Primordial black holes: fine-tuning and fine opportunities

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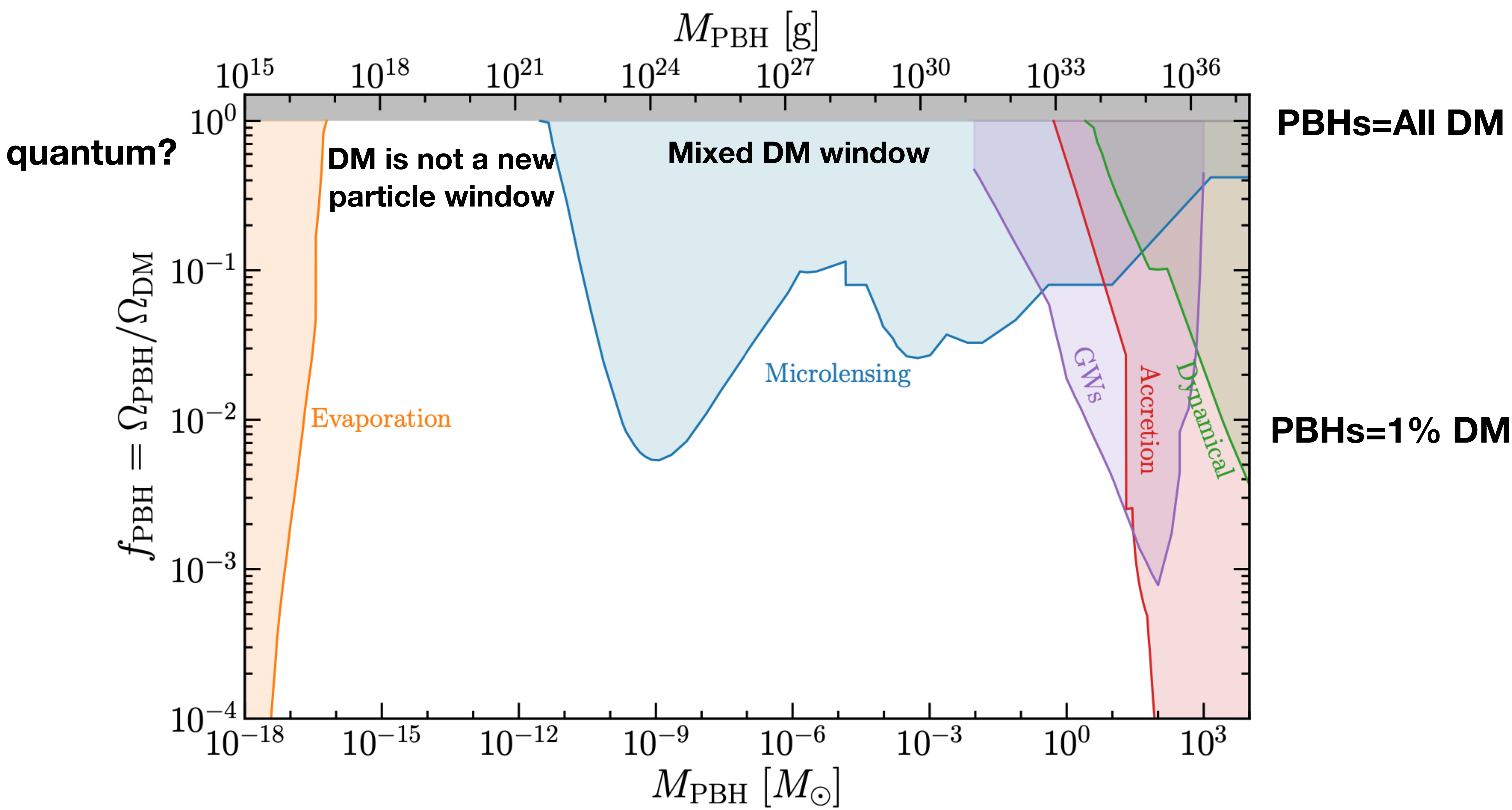
Contents and initial conditions of the universe

- Black holes are cold and dark, but hard to produce

- Primordial (unlike astrophysical) BHs are a DM candidate
- Alternatively, decaying BHs could probe quantum gravity
- Forming PBHs requires something special (perhaps)

Observational constraints



Caveats apply: *Green and Kavanagh review* *https://arxiv.org/pdf/2007.10722.pdf*

Contents

1. Fine-tuning
   Nearly complete with Pippa Cole, Andrew Gow and Subodh Patil

2. Fine opportunities
   Lot’s of past papers by myself and others

Probing all scales

- The CMB+LSS provide a precision test of large scale physics - provide limited info on the inflationary model

- They probe about 6 out of 60 efolds of length scales

- Primordial black holes (PBHs) and stochastic gravitational wave background (SGWB) constrain all scales - weakly

- What are the chances of a detection?

Stochastic GW background

- At second order, scalar and tensor perturbations couple

$$\boxed{\Omega_{\rm GW}^{\rm induced} \sim \frac{1}{12}\Omega_{r,0}\mathcal{P}_{\mathcal{R}}^2 \sim 10^{-6}\mathcal{P}_{\mathcal{R}}^2(k \gg k_{\rm CMB})}$$

- Each scale has corresponding horizon (PBH) mass and GW frequency - *Domenech 2021 review*

Fine-tuning

- $\epsilon = \dfrac{\partial \ln(\text{observable})}{\partial \ln(\text{parameter})}$

- If you vary the model parameters the observables will vary

- The question is by how much - *Azhar & Loeb 2018; Nakama & Wang 2019*

- A value <<1 implies the observable is robust, or the parameter is irrelevant

- An order unity value would mean the output changes by the same magnitude as the input

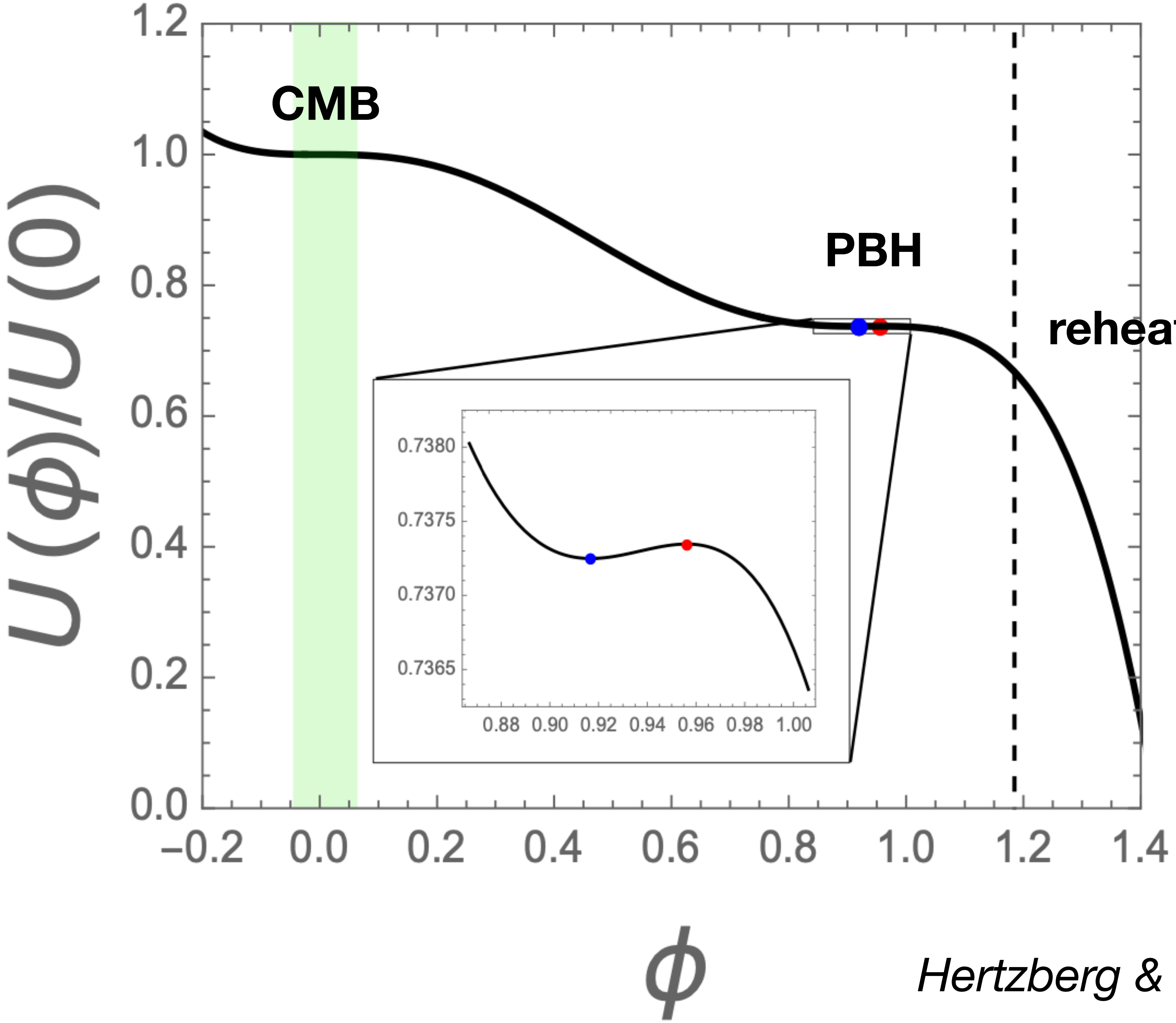
- A value of 100 means that the observable changes 100 times more quickly than the input value

- Let's start with the power spectrum amplitude

Questions

- If you take a model of inflation with a peak (to generate PBHs/SGWB) and vary one of the potential parameters at random, by how much would do you need to vary it such that the peak amplitude changes by order unity?
- We study single-field inflation with an inflection point (ultra slow roll). A feature is needed.
- Arguably the most economical model, but natural?
- We start with a polynomial potential

A polynomial potential



Hertzberg & Yamada 2017

A polynomial potential

$$V(\phi) = c_0 + \frac{c_1}{\Lambda}\phi + \frac{c_2}{2\Lambda^2}\phi^2 + \frac{c_3}{3!\Lambda^3}\phi^3 + \frac{c_4}{4!\Lambda^4}\phi^4 + \frac{c_5}{5!\Lambda^5}\phi^5$$

