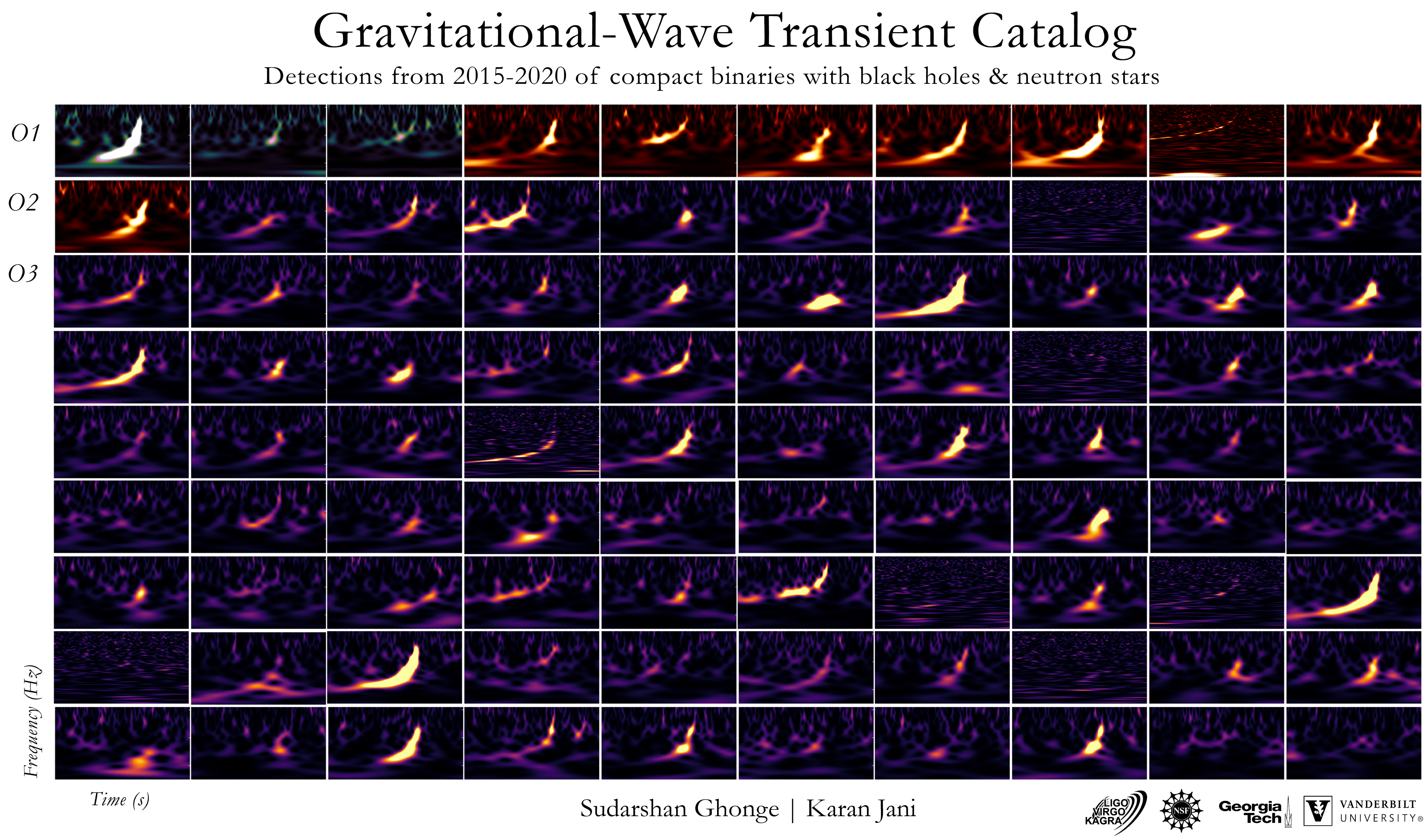
JOHNS HOPKINS UNIVERSITY

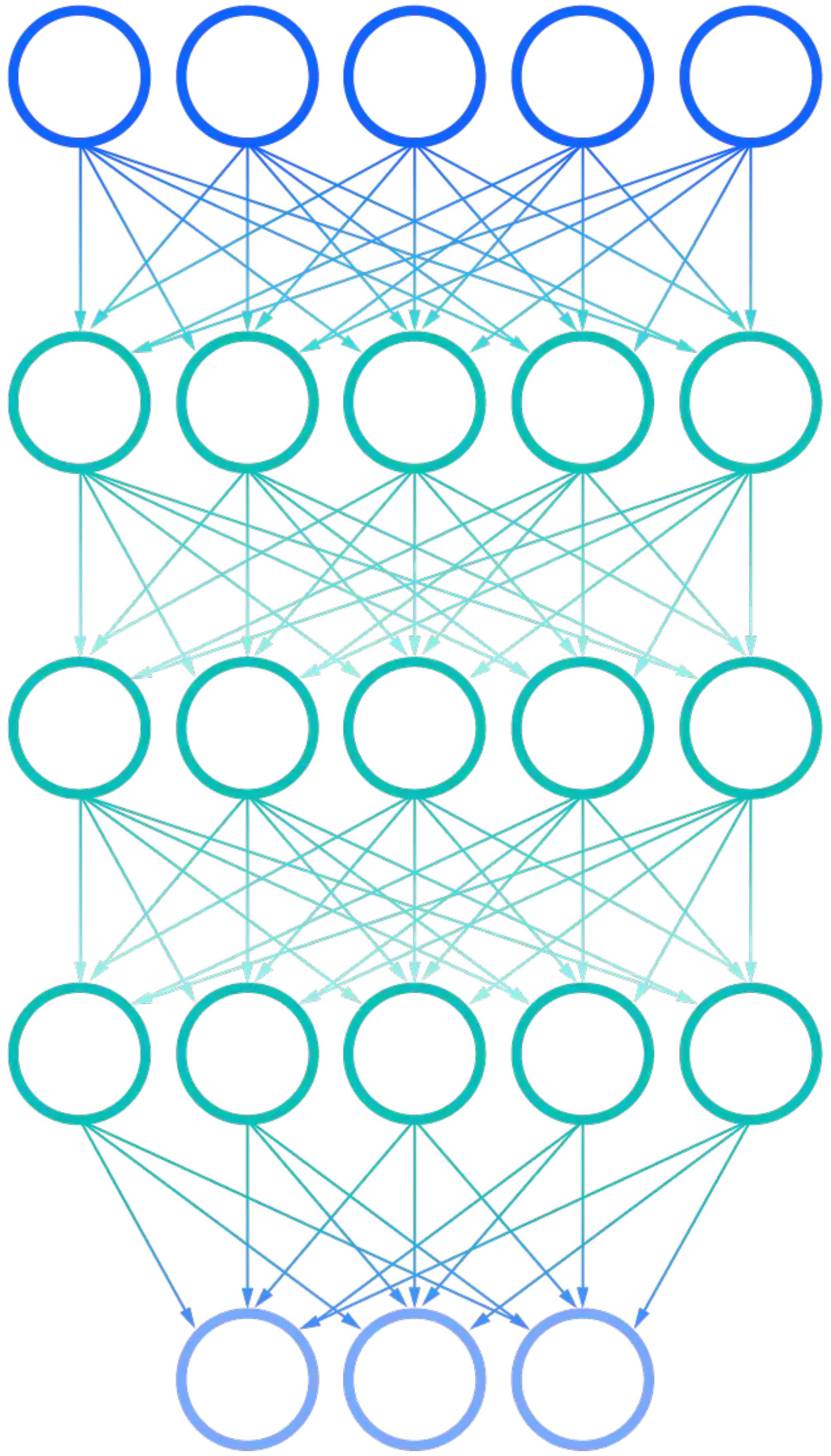
Enhancing Gravitational Wave Science with Differentiable Models and New Physics Searches

Thomas Edwards | King College London | 16th June 2023

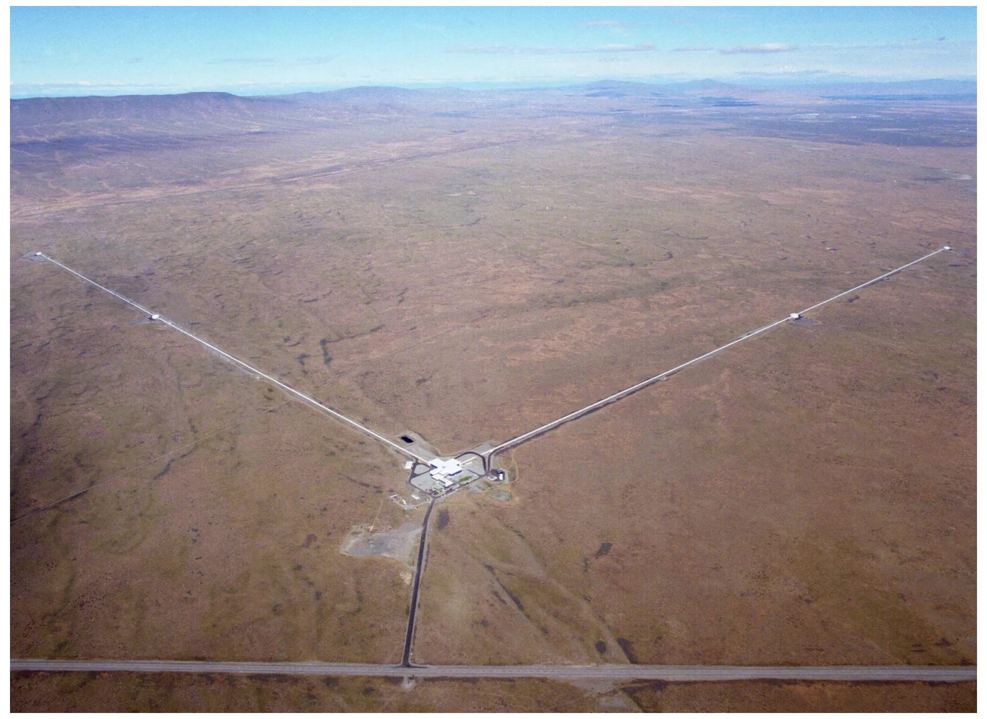
2302.05329
2302.05333
2306.00050
+ 2 upcoming papers

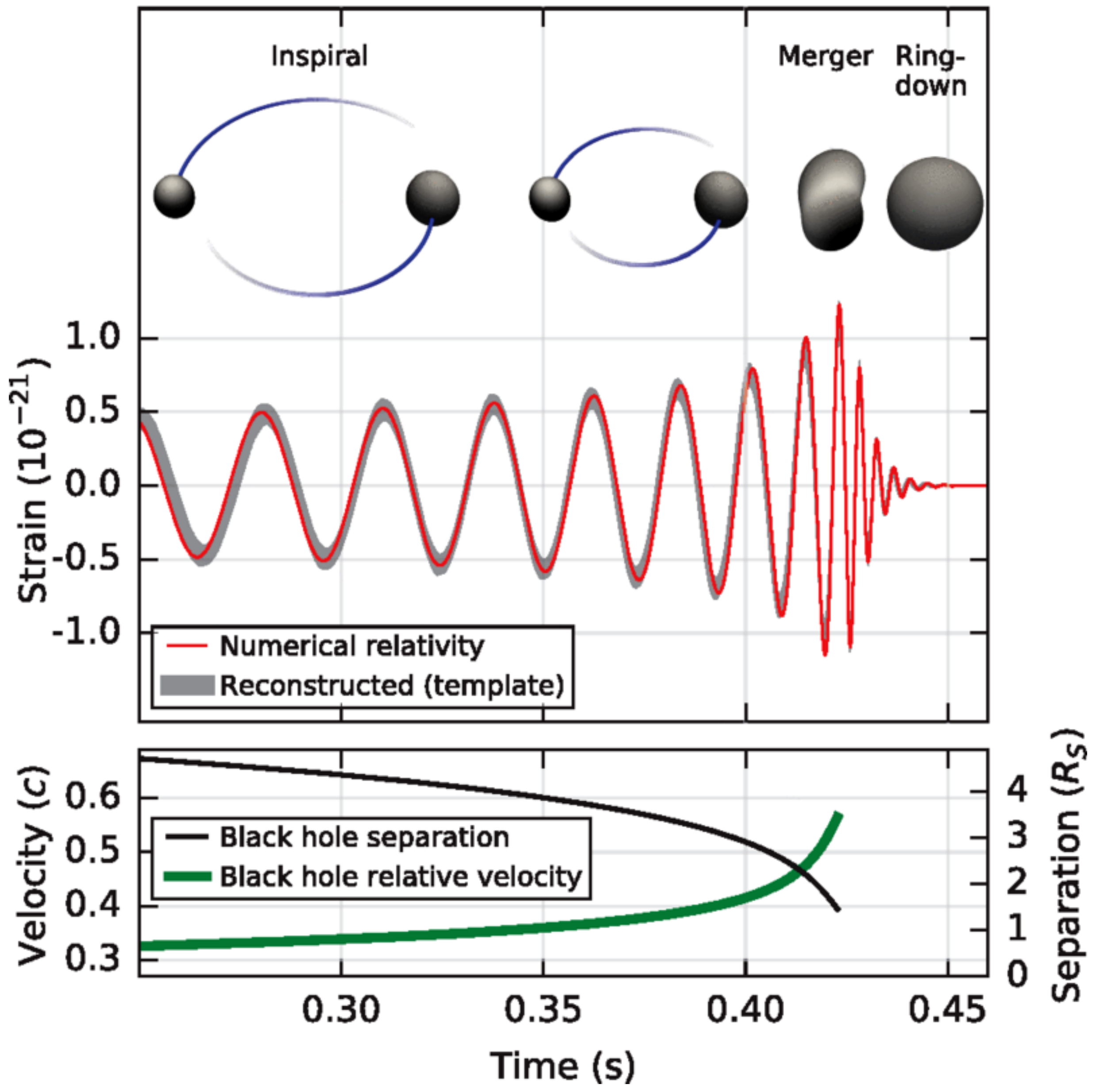
Gravitational Wave + ML Revolution





Gravitational Wave Analysis





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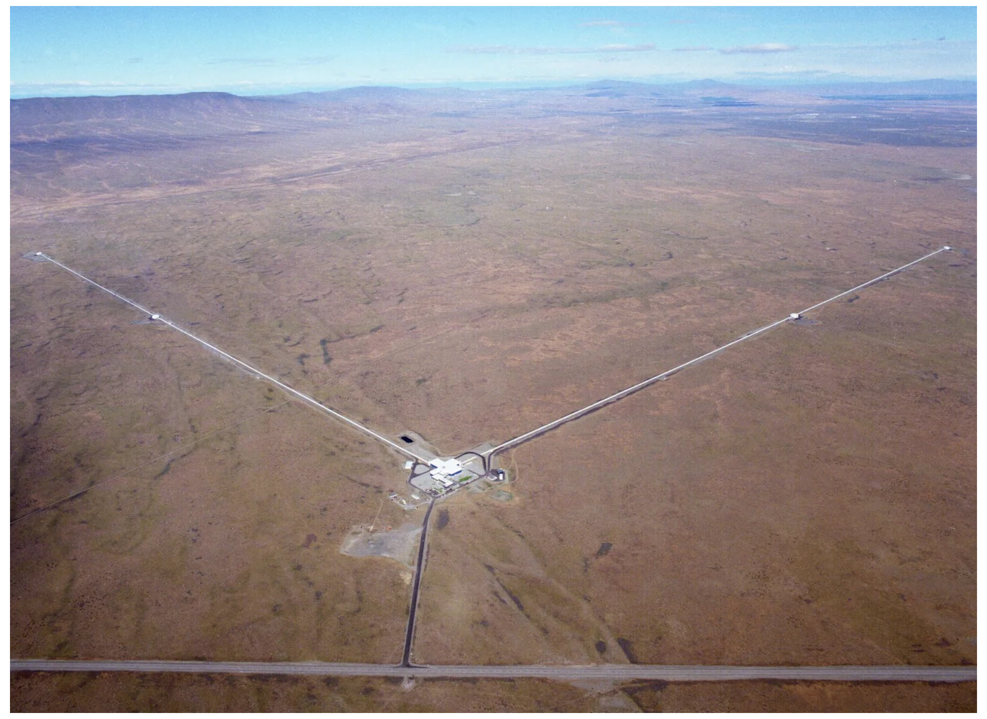

