPPED GUTS

* **QUANTUM NUMBERS**

$$SU(5)\times U(1)_X$$

$$\underbrace{5_{-3},\ 10_1,\ 1_5}_{\text{SM family} + \nu_R}$$

* **WEAK MIXING ANGLE**

$$\sin^2\theta_W = \frac{3/8}{1+\frac{5}{3}\left(\frac{\alpha_5}{\alpha_X}-1\right)}$$

↳ Axion coupled to $U(1)_X$ without $SU(5)$ → $\nexists$ common origin

↓

$\alpha_5 \neq \alpha_X$

i) $\nexists$ reason for SM charges
eg: fermion with electric charge $+\frac{1}{2}$?

ii) $\nexists$ prediction of $\sin^2\theta_W$!

} ← $\alpha_5 \neq \alpha_X$

↑ price to pay... :(

Kinetically Mixed Photons ?

- $G_{GUT} \times U(1)_{Dark}$ with 2 axions:

dark photon

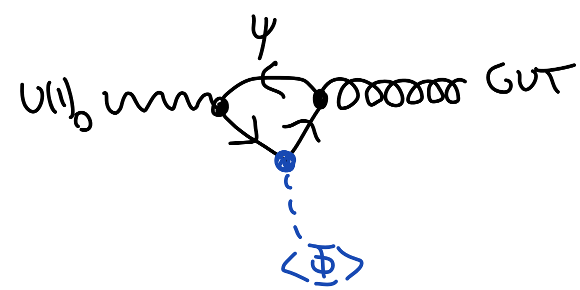
QCD axion

axion coupled to dark sector

$$\alpha_{GUT}\frac{a}{f_a}G\tilde{G}_{GUT} + \frac{\alpha_D}{f_b}b_D F\tilde{F}_D$$

- Gauge invariance forbids tree-level kin. mixing

↳ higher dim: $\frac{1}{M_P} F_D\, \Phi\, G_{GUT}$



$$\epsilon \sim \frac{\alpha_{GUT}\alpha_D}{16\pi^2}\frac{M_{GUT}}{M_{Pl}}$$

- After GUT SSB: $\frac{\epsilon^2}{8\pi}\alpha_D\frac{b}{f_b}F\tilde{F}$

expected to give a large suppression!

$\epsilon^2 \lesssim 10^{-8}$

CHARGE QUANTISATION IN GUTS

(see Polchinski ST vol. 2)

↳ Define $Q' = Q_{em} + \frac{T_{color}}{3}$ ⇝ accounts for triality

* Isolated states have $T_{color} = 0 \mod 3$

↳ All particles in 5-plet of SU(5) have integer Q'.

↳ All SU(5) reps. are obtained by tensor prod. of 5-plet.

↳ $Q' =$ integer ⟶ $Q_{em} =$ integer for isolated states, too!

NON-STANDARD EMBEDDING

* $G_{GUT} \supset SU(5) \supset SU(3) \times SU(2) \times U(1)$
  corresponds to the canonical/standard embedding
  If G_{GUT} is larger, other embeddings are possible

* k-level embedding: $G_1 \times \ldots \times G_k \to G_{diag}$

* GUT predictions: $\left\{ \sin^2\theta_W = \dfrac{k_2}{k_1+k_2} \right.$, $E/N = \dfrac{k_1+k_2}{k_3}$

* Problems: $\left\{ \begin{array}{l} \text{- replication of representations; "k - copies"} \\ \text{- "fractional" instantons: } S_k = \dfrac{S_{k=1}}{k} \\ \text{- Exotic, chiral fermions} \end{array} \right.$