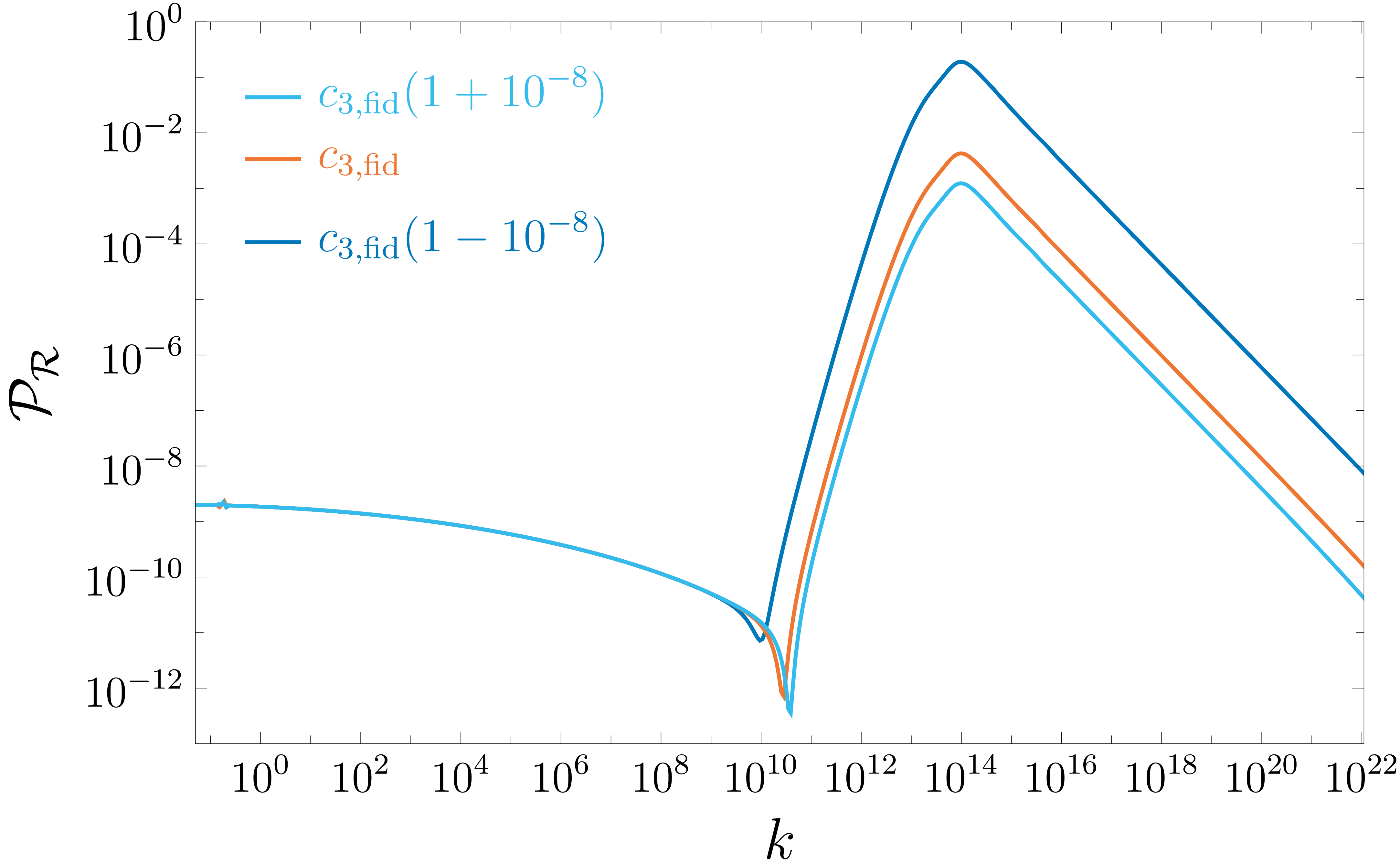
	Fiducial
ϕ_0	$-\lvert\Lambda^3/c_3\rvert$
$\phi_{\rm CMB}$	5.51674×10^{-4}
Λ	0.3
c_0	1
c_1	$(2\pi^2\Lambda^4)/(4225c_3)$
c_2	0
c_3	-0.52
c_4	1
c_5	-0.640725043

Clearly c_5 has been tuned to get the desired PBH abundance

Is this the only tuning required?

Hertzberg & Yamada 2017
but we use slightly different parameter values

A polynomial potential



Hertzberg & Yamada 2017

Polynomial problems

- A cubic potential is enough for an inflection point, why go further?
- One needs an inflection point for ultra-slow-roll inflation.
- Need a second “nearly” inflection point for the CMB perturbations, because polynomial inflation has too large tensor perturbations
- This represents an additional fine tuning
- Also need to fine tune initial velocity to stop overshooting the CMB flat part
- I had no idea it would be this hard

Hertzberg & Yamada 2017

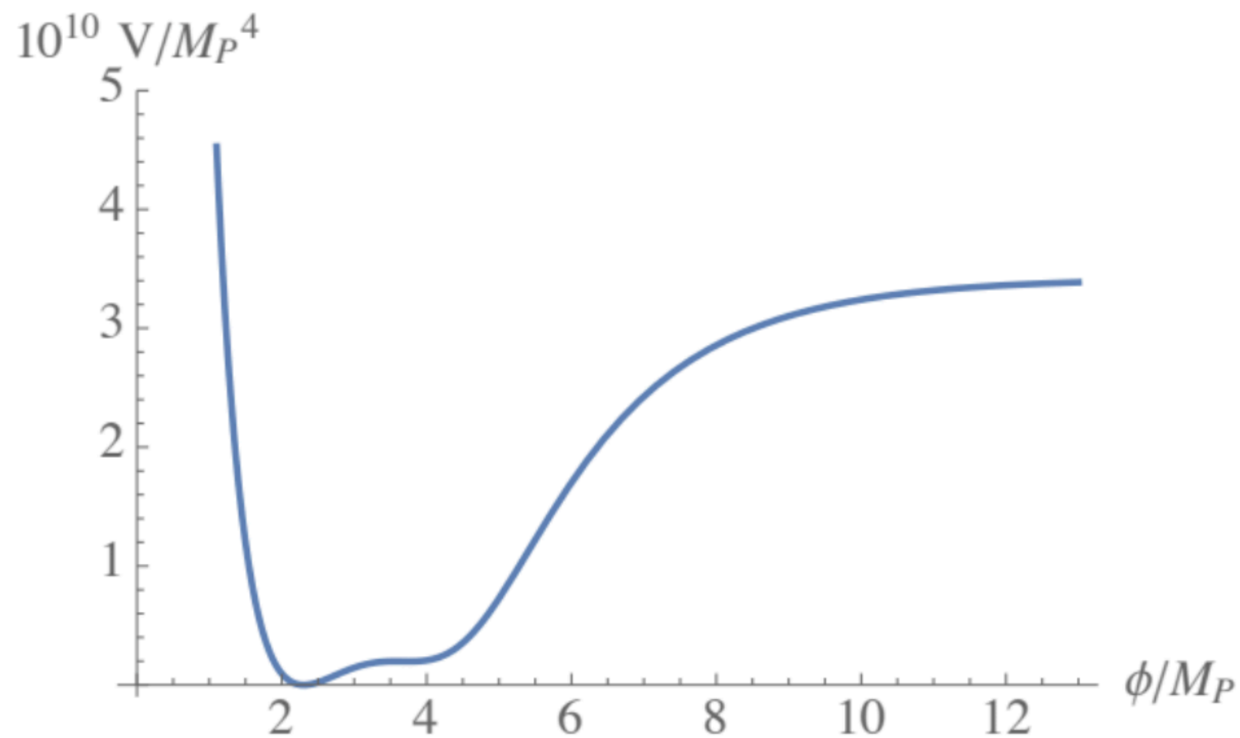
String inflation model

$$V(\phi) = V_0 \left[C_1 - e^{-\frac{1}{\sqrt{3}}\hat{\phi}} \left(1 - \frac{C_6}{1 - C_7 e^{-\frac{1}{\sqrt{3}}\hat{\phi}}} \right) + C_8 e^{\frac{2}{\sqrt{3}}\hat{\phi}} \left(1 - \frac{C_9}{1 + C_{10} e^{\sqrt{3}\hat{\phi}}} \right) \right]$$

$$C_6 = \frac{A_W}{C_W} \qquad C_7 = \frac{B_W}{\langle \tau_{K_3} \rangle^1 / 2} \qquad C_8 = 0$$