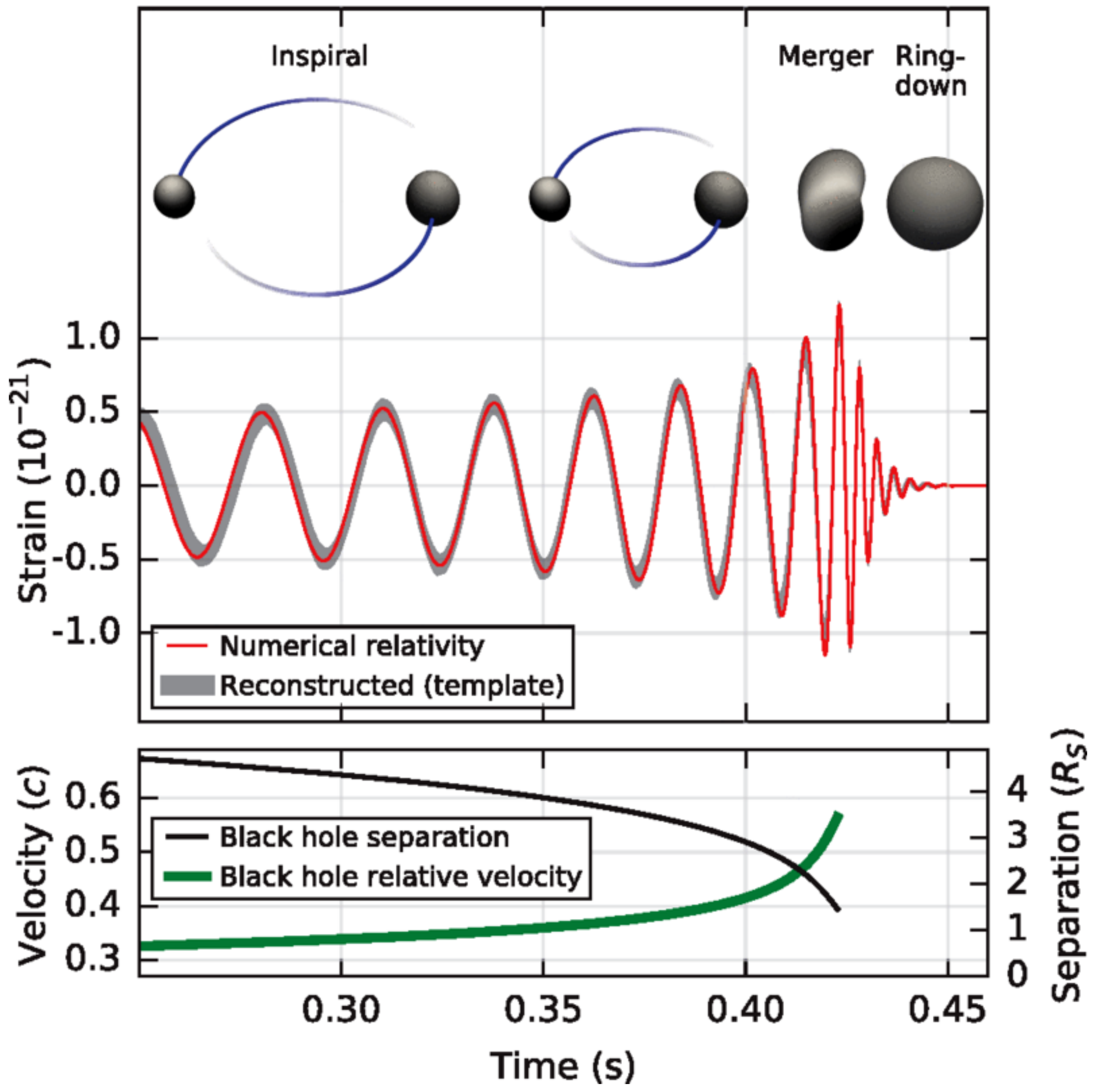

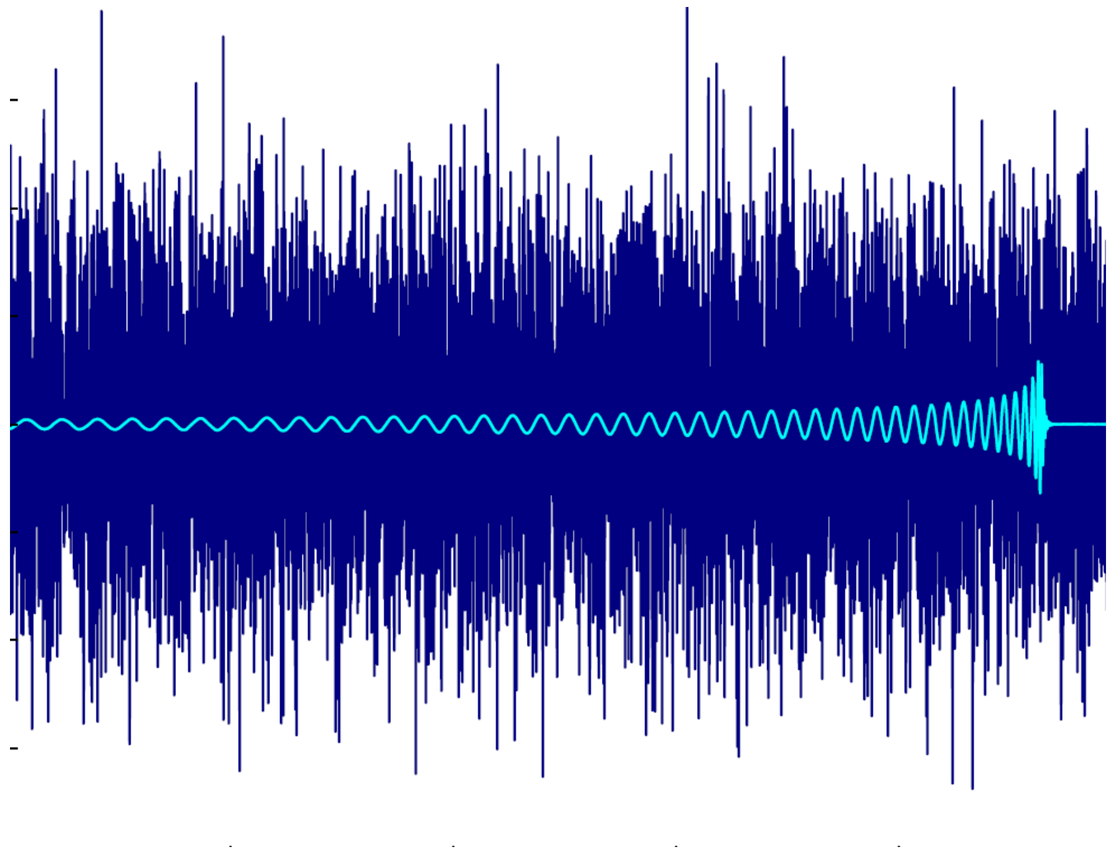

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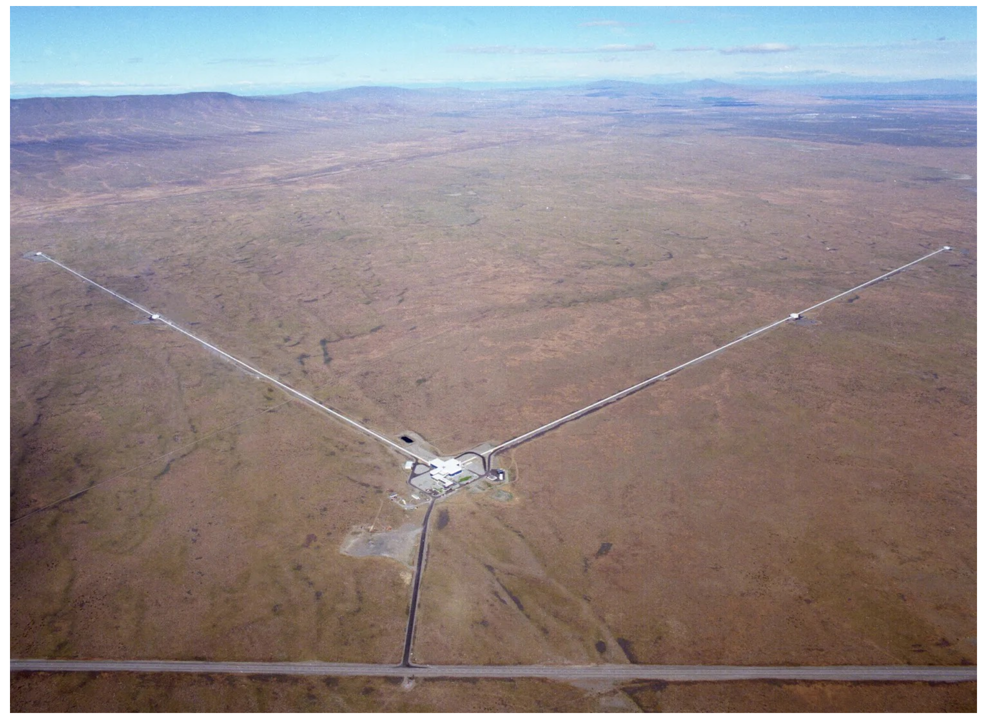