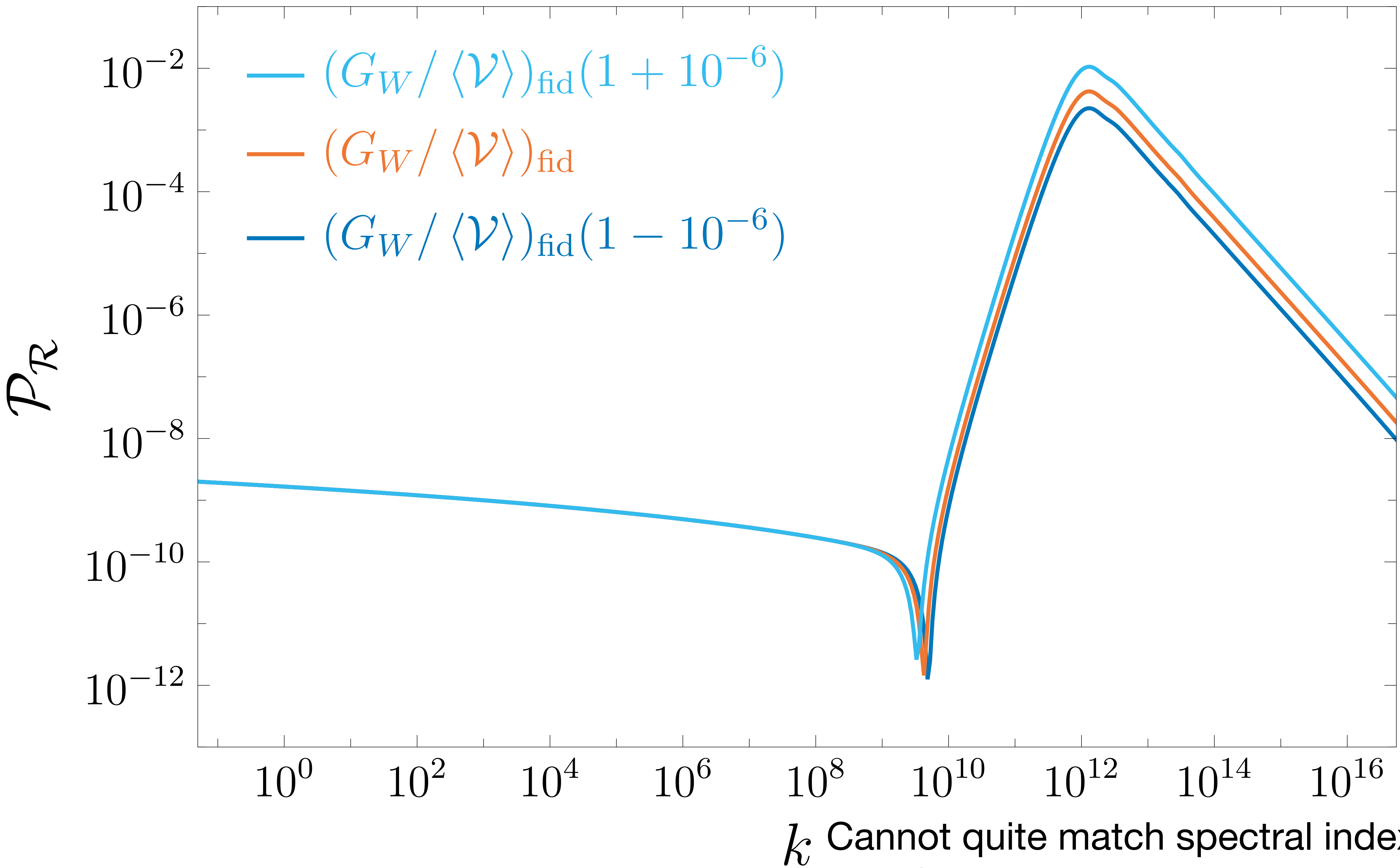
$$C_8 C_9 = \frac{G_W}{\langle \mathcal{V} \rangle} \frac{\langle \tau_{K_3} \rangle^3 / 2}{C_W} \qquad C_{10} = \frac{R_W}{\langle \mathcal{V} \rangle} \langle \tau_{K_3} \rangle^3 / 2$$

	Fiducial
ϕ_0	12
$\phi_{\rm CMB}$	9.33731
A_W	2/100
B_W	1
C_W	4/100
$\langle \tau_{K_3} \rangle$	14.30
$G_W / \langle \mathcal{V} \rangle$	3.08054×10^{-5}
$R_W / \langle \mathcal{V} \rangle$	7.071067×10^{-4}



Plot with their parameters
Cicoli, Diaz & Pedro 2018

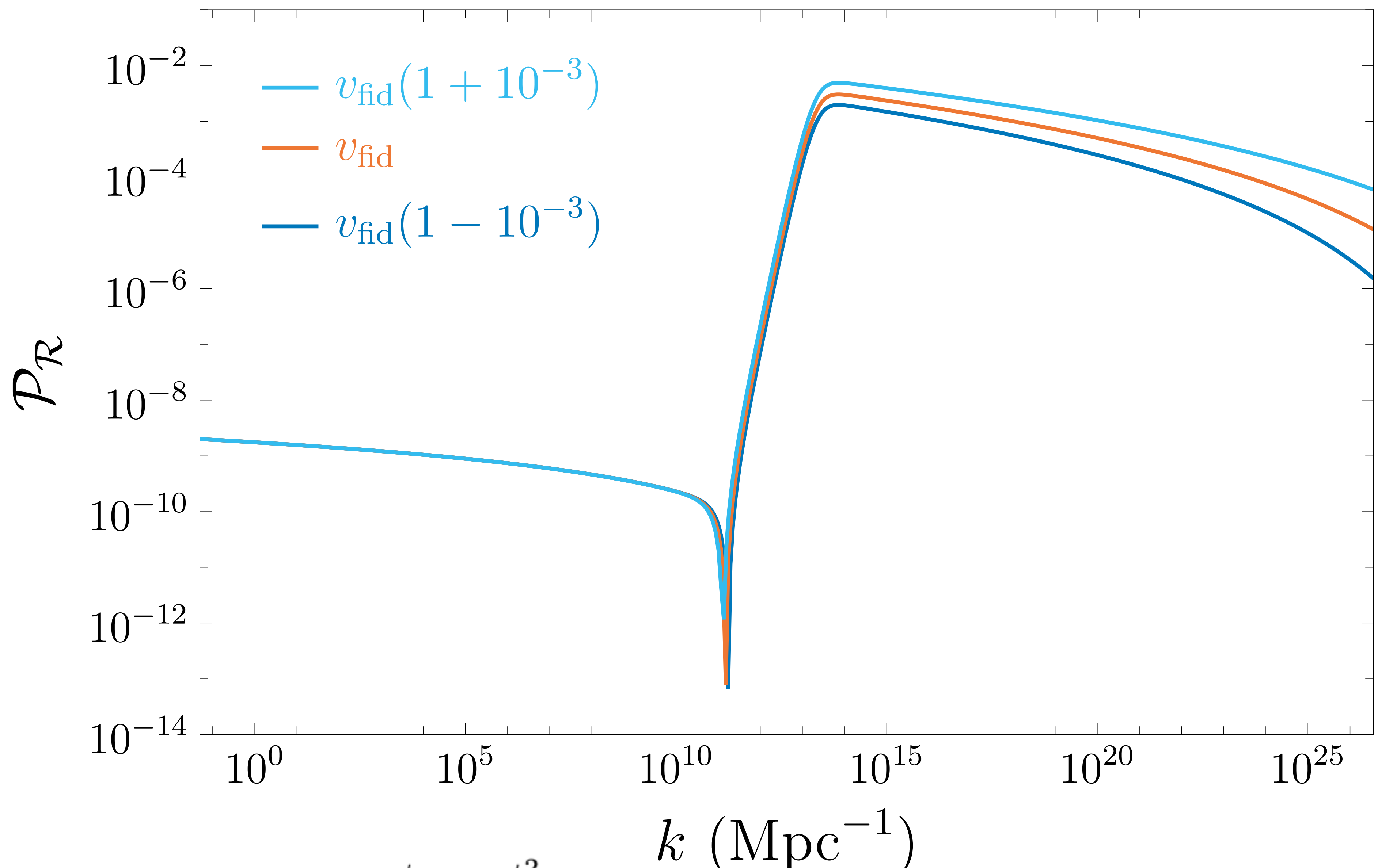
String inflation model



Cannot quite match spectral index

Cicoli, Diaz & Pedro 2018

“Least tuned” model



$$V(\phi) = \frac{\lambda}{12}\phi^2 v^2 \frac{6 - 4a\frac{\phi}{v} + 3\frac{\phi^2}{v^2}}{\left(1 + b\frac{\phi^2}{v^2}\right)^2}$$

Cannot quite match spectral index
Germani & Prokopec 2017

Designer potential

$$V(\phi) = V_0 \frac{\phi^n}{\phi^n + M^n}\left(1 + A \exp\left[-\frac{(\phi - \phi_d)^2}{2\sigma^2}\right]\right)$$

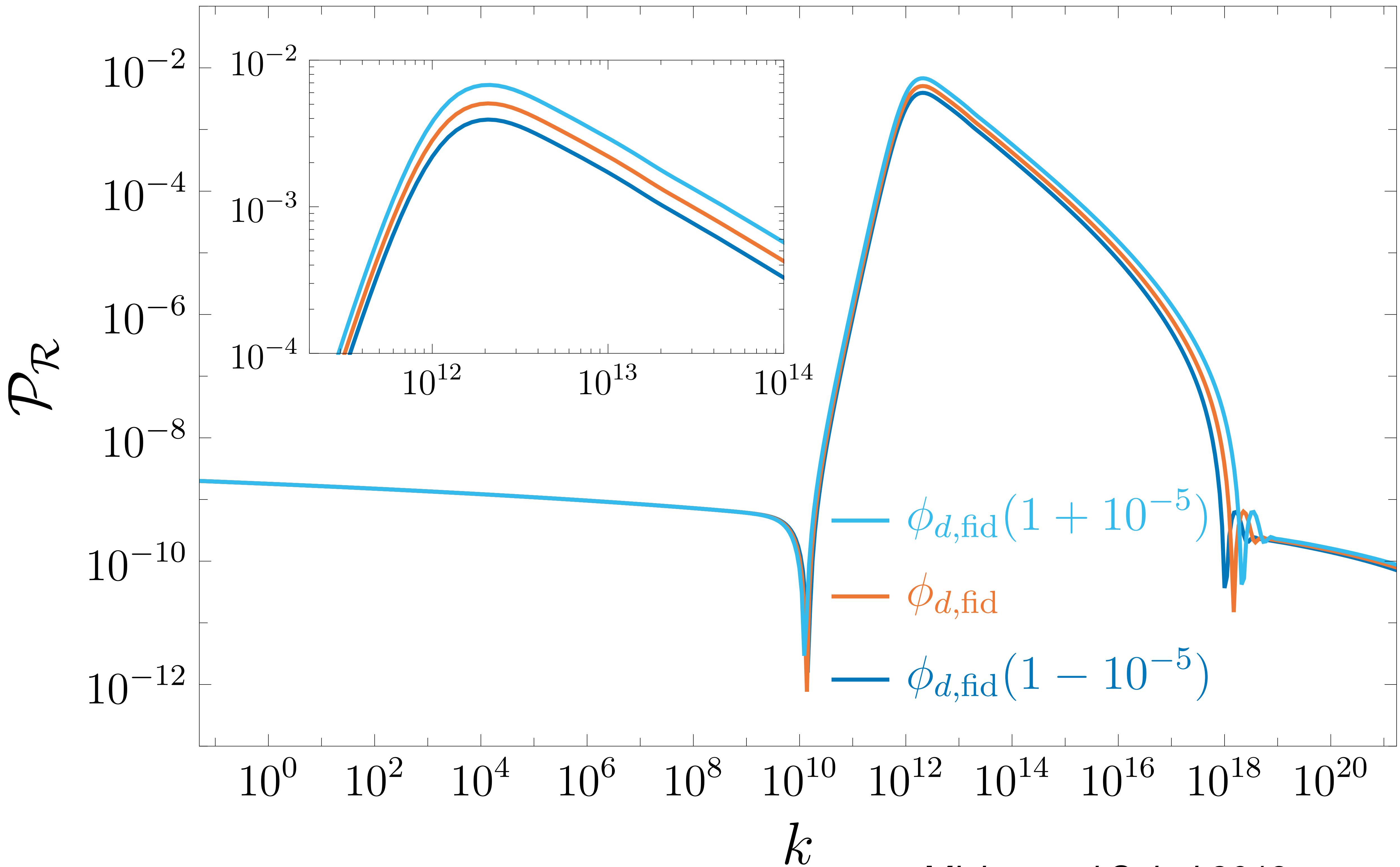
	Fiducial
ϕ_0	4
ϕ_{CMB}	3
n	2
M	$M_{\mathrm{Pl}}/2$
A	1.17×10^{-3}
σ	1.59×10^{-2}
ϕ_d	2.18812

phi_d, sigma and *A* set the details of the bump which are added by hand

This allows the bump to be added to any model which otherwise matches observations

Does the fine tuning only depend on the amplitude of the bump?

Mishra and Sahni 2019



Mishra and Sahni 2019

Fine-tuning

- Is multifield inflation better?
- How about completely different formation mechanisms? Arguably yes, exponentially better from preheating for light PBHs! *Martin et al 2019*
- A smaller peak requires less tuning
- Non-Gaussianity?
- Reduction in equation of state?
- You can say nature just picks a value, which we can't predict

General lessons

- Forming light PBHs is “easier” since the PBH and CMB scales are more widely separated
- A discontinuous potential separates scales but is unmotivated
- Matching CMB observations to an inflection point smoothly is hard
- Fine-tuning varies significantly between models but we never find a WIMP miracle
- Fine-tuning is insensitive to PBH mass

Interlude

EDI

- Some personal opinions, discussion and feedback is welcome

How you can help

- Undertake unconscious bias training and actually think about the lessons - how can you apply them in your role?
- Politely call out bad behaviour if possible - undertake bystander training
- Fill in staff/culture surveys and ideally don’t tick “prefer not to say” options, to help fight against small number statistics issues
- Actions >> promised actions

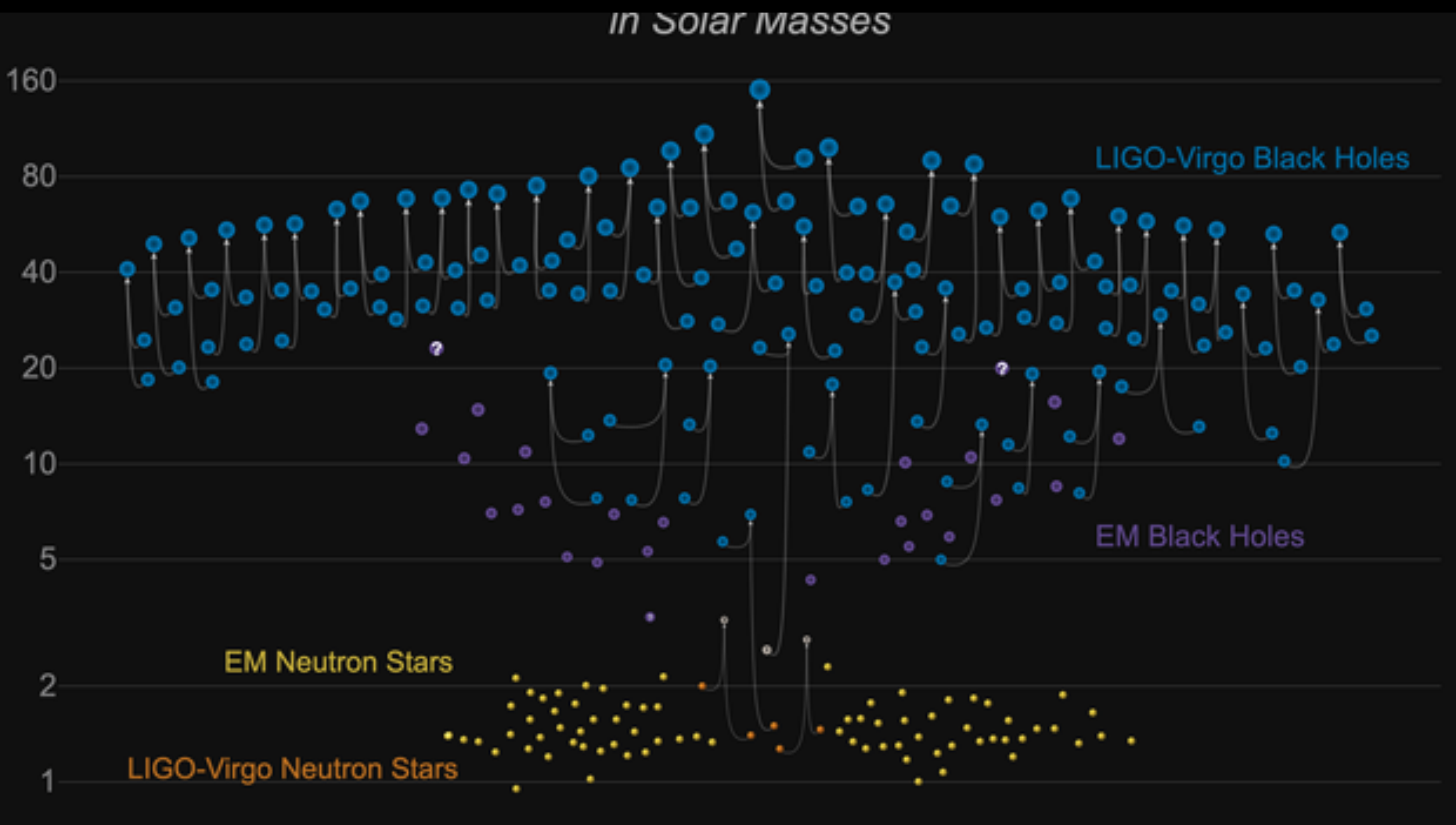
How can management help

- Don’t ask for pointless reports, and do share best practice between groups
- If you do request reports, surveys, etc, then make sure there are resources available to act on the results!
- Don’t oversimplify - correlation is not causation
- See the bigger picture
  Diversity on panels is good, but overloading underrepresented groups with panel work isn’t
- Actions >> promised actions

Part 2: Fine opportunities

The LIGO-Virgo-Kagra events

- It appears unlikely that more than 1% of the dark matter can be made out of LIGO mass PBHs
- But could the LIGO BHs be primordial?
- Black holes have no hair, so how can we know?