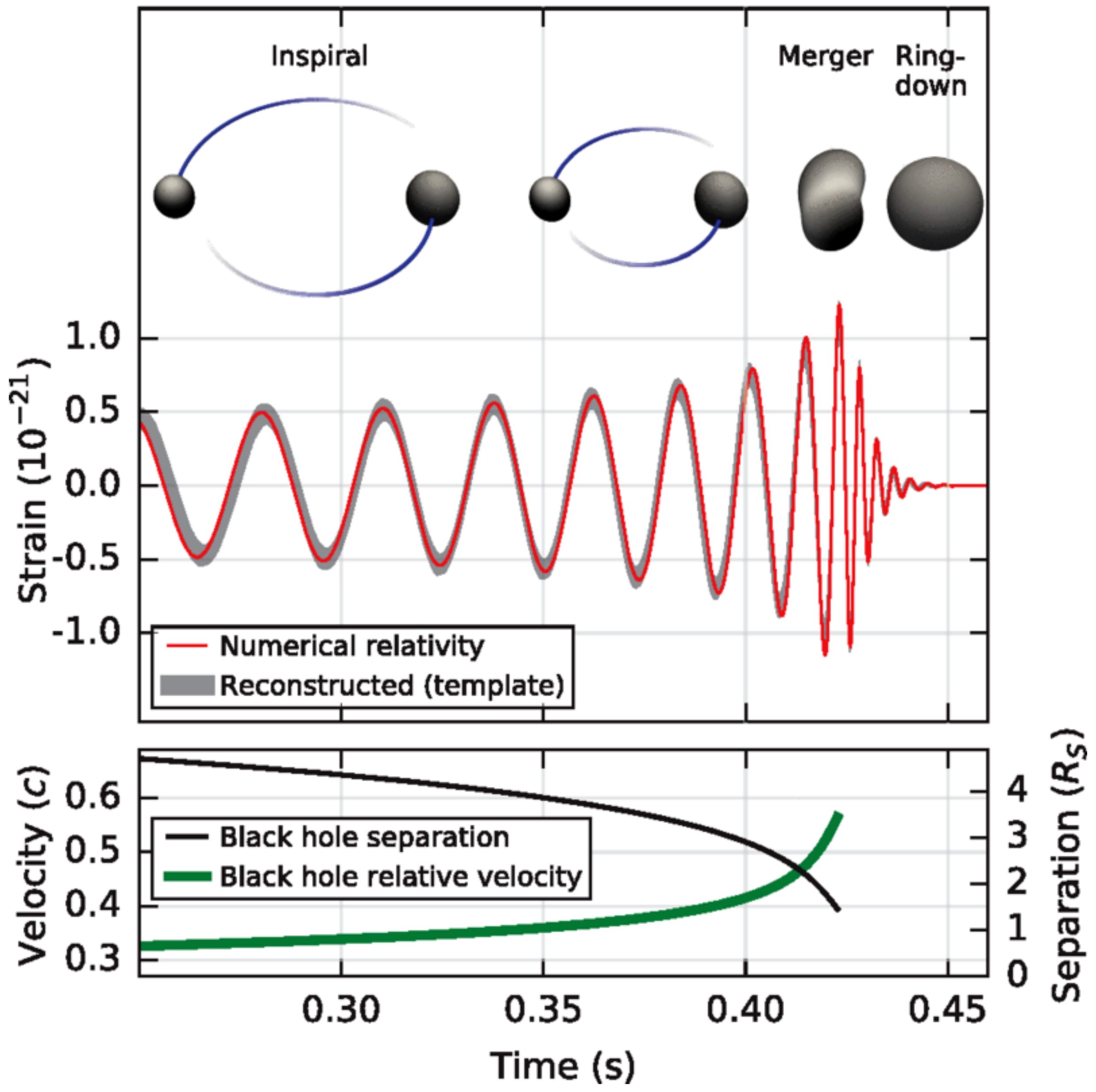

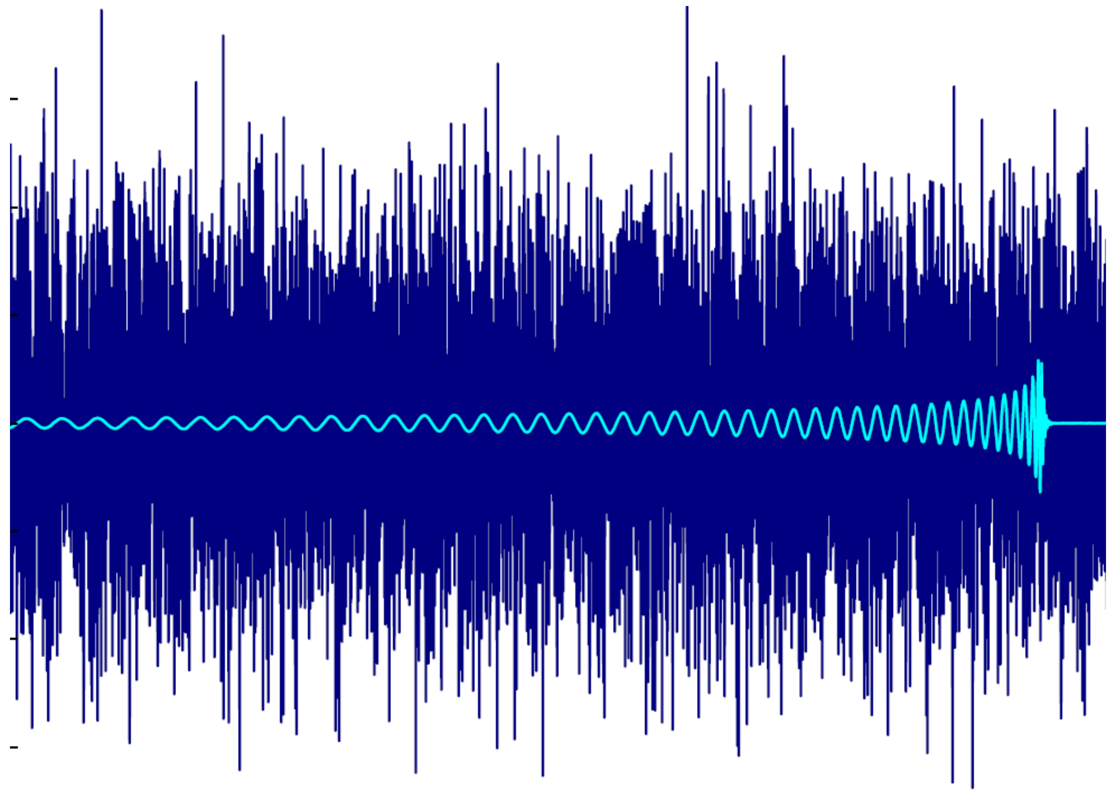

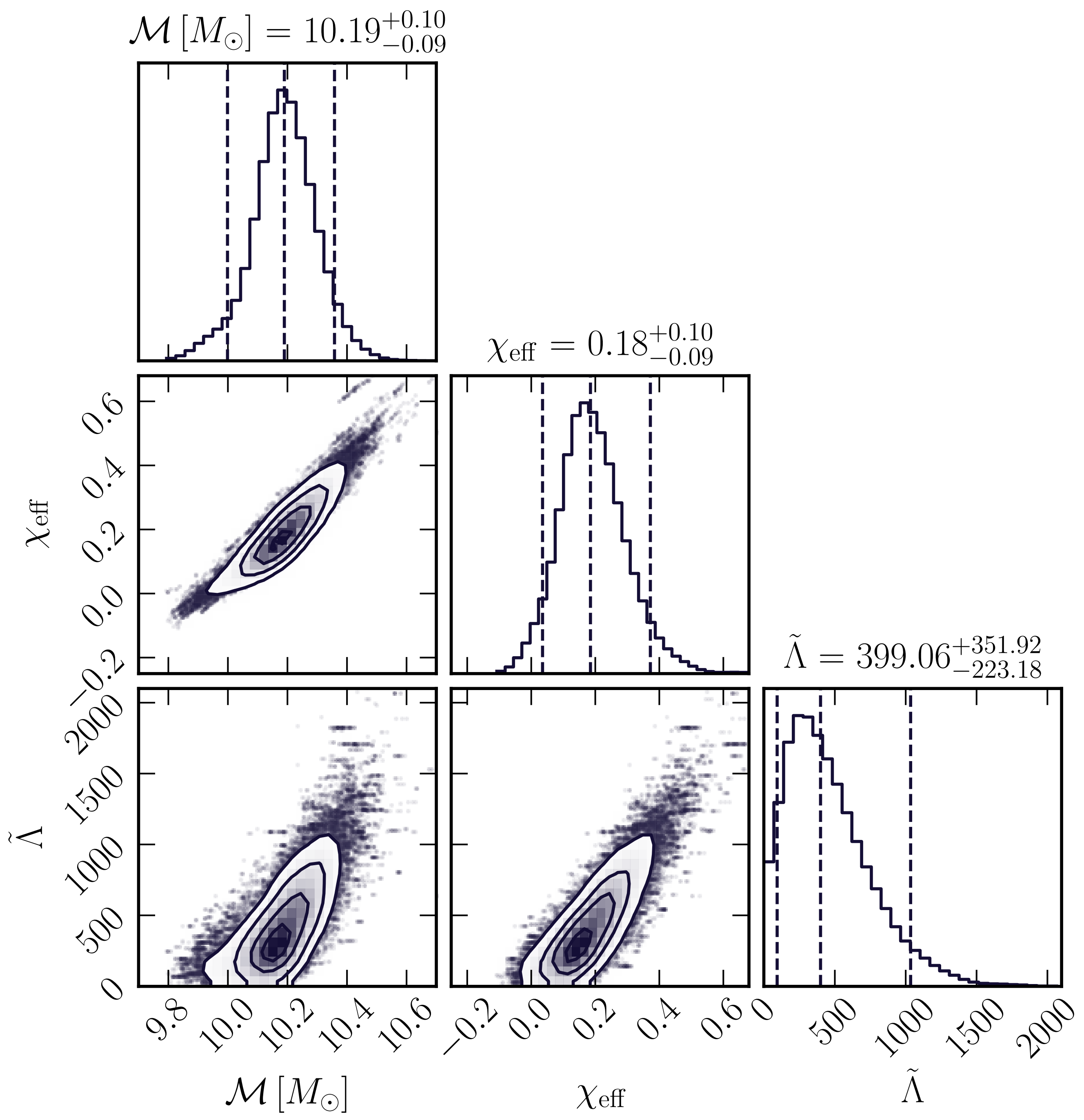

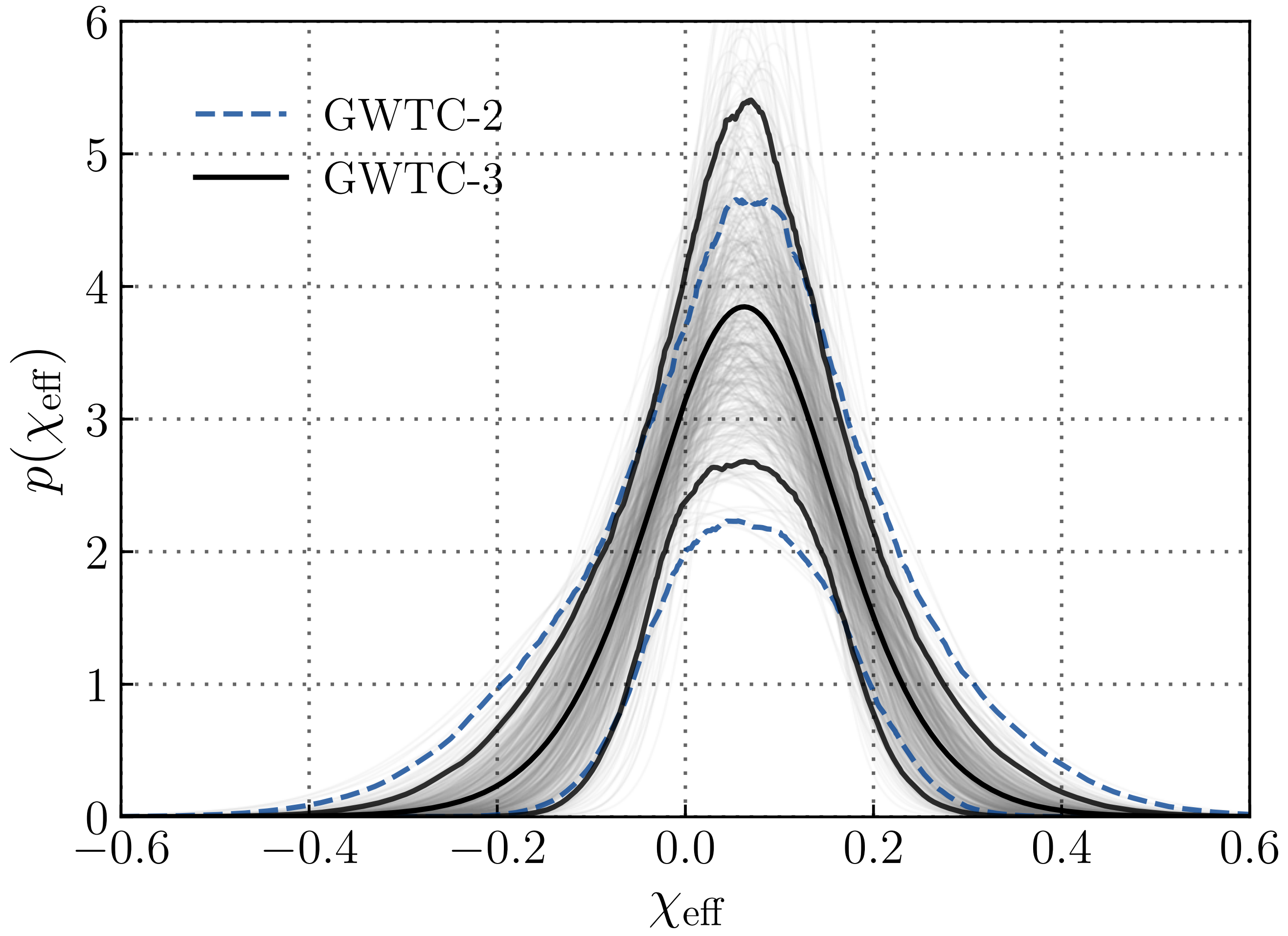

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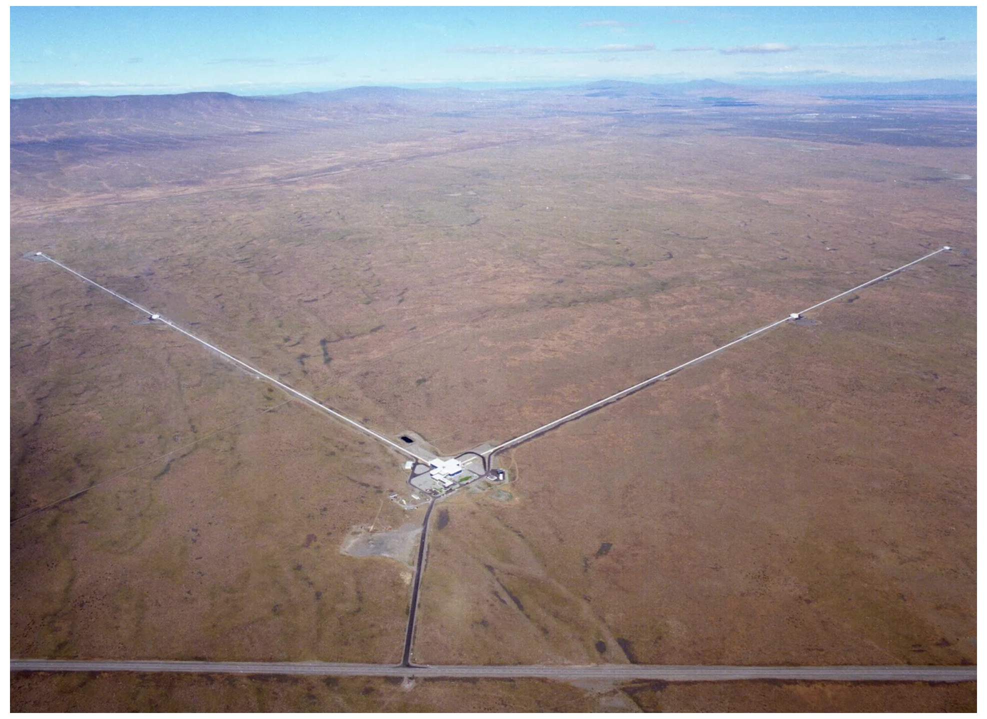

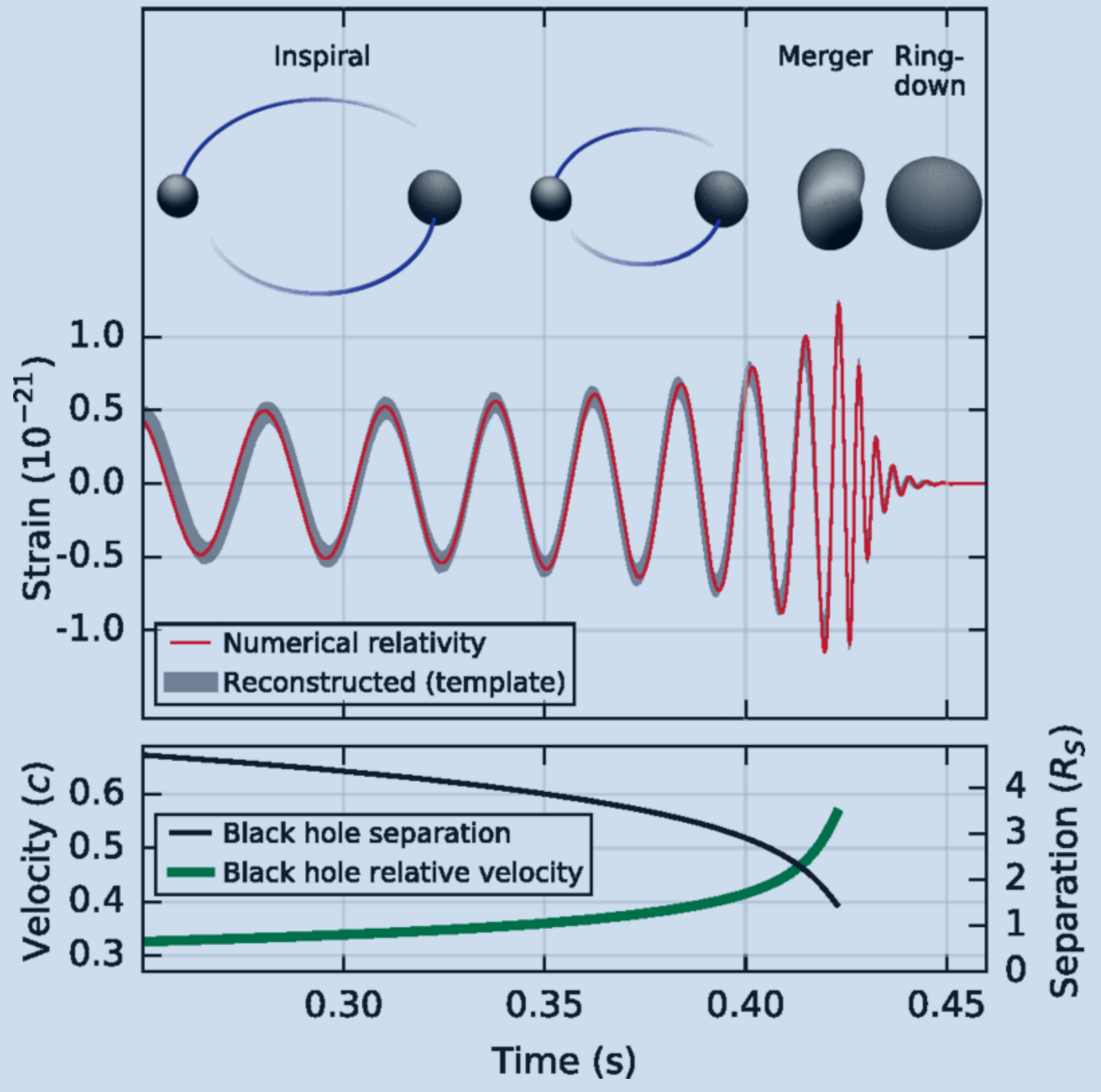

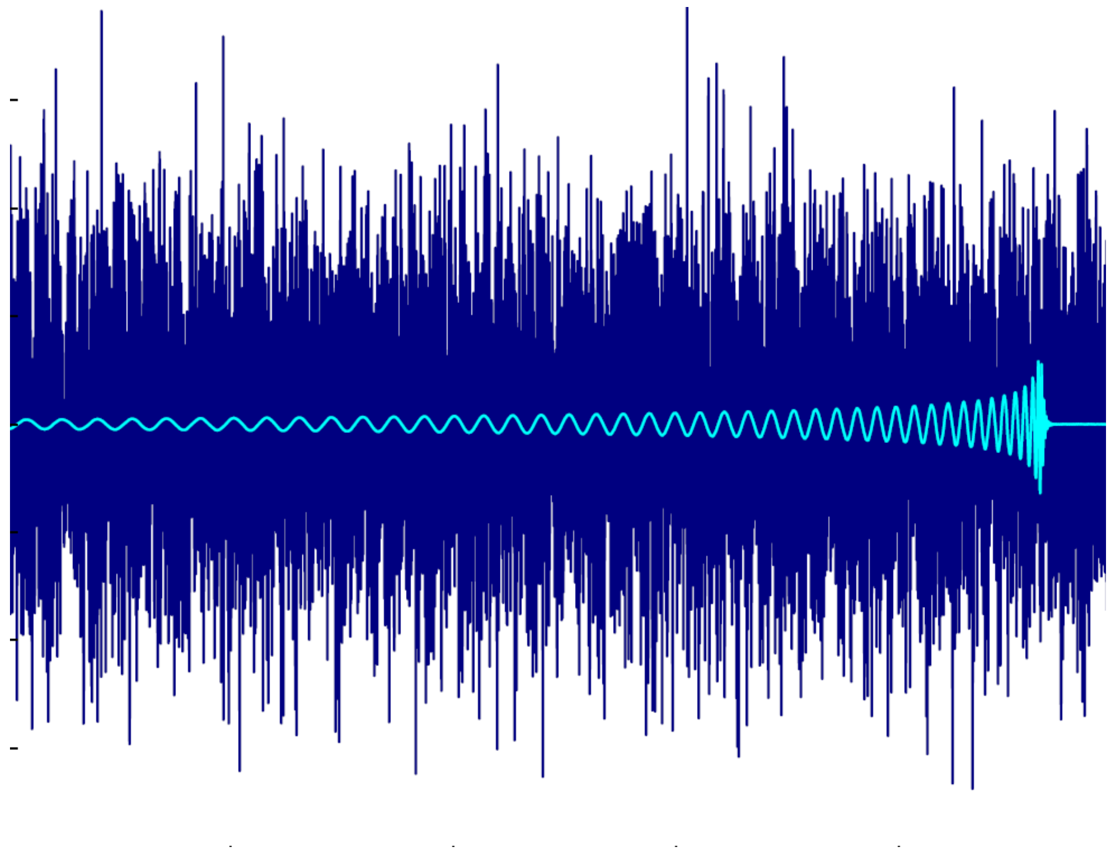

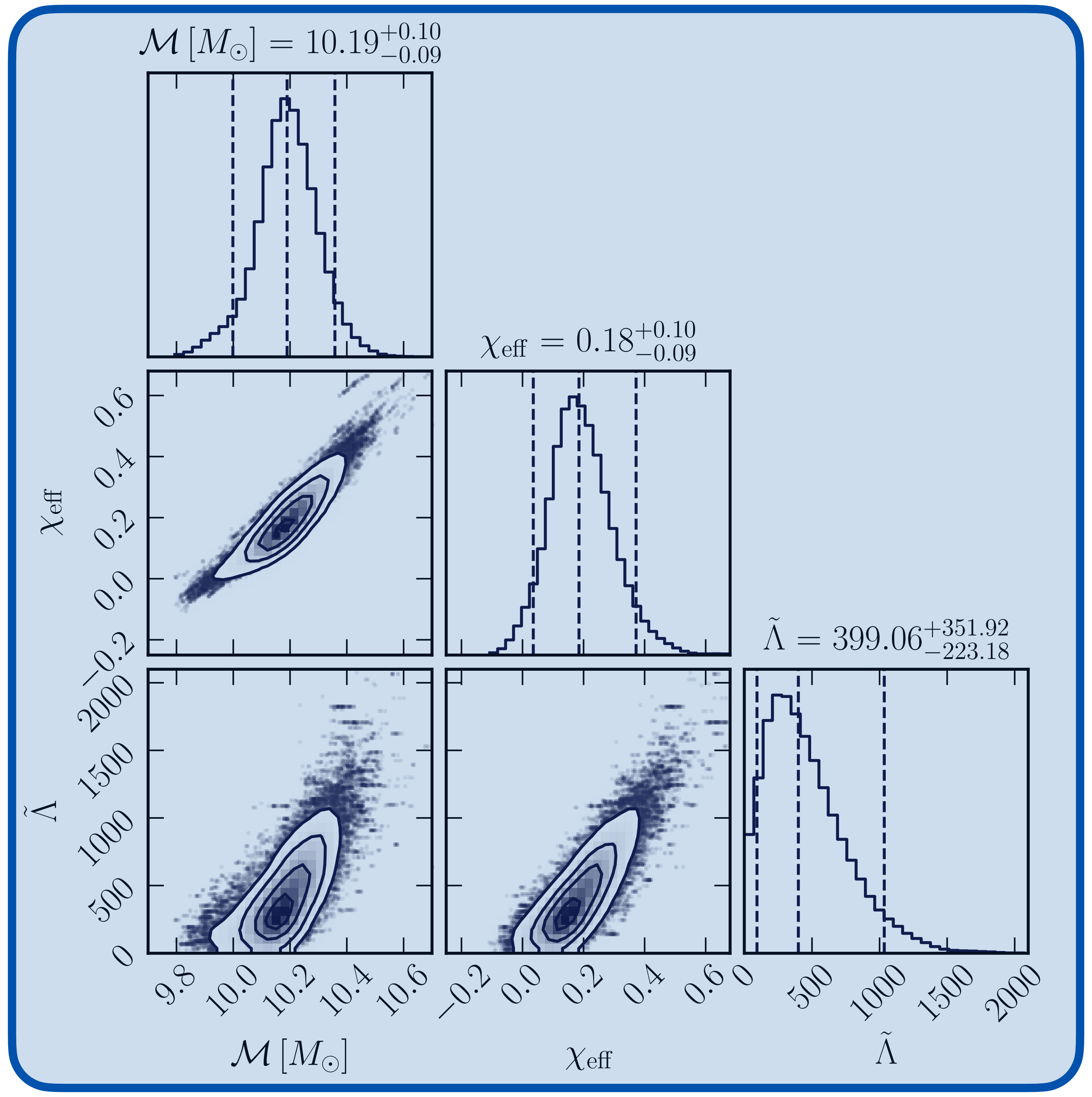

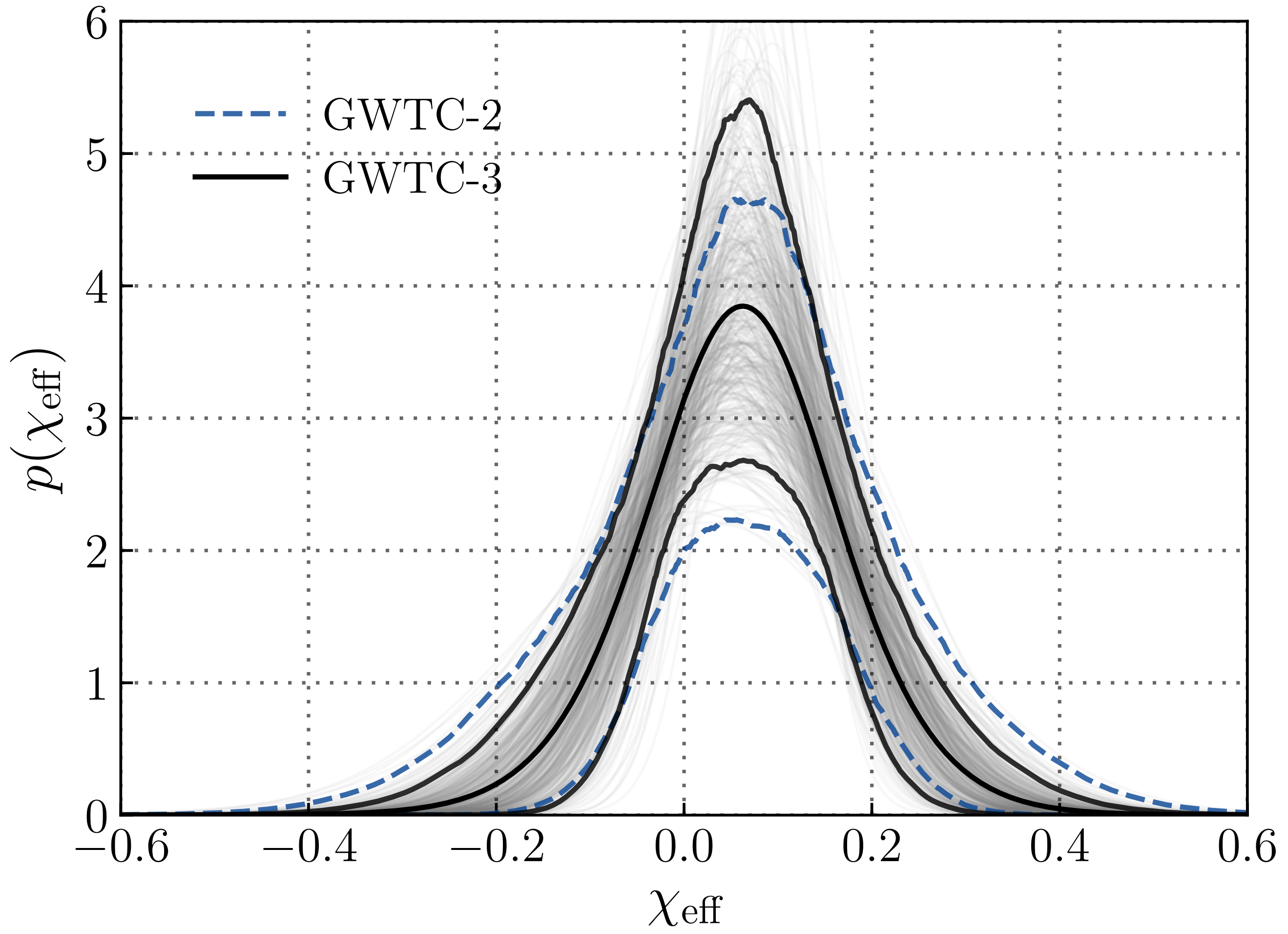

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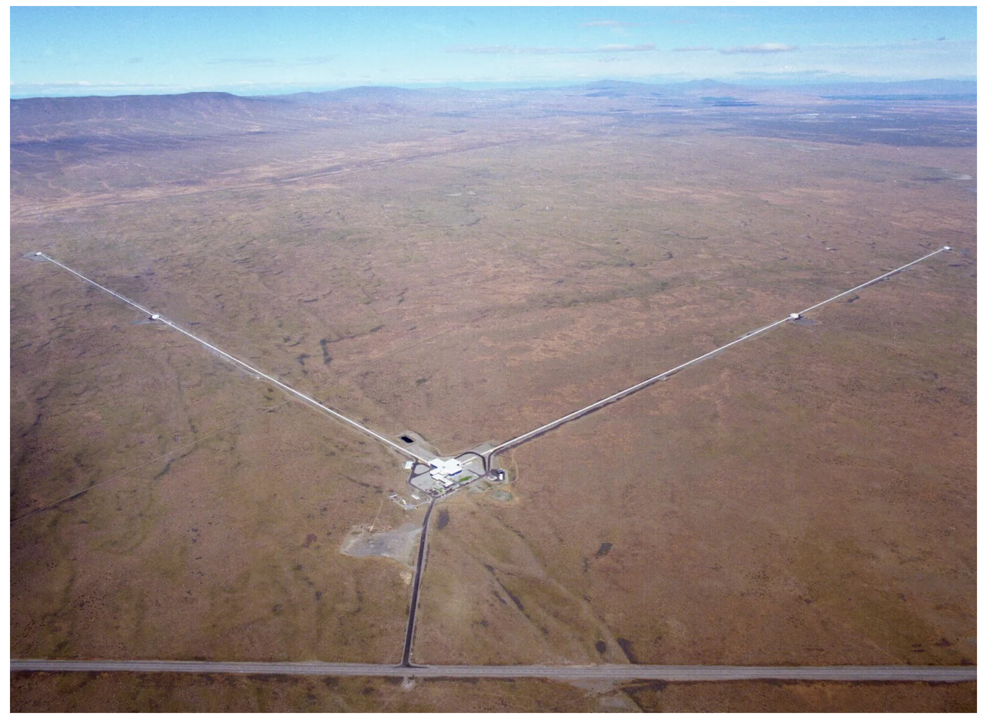