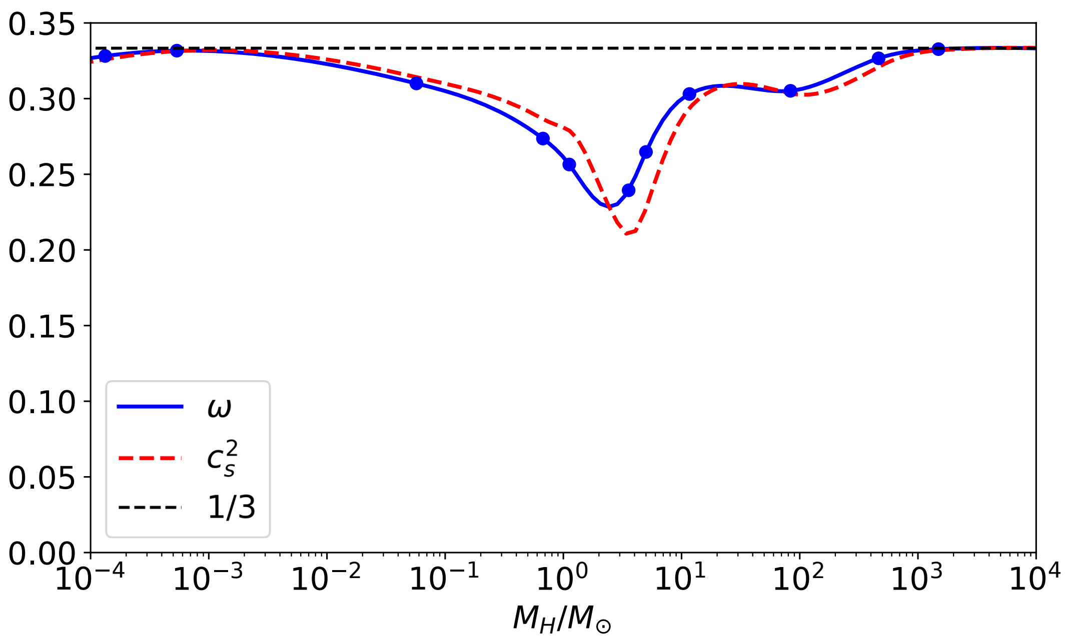
LIGO & Virgo collaboration

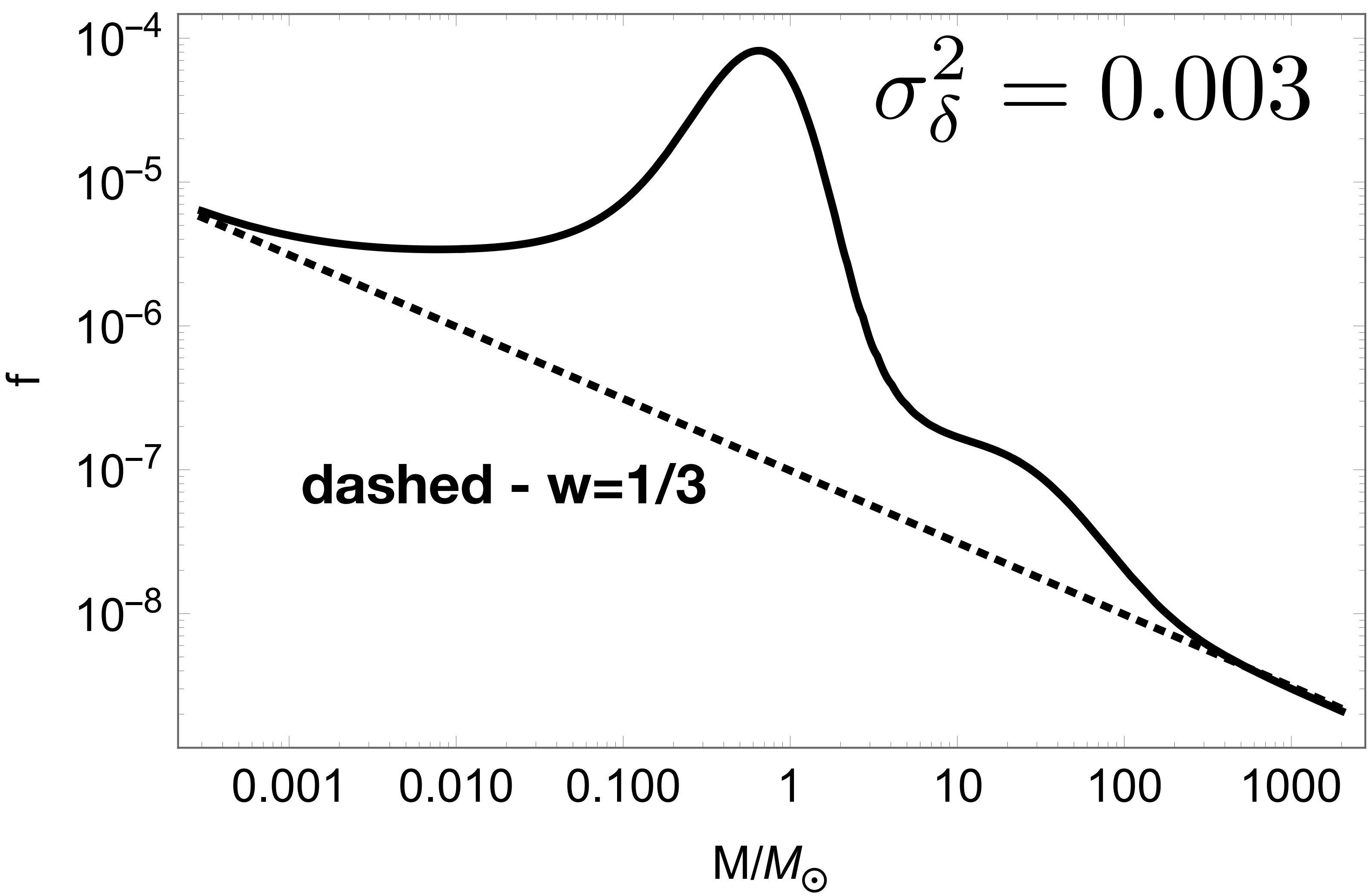
Fine opportunities

- More and better LIGO-Virgo-KAGRA data soon
- Current detections must be mainly or purely astrophysical compact objects - *Hall, Gow, CB, ’20; Hutsi et al 2021; de Luca et al 2022; Franciolini 2022*
- Opportunity to observe sub-Chandrasekhar mass object
- “Cleanest” PBH vs astrophysical BH signature
- QCD connection motivates low mass objects within SM physics - arguably the cleanest PBH signature
- PTA constraints are complementary GW probe - at the right frequency and sensitivity

The PBH-QCD mass function

$$f(M) \propto M^{-1/2} e^{-\frac{\delta_c^2}{2\sigma_\delta^2}}$$





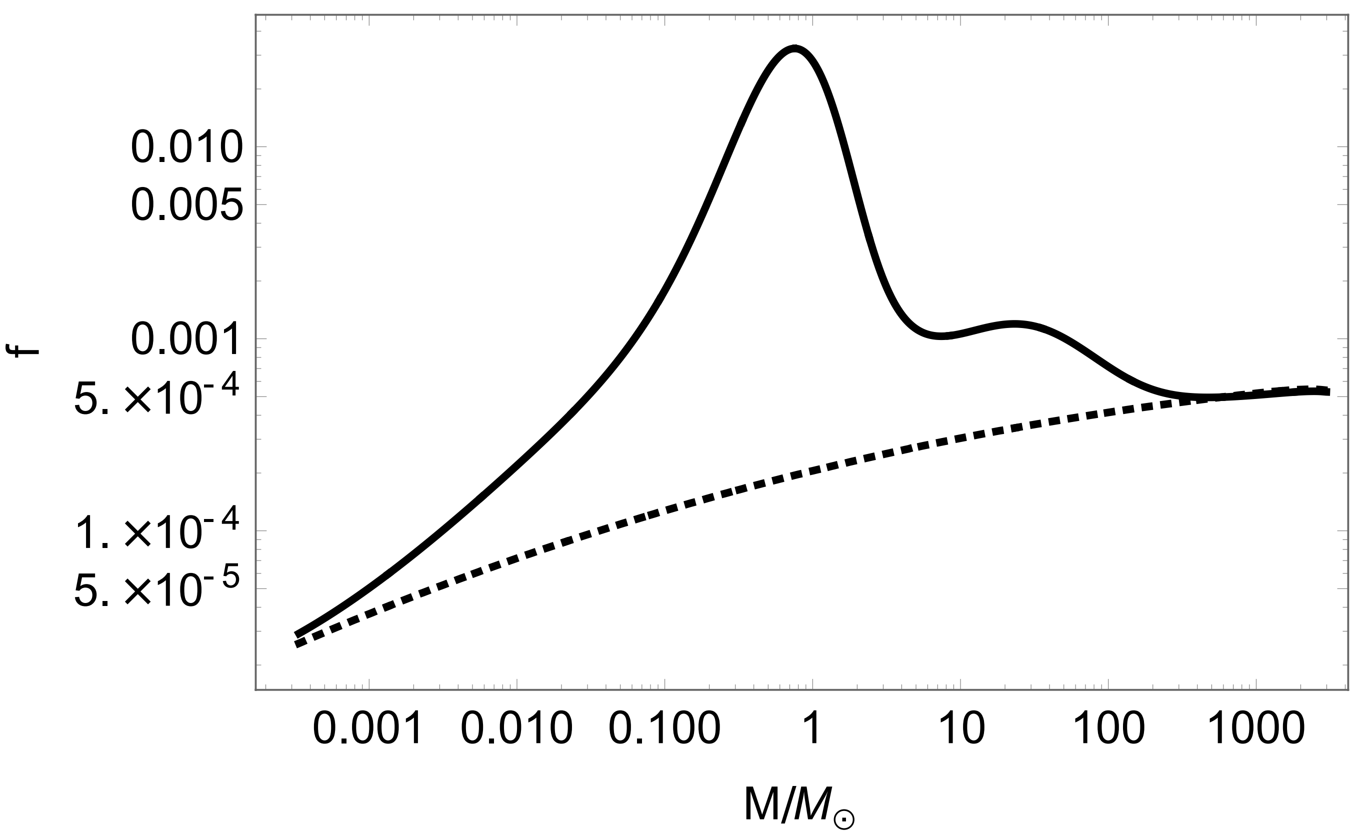
CB, Hindmarsh, Young & Hawkins 2018

The QCD phase transition took place during the time when LIGO mass PBHs would have formed - *Carr et al '19; Franciolini et al; Escriva et al 22*

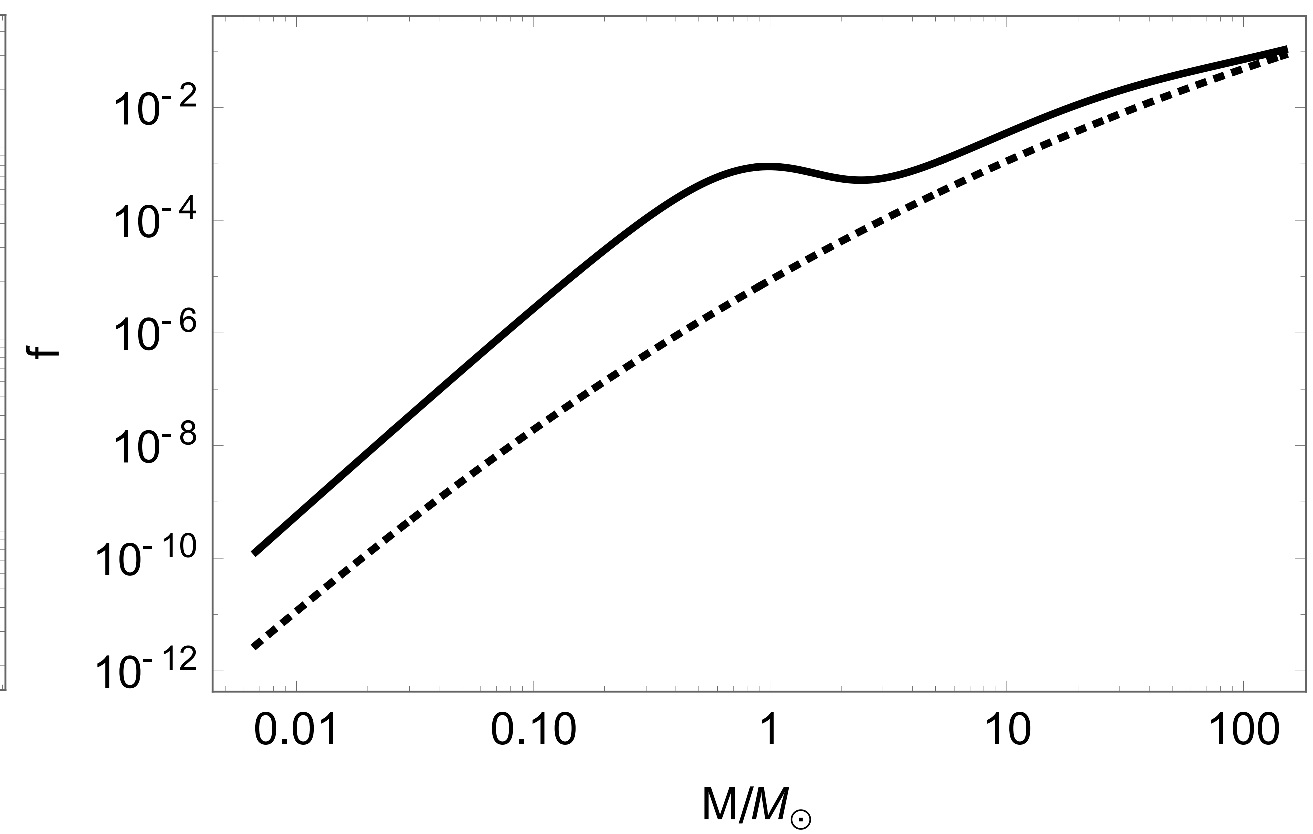
Varying the primordial perturbations

If the primordial power spectrum is not scale invariant on the relevant scales then the mass function changes, but a peak remains