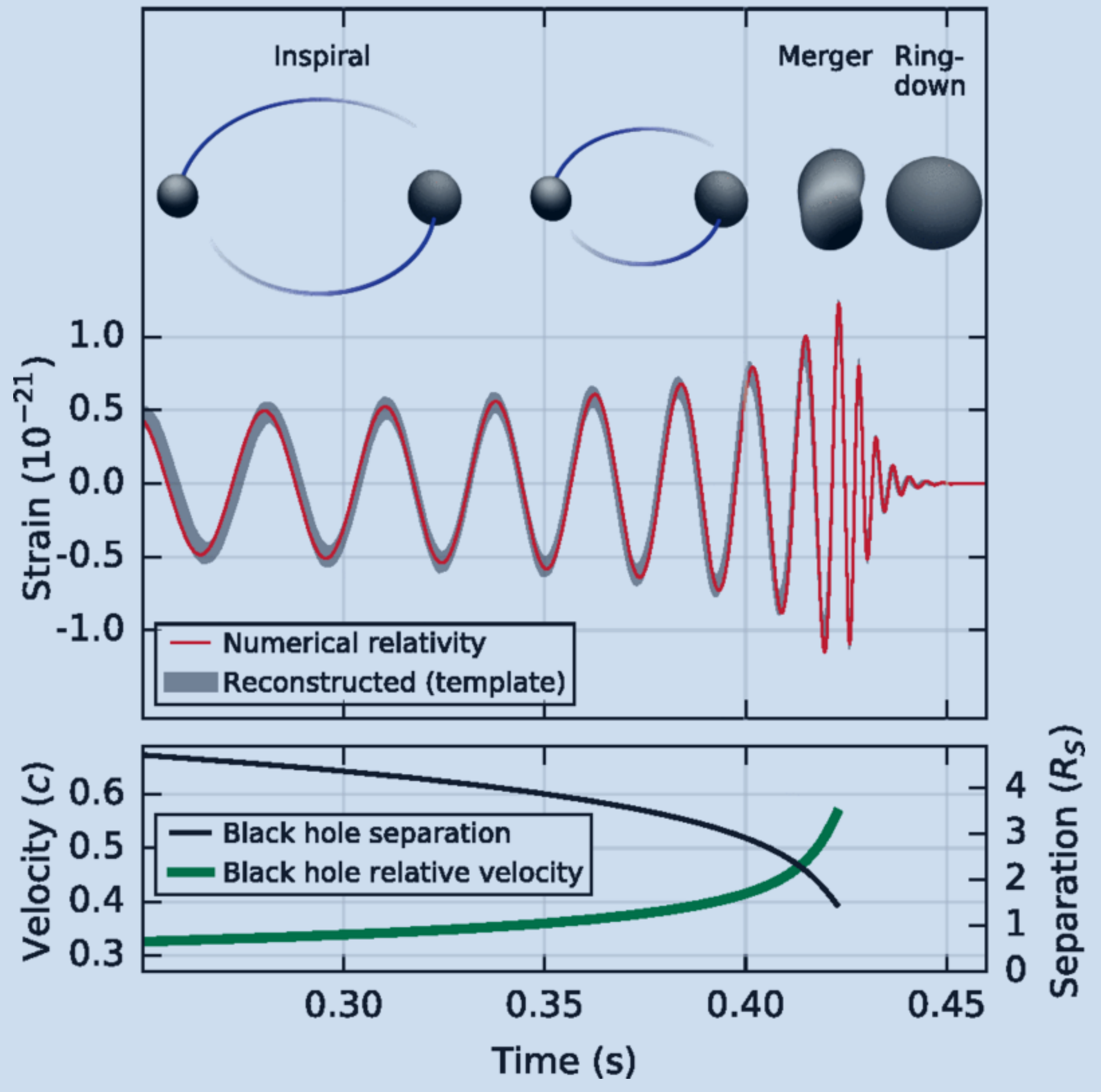

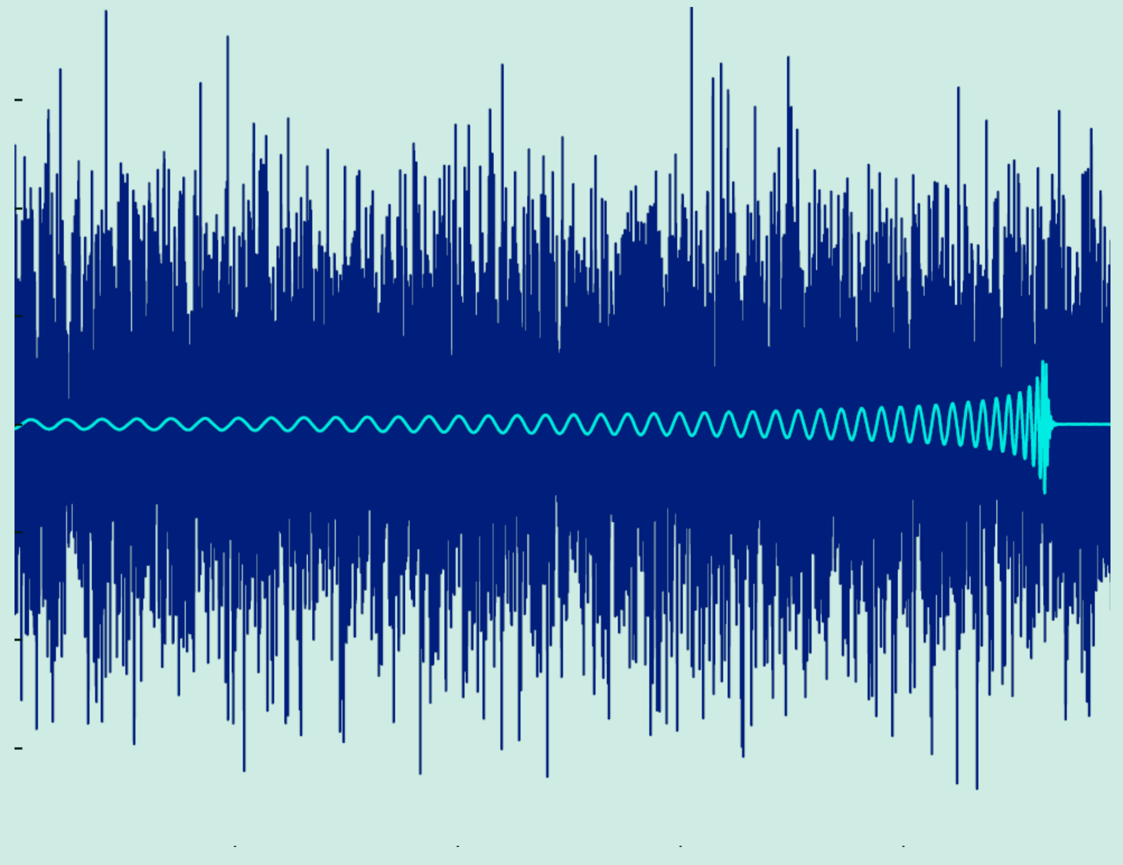

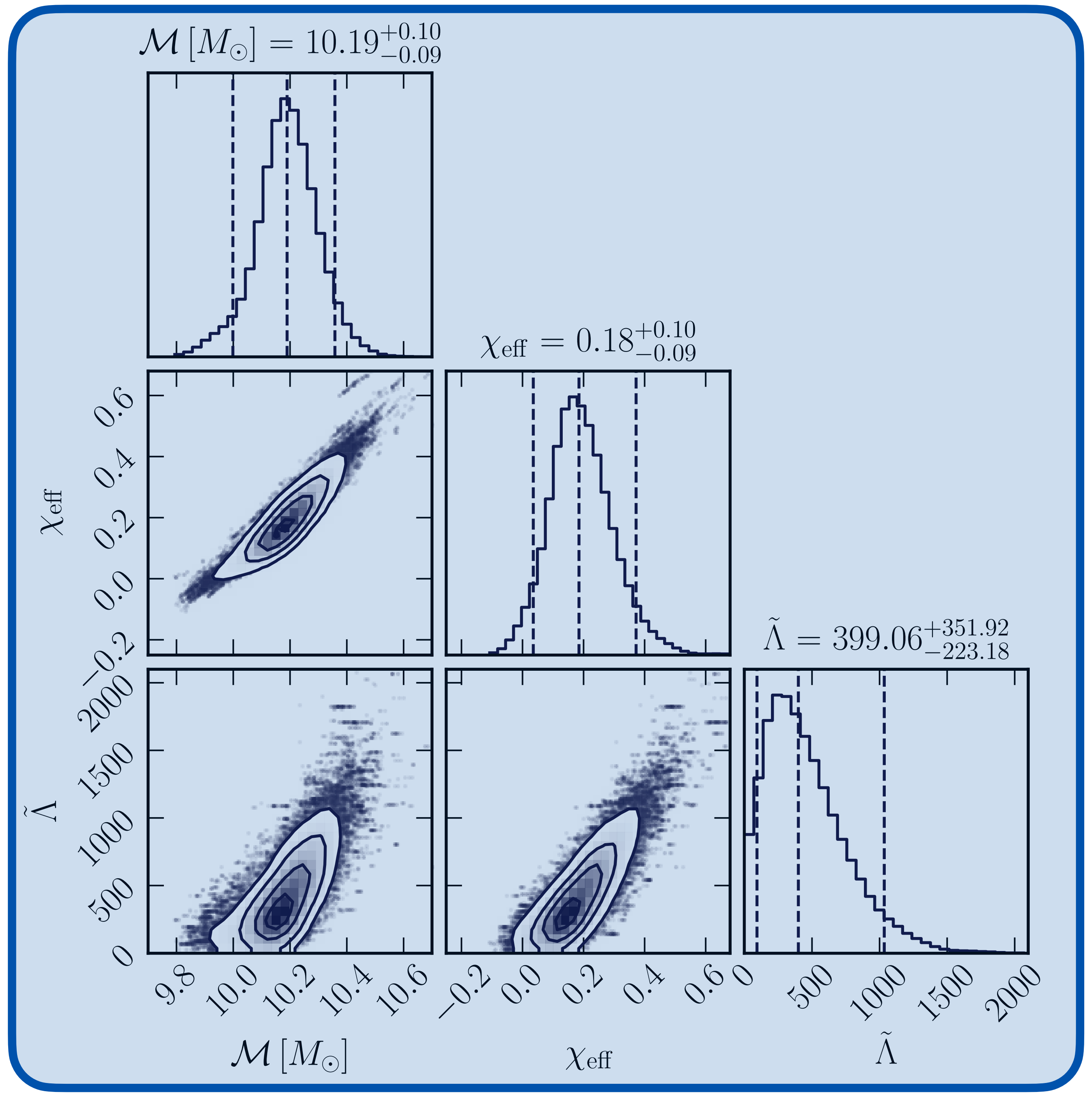

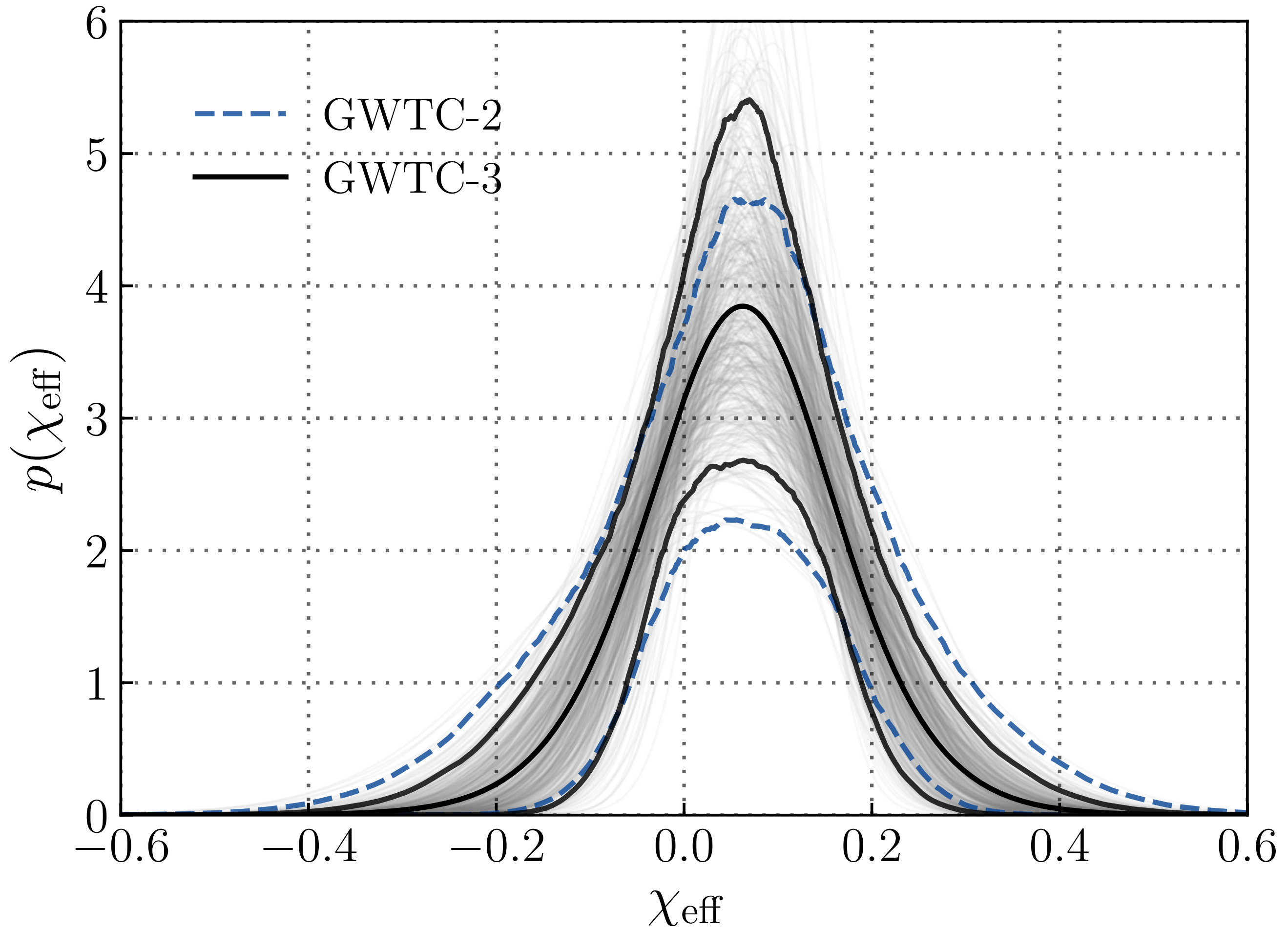

Overview

How can automatically-differentiable models improve GW analysis tasks?


Kaze Wong


Max Isi

+ Kelvin K. H. Lam, Adam Coogan, James Alvey, and Daniel Foreman-Mackey

What could current searches be missing?


Horng Sheng Chia


Jay Wadekar


Aaron Zimmerman

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Symbolic and numerical differentiation

Symbolic and numerical differentiation

$$z = y\sin(x) + y^3$$

$$\frac{\partial z}{\partial x} = y\cos(x) \qquad \frac{\partial z}{\partial y} = \sin(x) + 3y^2$$

- Requires closed-form expressions
- Can lead to “expression swell”

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Requires O(n) calls of the function where n is the number of parameters of the function
- Leads to numerical inaccuracies (rounding and truncation error)

What is Automatic Differentiation (AD)?

- Automatic differentiation is a family of methods which allows one to compute machine precision derivatives with little computational overhead for arbitrary computational programs
- Its foundation is that each mathematical step is itself differentiable and composable using the chain rule

$$\frac{\partial w}{\partial t} = \sum_{i}^{N} \frac{\partial w}{\partial u_i} \frac{\partial u_i}{\partial t}$$

Forward mode AD

- Forward mode breaks down a function into intermediate steps and simultaneously evaluates both the value of the intermediate variable and its derivative (also known as using dual numbers)
- Discussed extensively here
- Runs in ~n x O(f)

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$$f : \mathbb{R}^n \to \mathbb{R}^m, \quad \text{where} \quad n \ll m$$

Why JAX?

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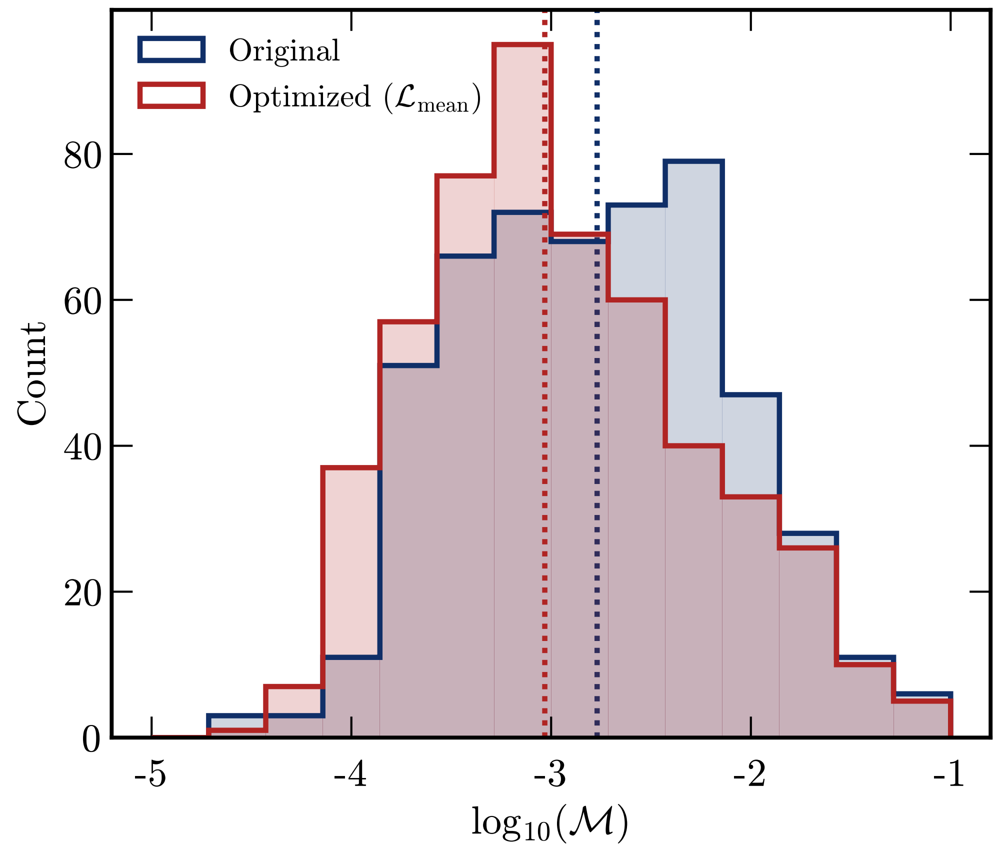
Use XLA compilation to speed up code substantially

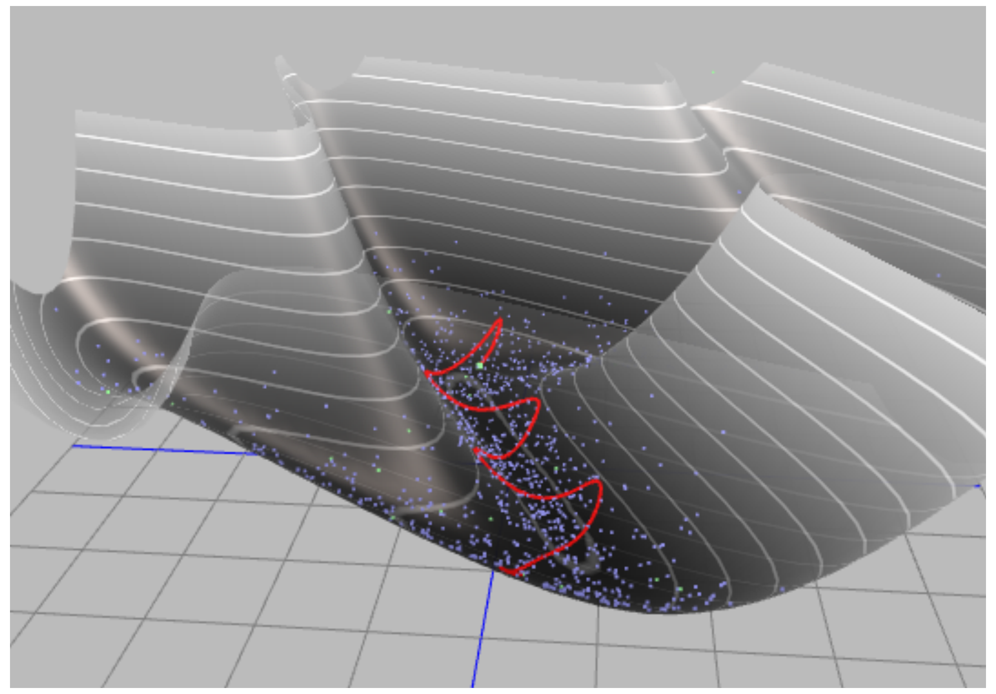
Scales to different computing architectures and multiple cores or a cluster

[https://github.com/google/jax]

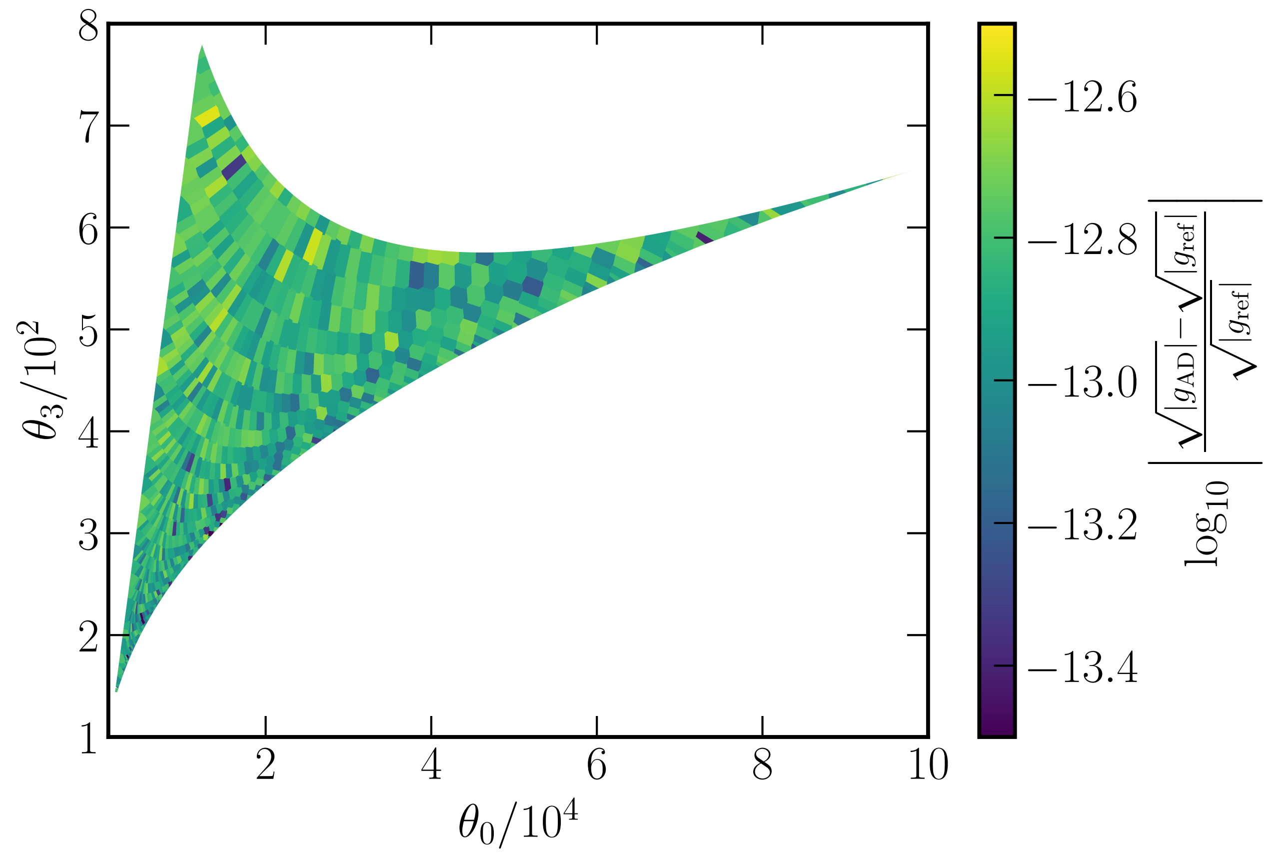
Ripple



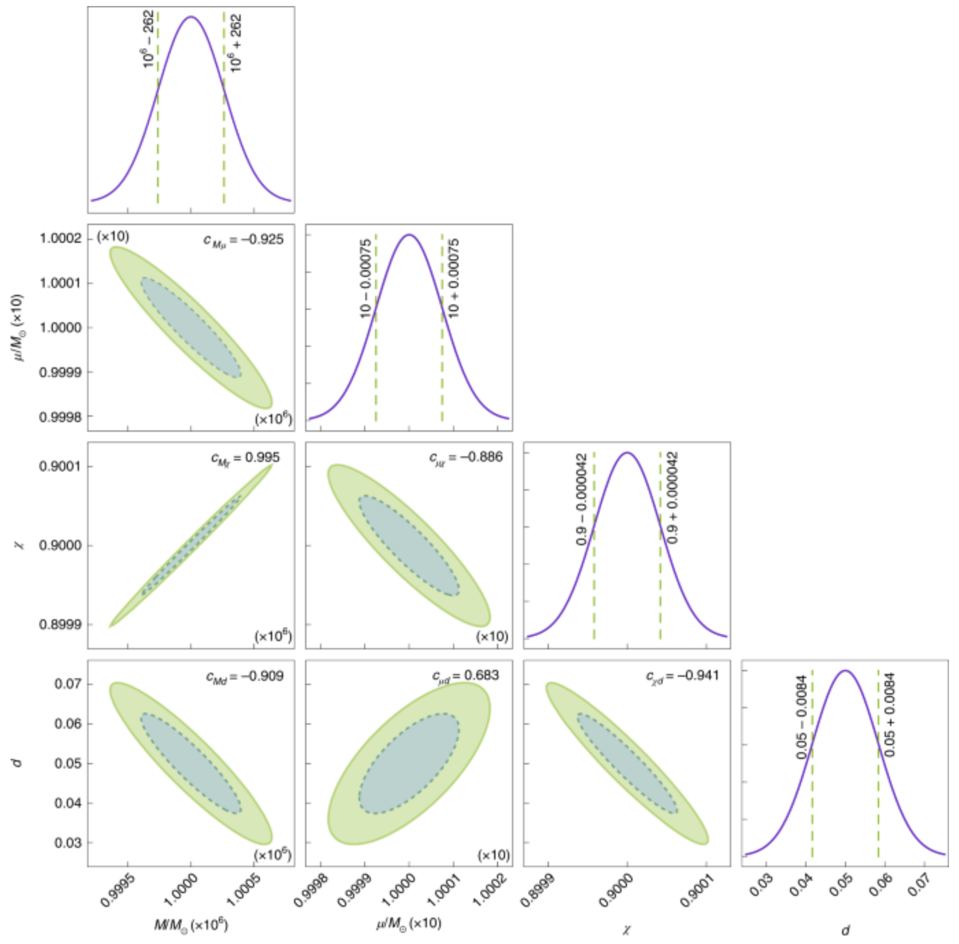
Waveform Optimization



Gradient-based Parameter Estimation



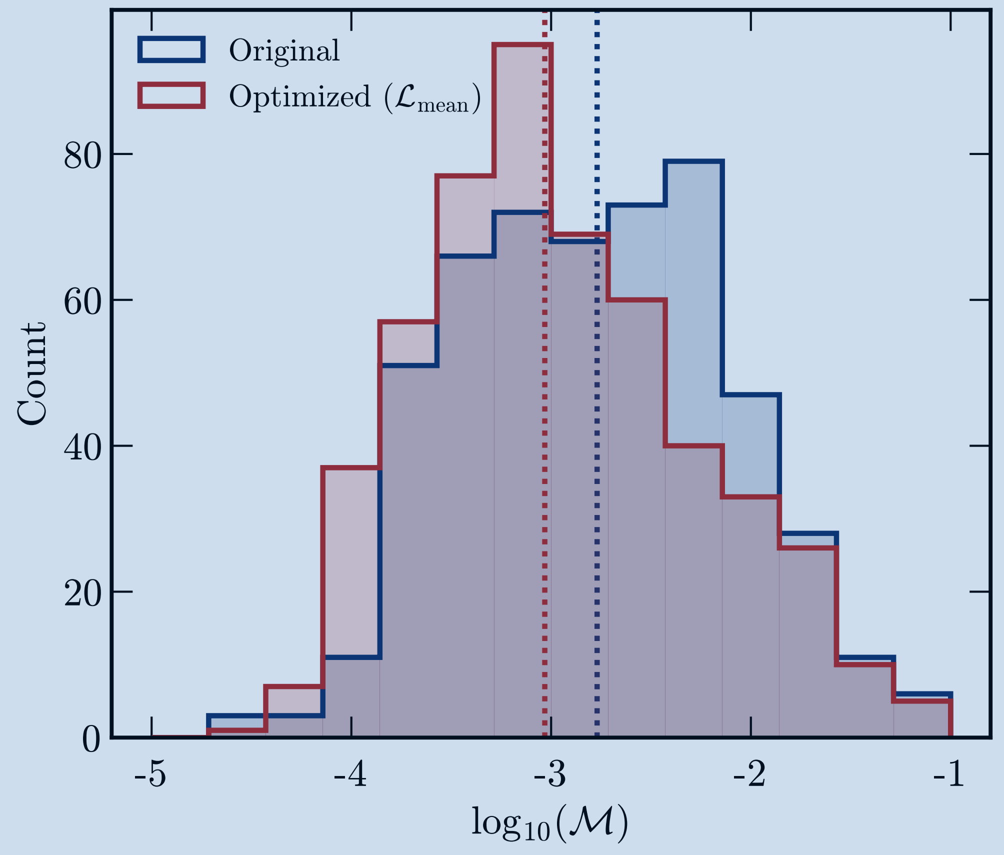
Template Bank Generation



Fisher Analyses

[github.com/tedwards2412/ripple]

Ripple



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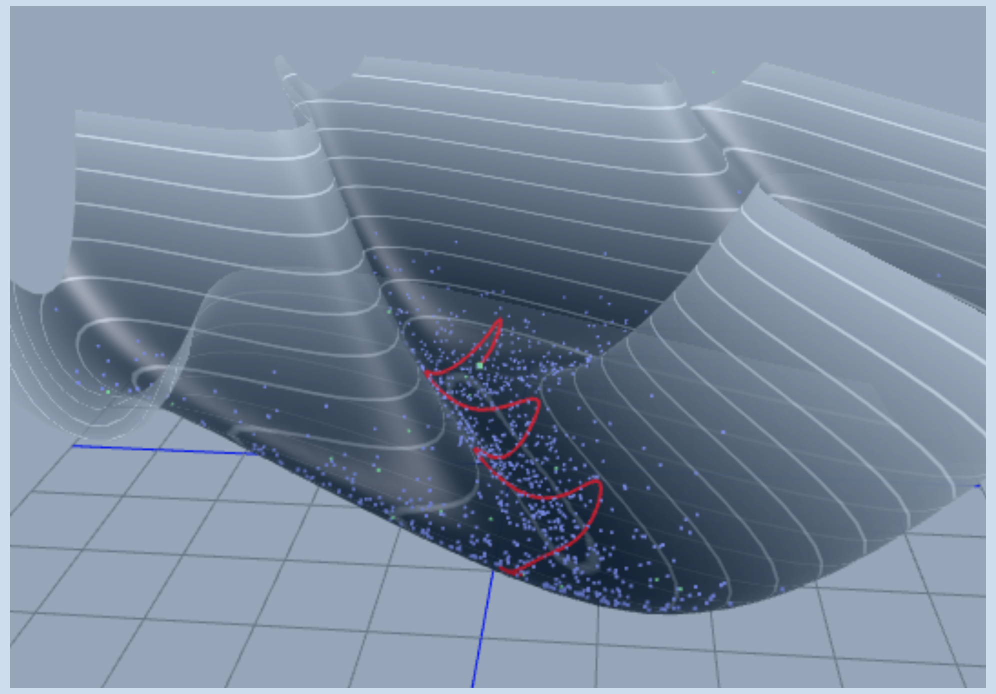

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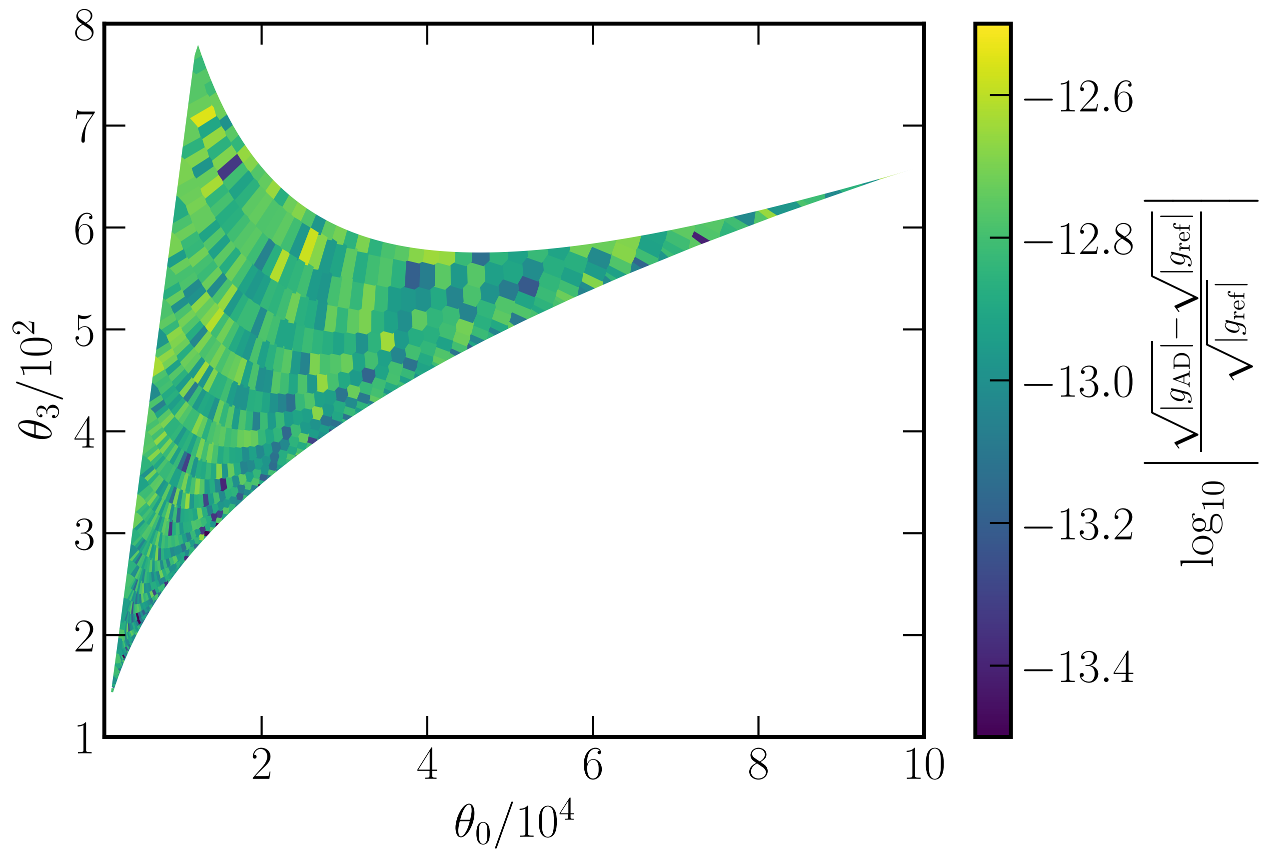