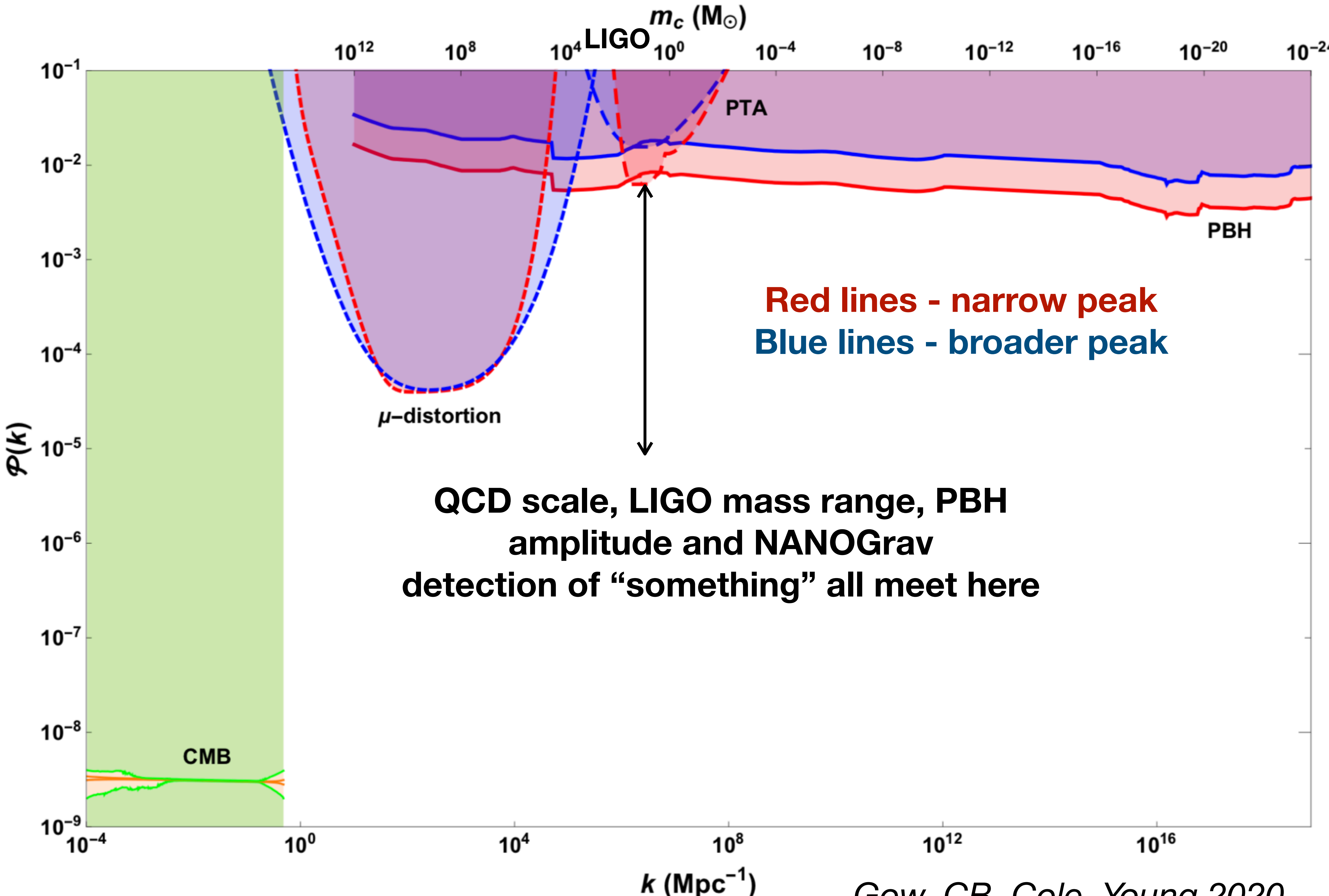
$$n_s - 1 = -0.05$$



$$n_s - 1 = -0.2$$



The initial conditions of the universe



Gow, CB, Cole, Young 2020

Power spectrum messages

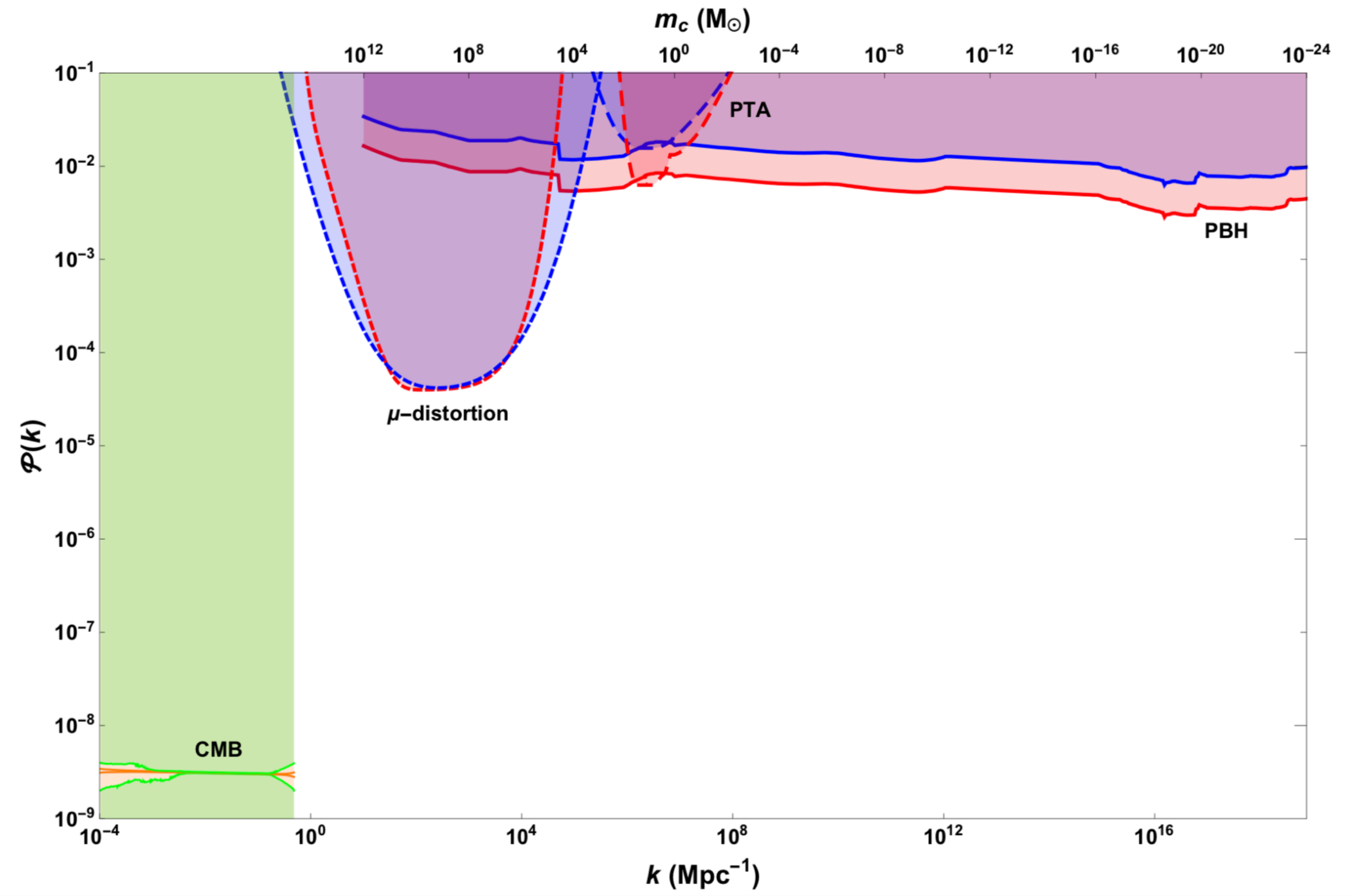
- Assuming Gaussian perturbations and that PBHs form from the direct collapse of large overdensities

    1. The formation of supermassive PBHs is ruled out

    2. LIGO-Virgo mass BHs produce a stochastic GW background which the PTA experiments should detect now/soon

    3. No competitive power spectrum constraints on even smaller scales, yet

Reflections



- Constraints need to be made for a consistent choice of power spectrum peak.