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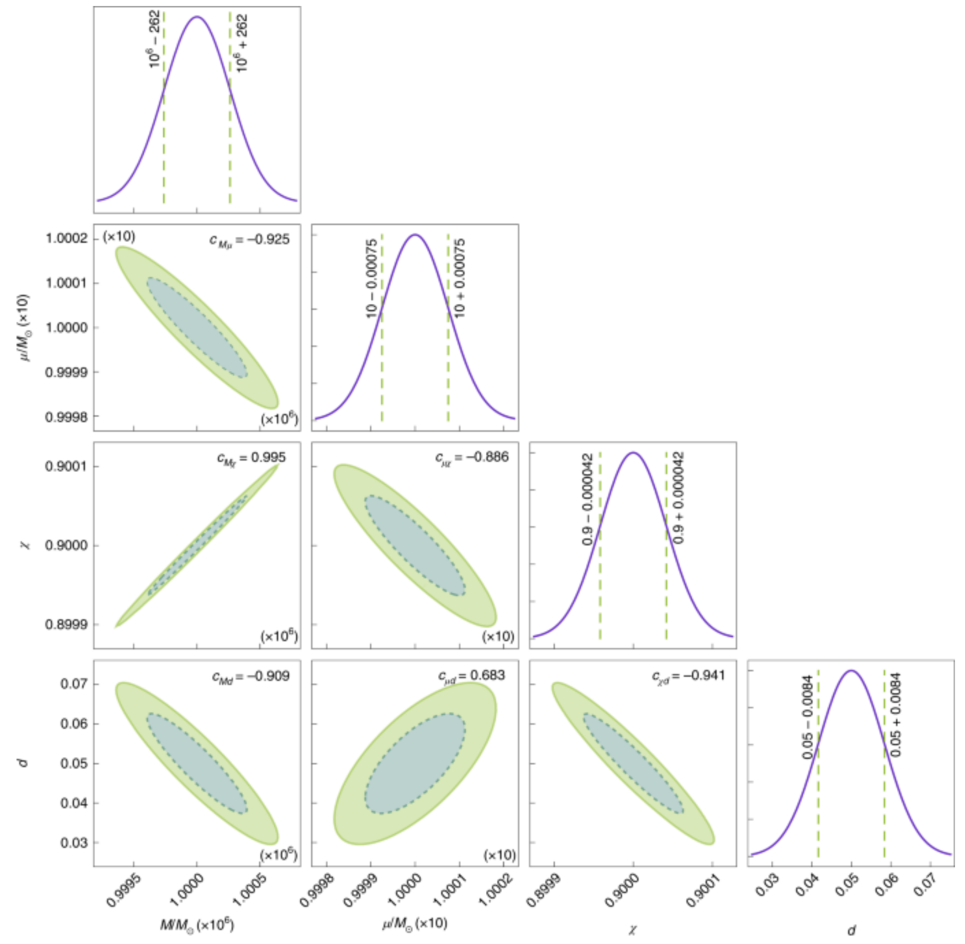

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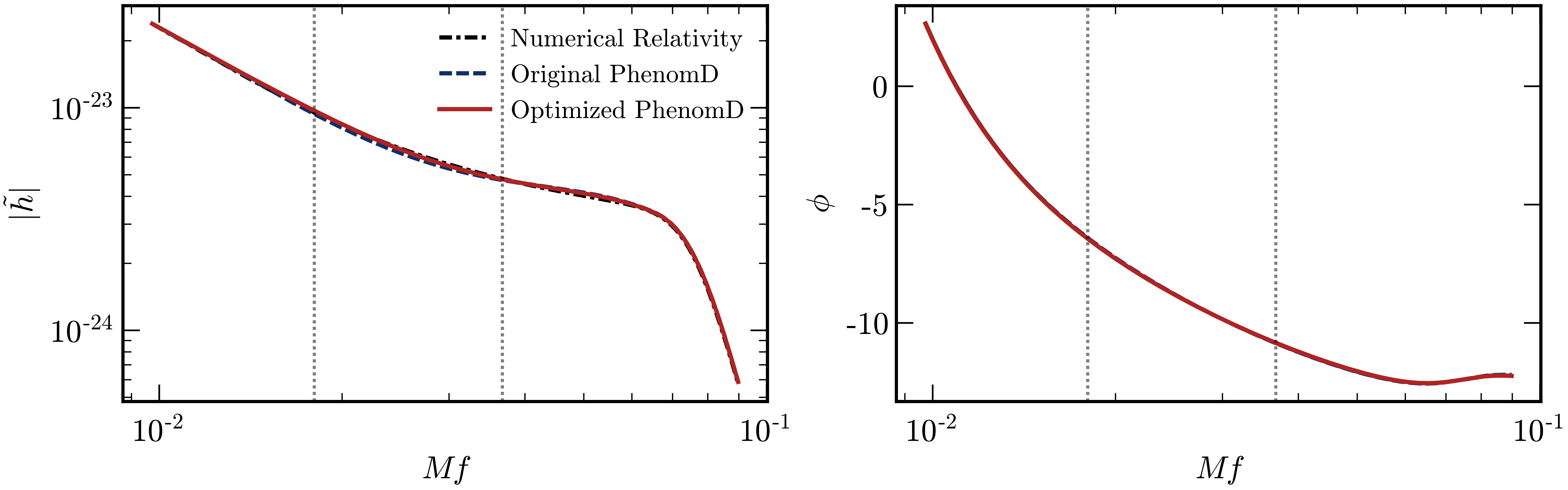

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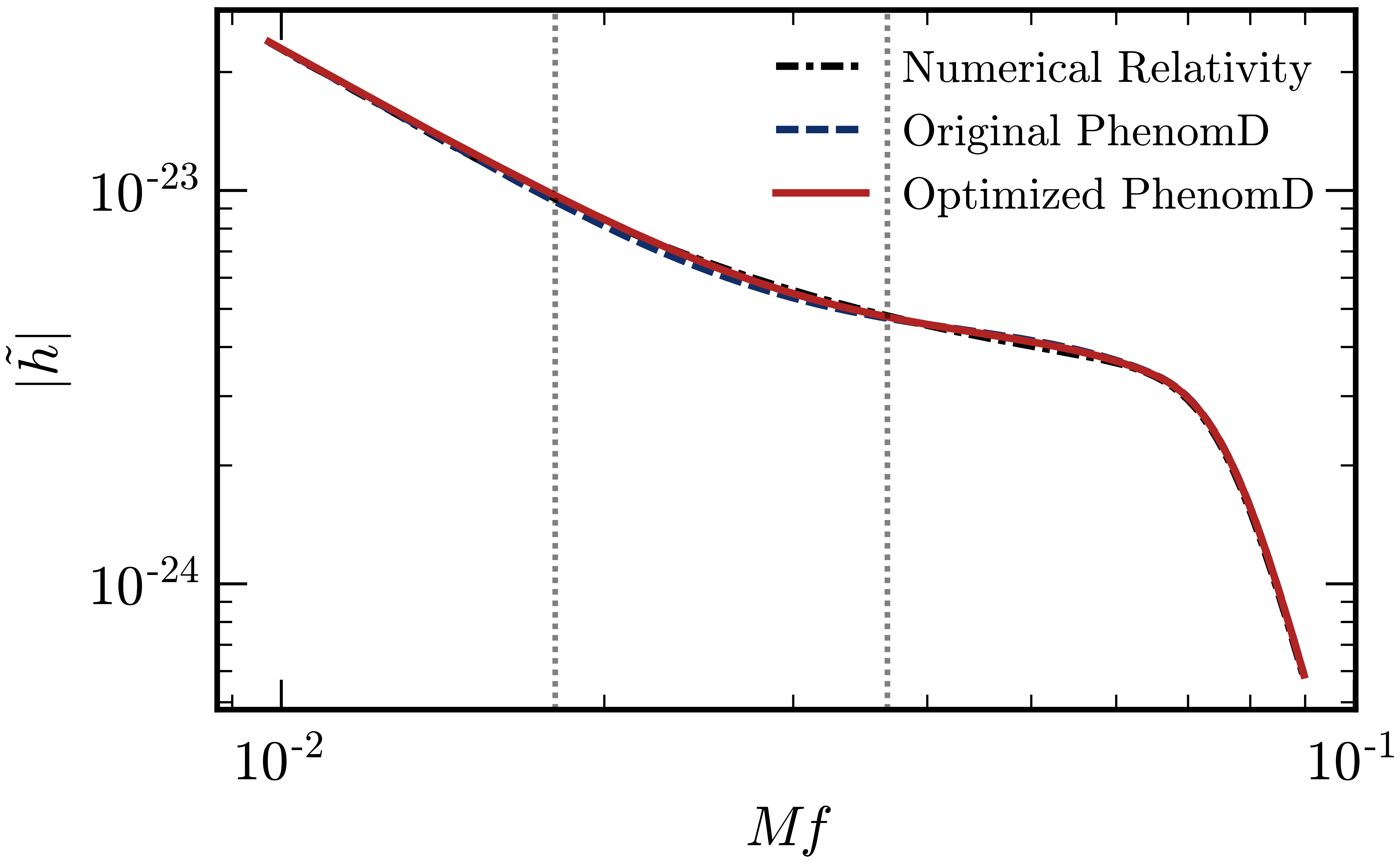

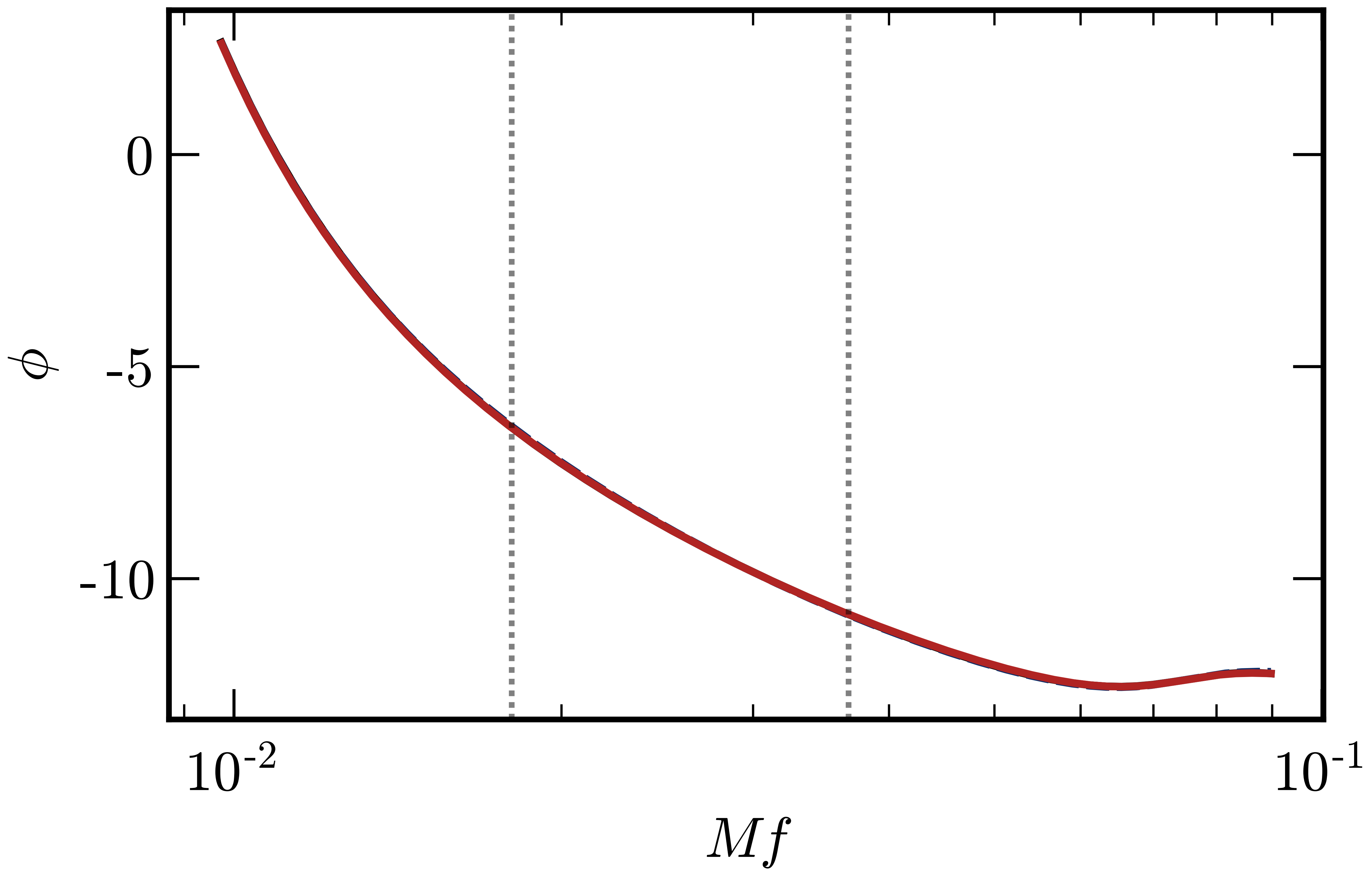

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Gradient Descent?

To use gradient descent we need to define a loss function, update rule, and stopping criteria

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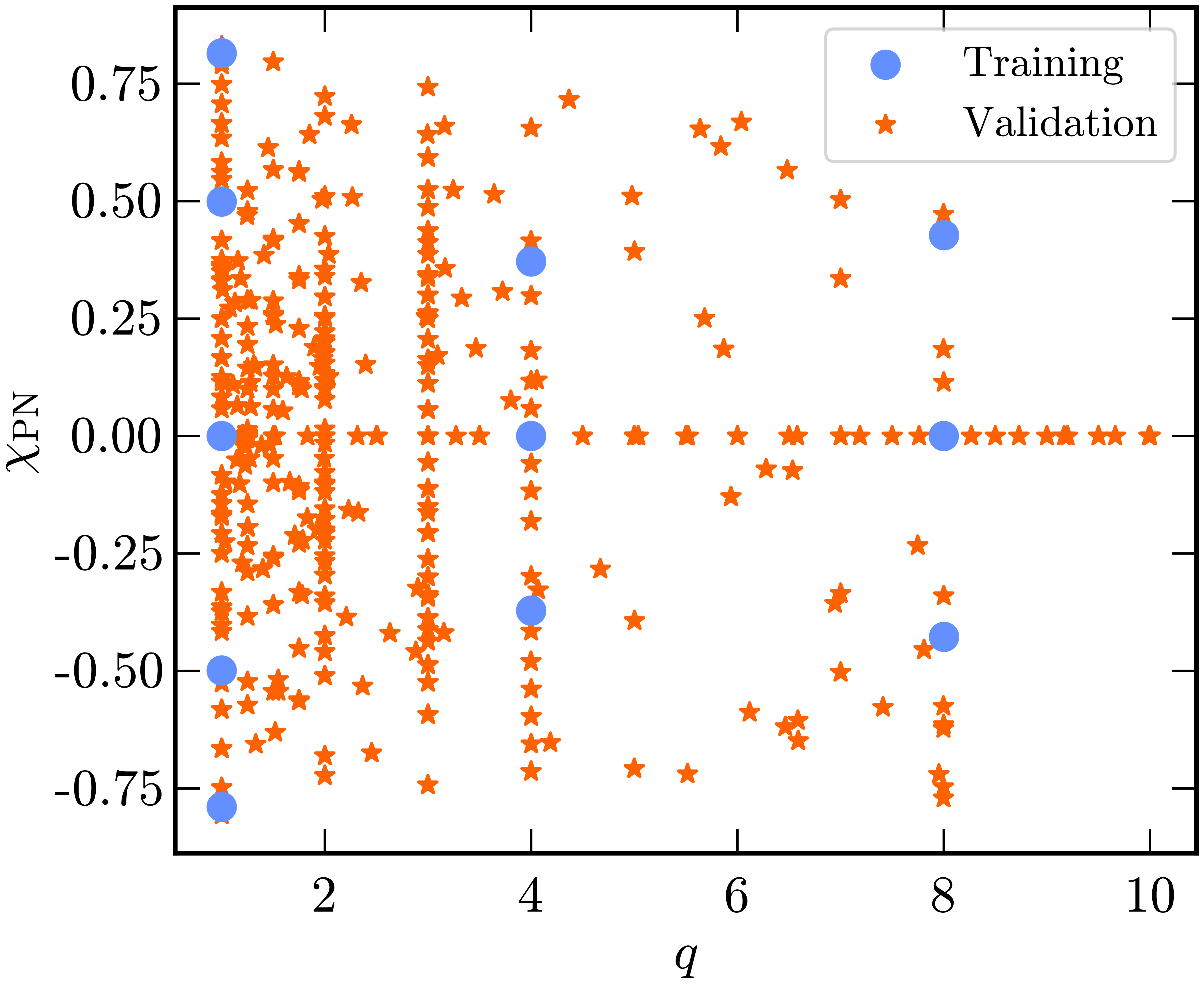
$$\mathcal{L}_{\text{mean}} = \frac{1}{N}\sum_{i=1}^{N}\mathcal{M}_i$$