- Choice of Press-Schechter vs peaks not very important, likewise for the window function.
  Non-Gaussianity is a degeneracy

- Beware the simple relations between horizon and PBH mass - we find an order-of-magnitude shift to heavier PBH masses for any given *k* value

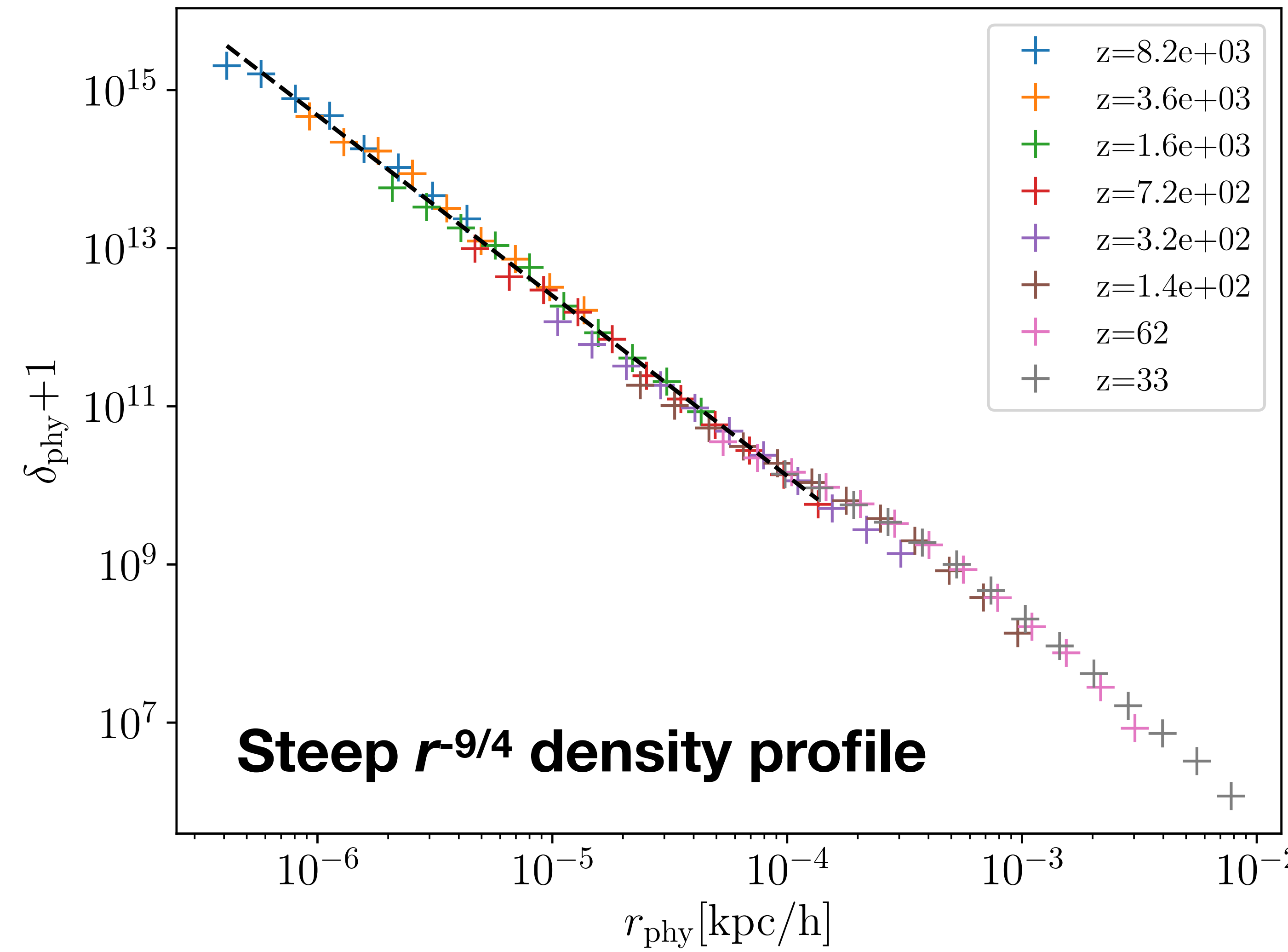
Gow, CB, Cole, Young 2020

- Accurate calculations are (finally) required

WIMPs and PBHs are incompatible

- Assuming WIMPs have the standard, velocity independent cross section which gets the right abundance, and $M_{PBH}>10^{-6}$ M_{sun}.
- If $f_{PBH}<1$, then another DM component is inevitable
- Steep and high density profiles form around PBHs (density~ $r^{-9/4}$). WIMPs would rapidly annihilate to gamma rays.
- In contrast to ultracompact minihalos without a PBH seed. *Gosenca et al '17, Delos et al '17*

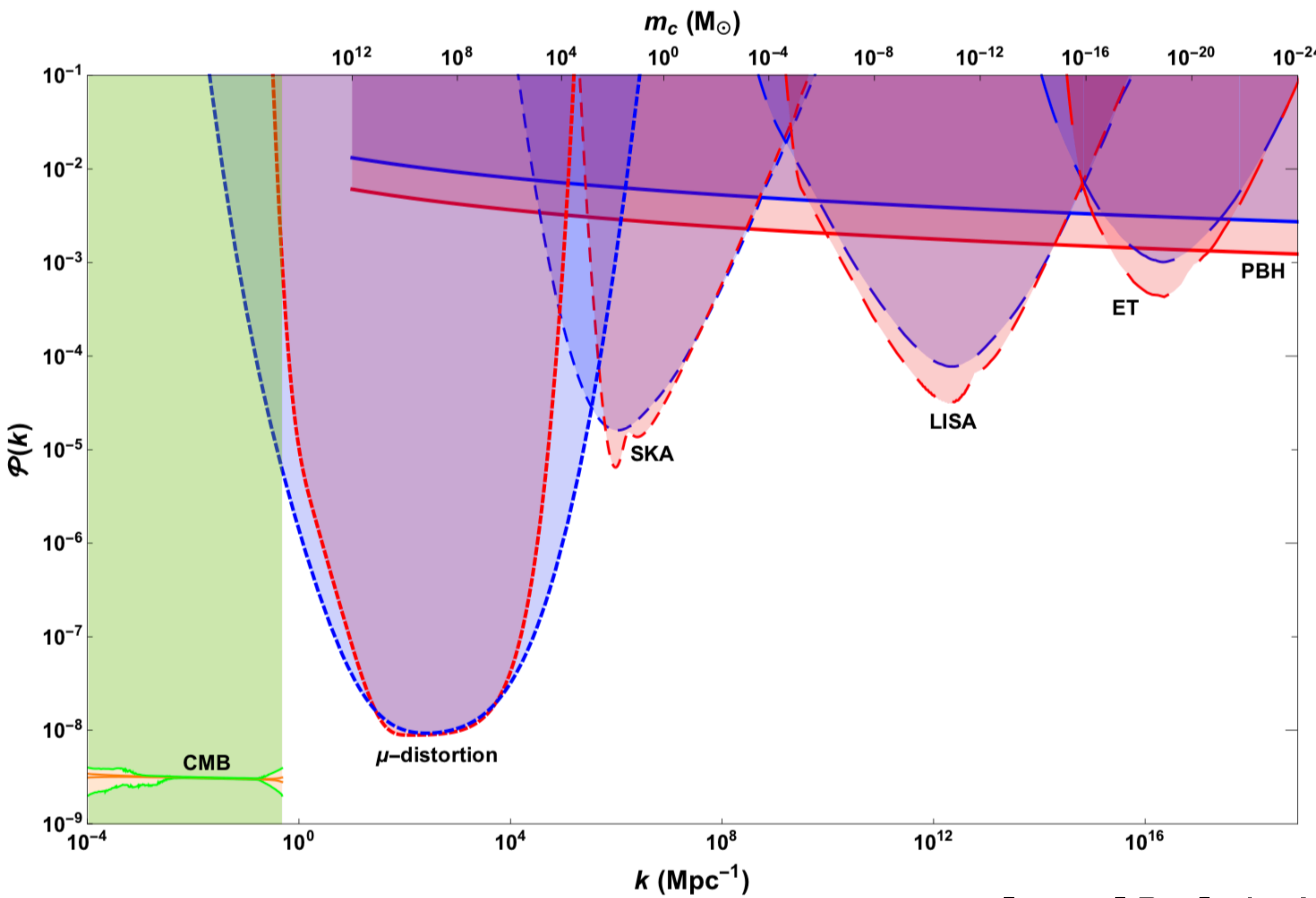
- A detection of WIMPs or PBHs may effectively rule out the existence of the other



Adamek, CB, Gosenca & Hotchkiss 2019;

Lacki & Beacom 2010; Eroshenko 2016; Boucenna, Kühnel, Ohlsson & Visinelli 2017
The 3 papers above all find different profiles.
We made the first simulations of this scenario

A bright future (forecast)



The PBH lines correspond to zero PBHs *Cole & CB '17*

Gow, CB, Cole, Young 2020

Summary

- PBHs are hard to produce
- Any detection would transform our knowledge of the contents and initial conditions of the universe
- LIGO, QCD and PTA coincidence

Relating PS amplitude to f_{PBH}

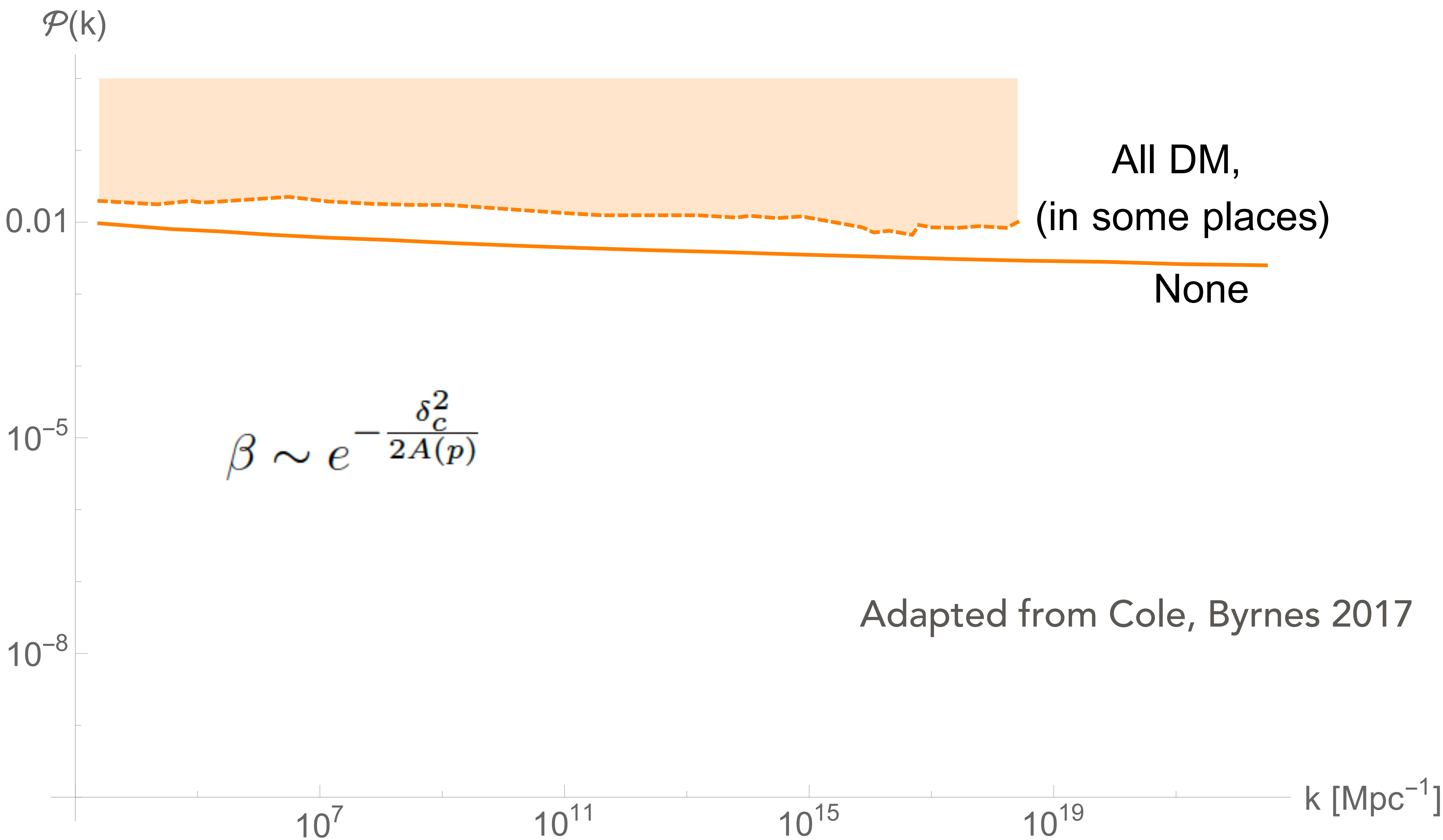
$$\beta \sim e^{-\frac{\delta_c^2}{2A(p)}}$$

$$\epsilon_{AL} = \frac{d\ln(\beta)}{d\ln(p)} = \frac{2\delta_c^2}{A(p)} \frac{d\ln(A(p))}{d\ln(p)}. \qquad \frac{2\delta_c^2}{A(p)} \simeq 10 - 100$$

- f_{PBH} values are fine-tuned by 1-2 orders of magnitude more than the power spectrum amplitude - only
- We have confirmed this numerically
- SGWB and PBH production both require tuning
- Makes a Bayesian model comparison disfavour PBHs (via this mechanism)

Required peak amplitude?

Hardly varies for any mass/f_{PBH}



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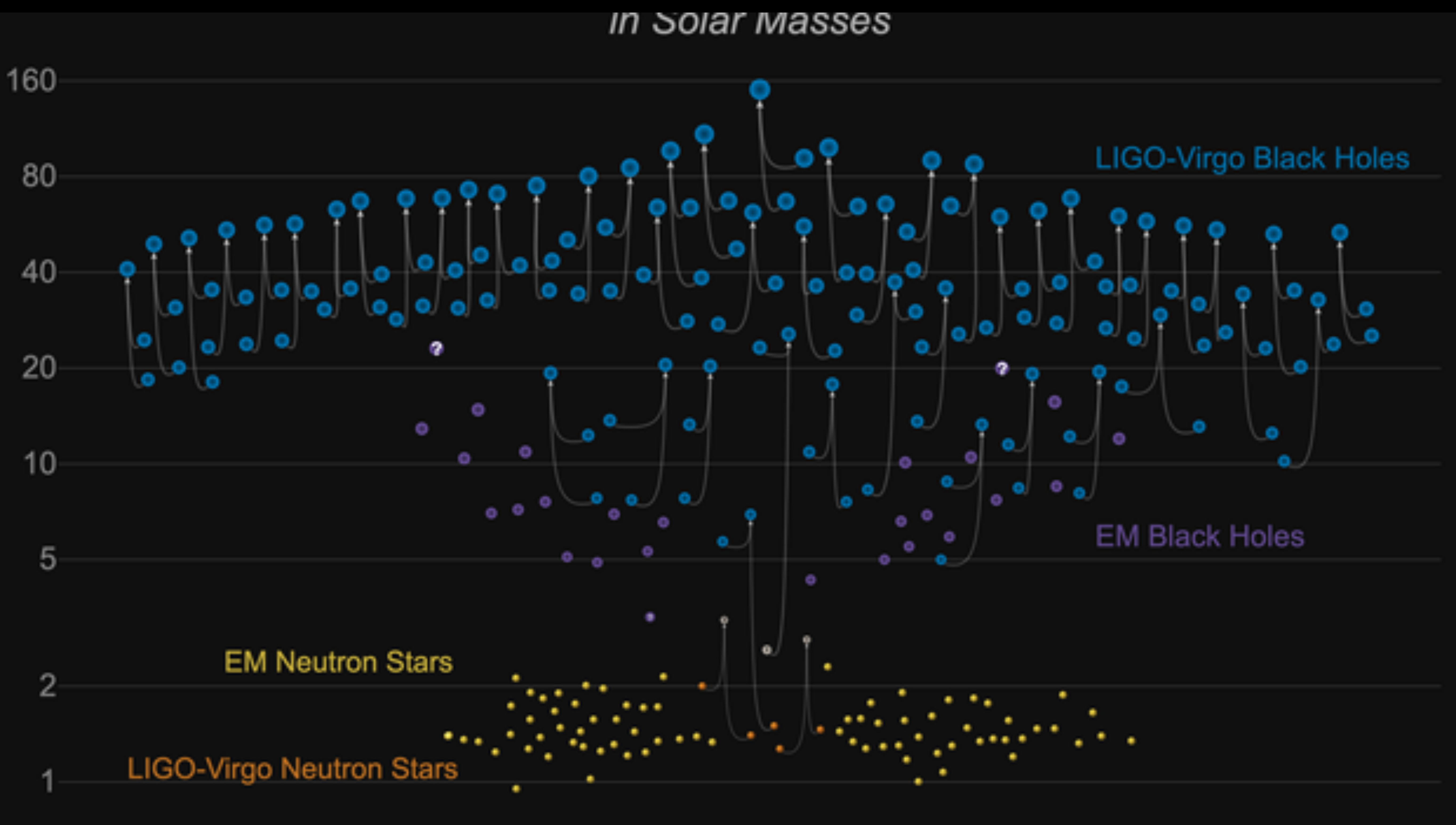
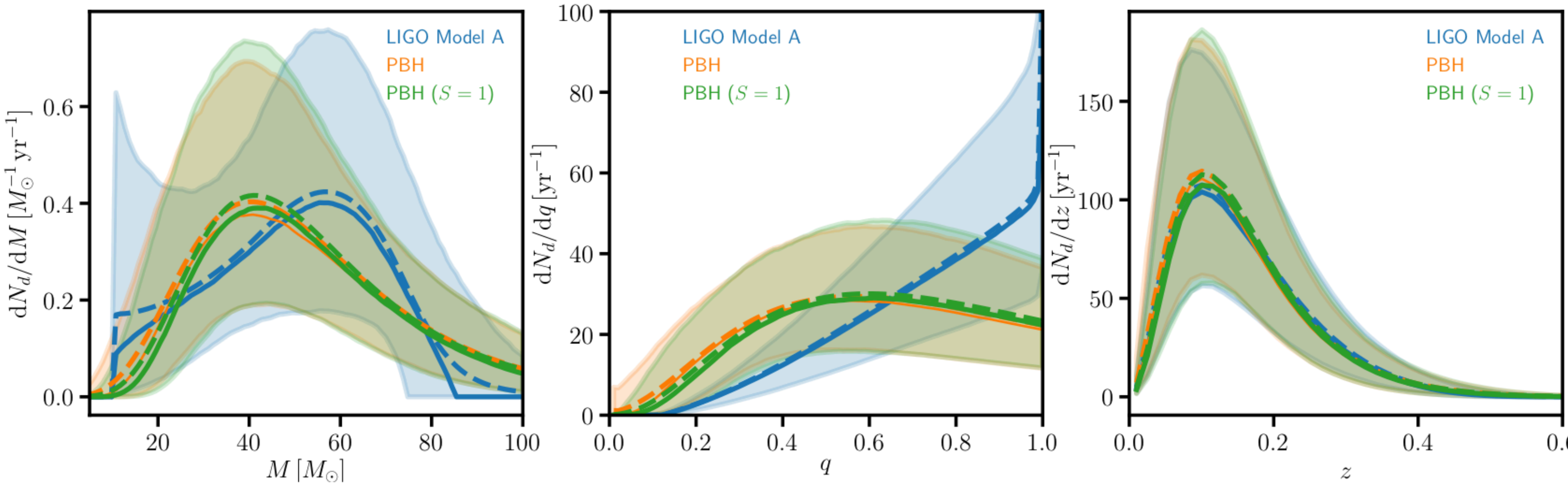


LIGO & Virgo collaboration

total mass **mass ratio** **redshift**



Hall, Gow, CB, 2020: Bayesian comparison

PBHs are more flexible at explaining individual events.

Total/chirp mass information dominates the signal.

Spin in isolation favours PBHs: *Fernandez & Profumo `19, Garcia-Bellido et al ’20, Wong et al 2020*

Ligo-Virgo BH lesson

- PBHs alone strongly disfavoured against only stellar BHs: Even when attempting to fit arbitrarily tuned PBH mass functions: *Hall, Gow, CB, '20*

- However, some evidence that a subdominant PBH population improves the fit: e.g. *Hutsi et al 2021, de Luca et al 2022, Franciolini 2022 - all agree it must be subdominant*

- This evidence depends on the astrophysical formation channels, which are highly uncertain and regularly updated

Sub-solar mass compact objects

- Second generation compact objects can only be heavier
- A sub Chandrasekhar/solar mass compact object cannot form within standard model astrophysics
- This mass scale corresponds to the QCD transition when quarks bind into hadrons:
  $t \sim 10^{-6}$ s, $T \sim 200$ MeV, $M \sim 1\ M_{\odot}$, $k \sim 10^{7}$ Mpc^{-1}

LVK PBH lesson II

- Almost a consensus that $f_{PBH}<1$ for LVK masses, even without including the GW merger rate constraints
- So we are necessarily studying a mixed DM model
- No reason why DM should all be the same thing
- Another opportunity is to probe the rest of the DM, which is expected to form a dense PBH dress
- The detection of **just 1 PBH** probes particle DM annihilations