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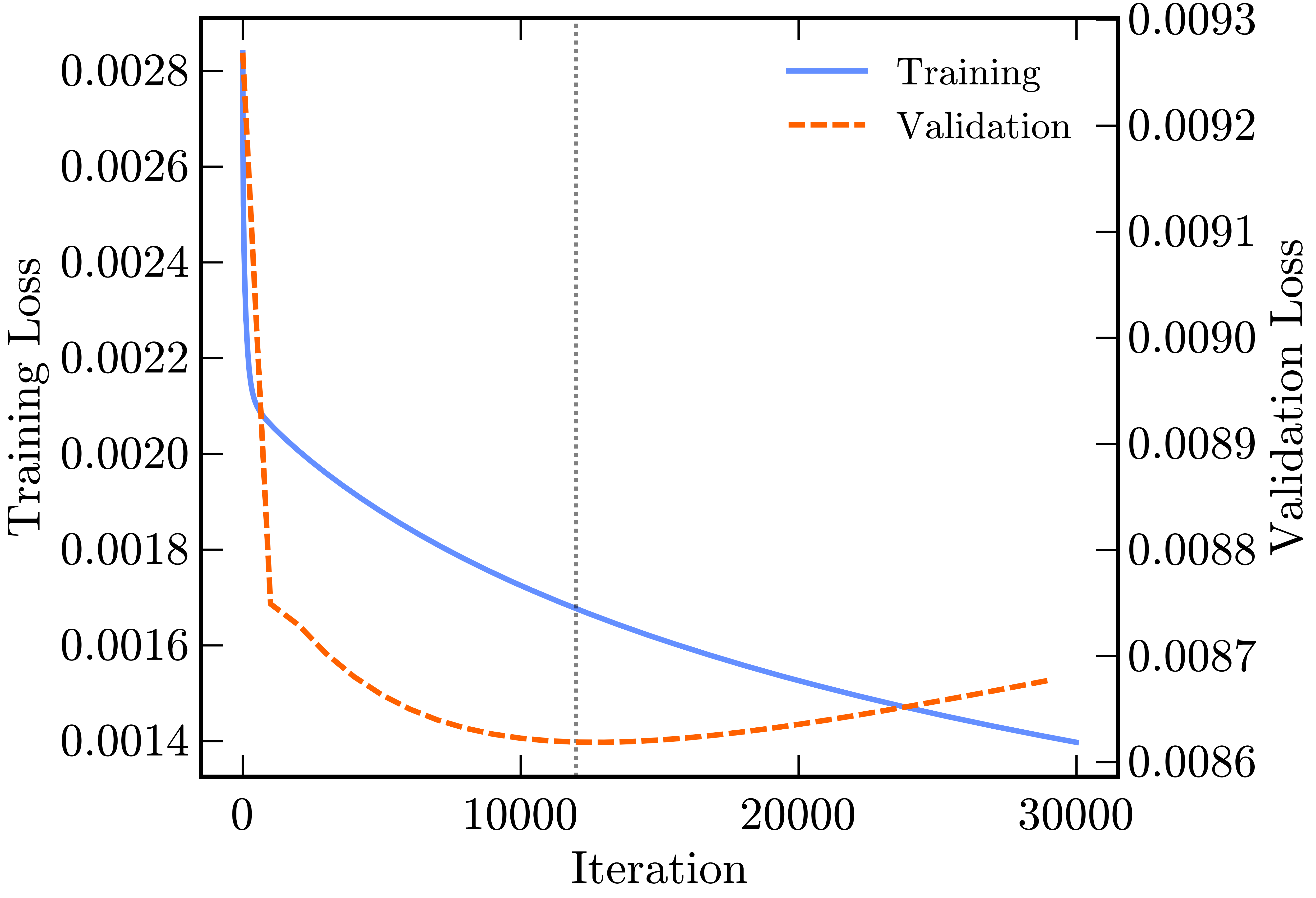
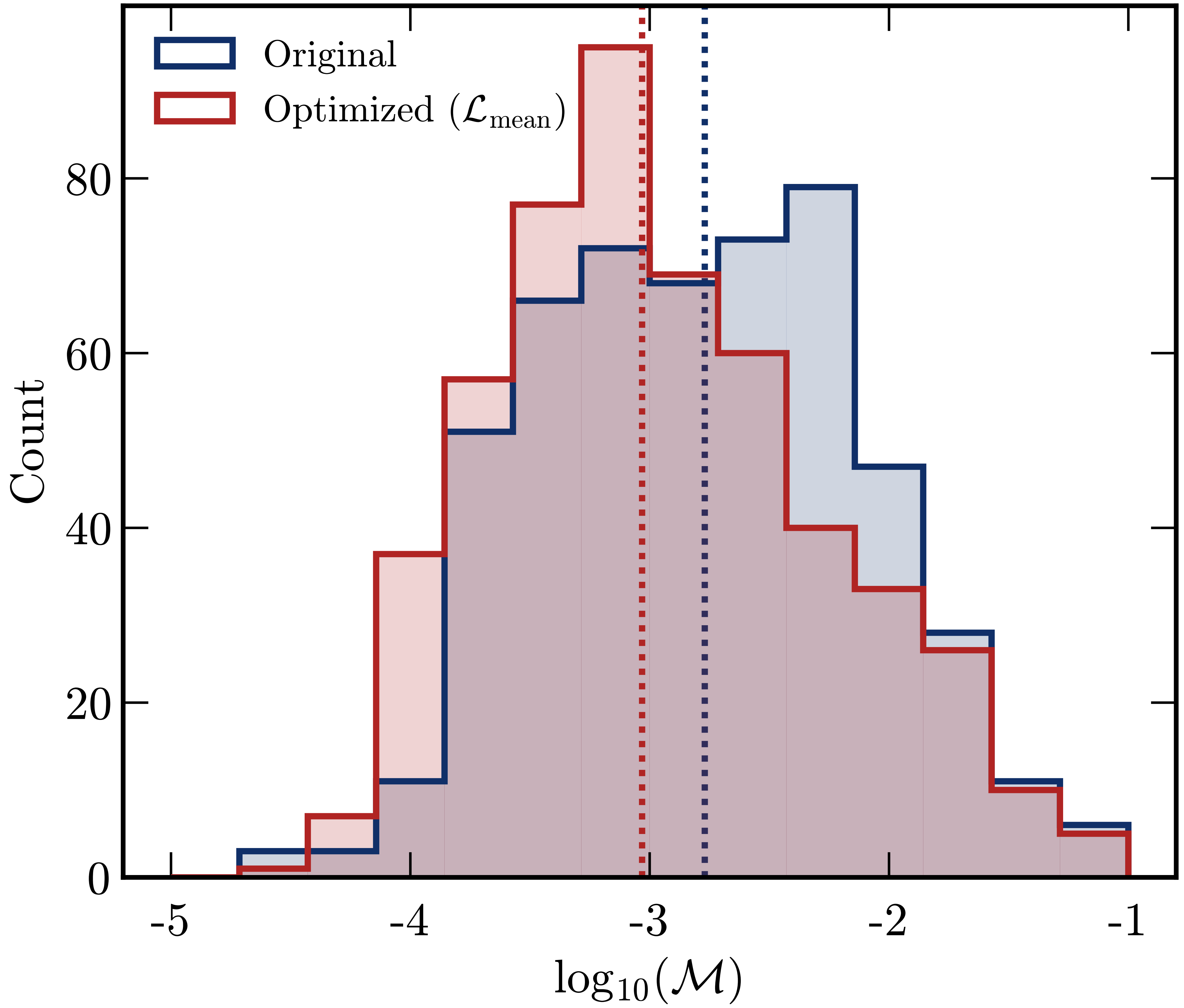


Up to 50% Better Waveforms For Free



Parameter Estimation

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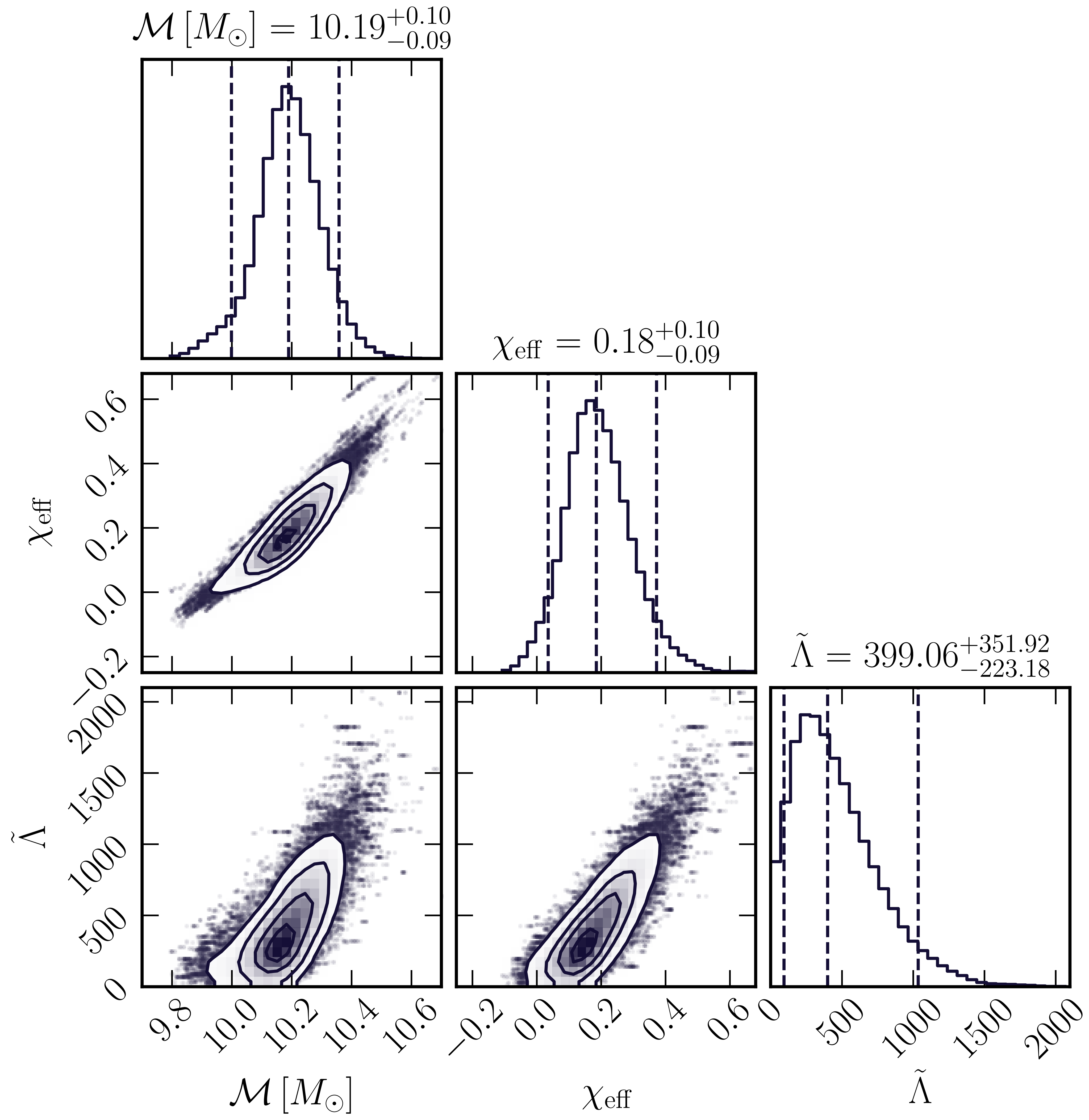

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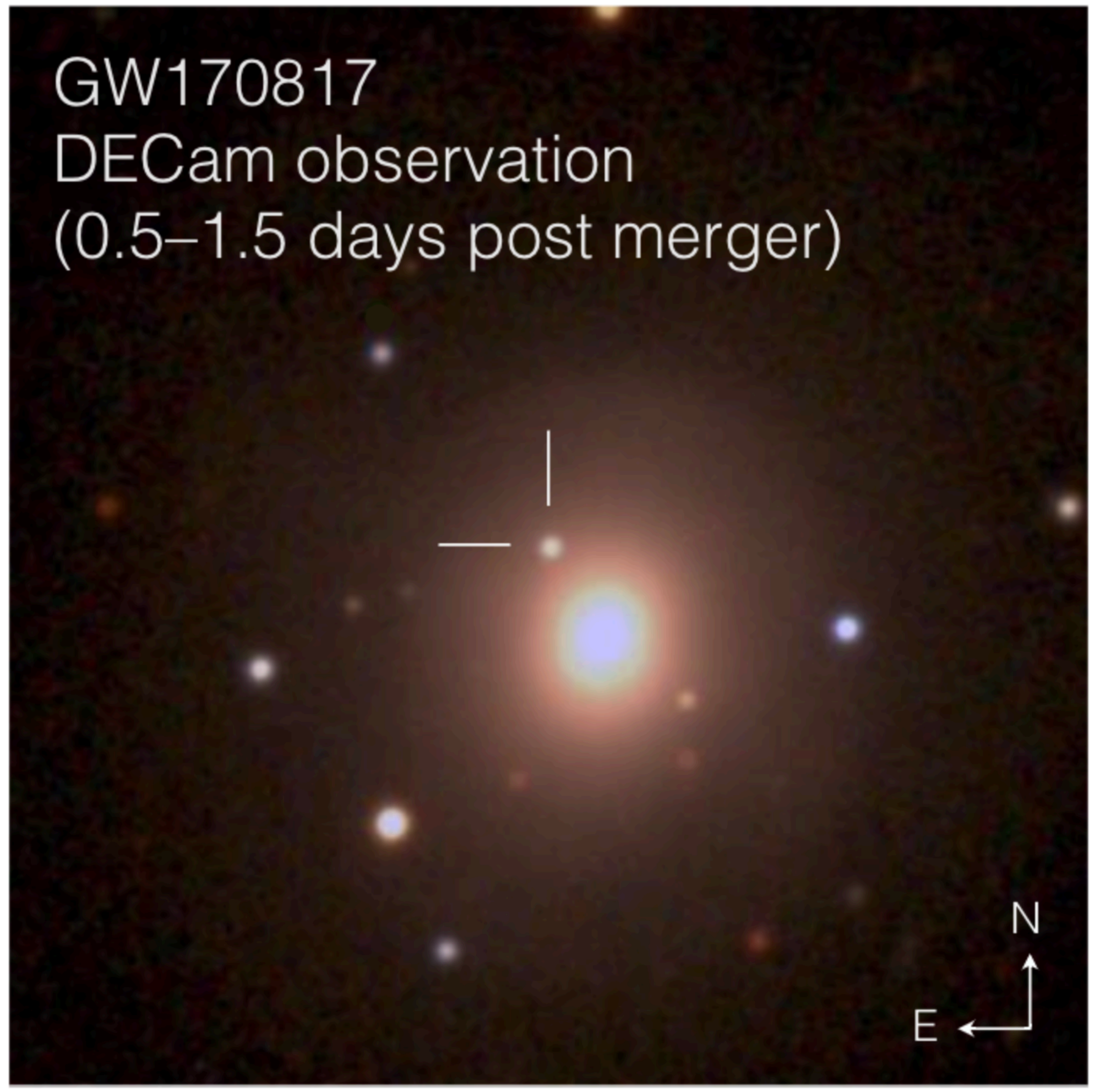
Low-latency follow up



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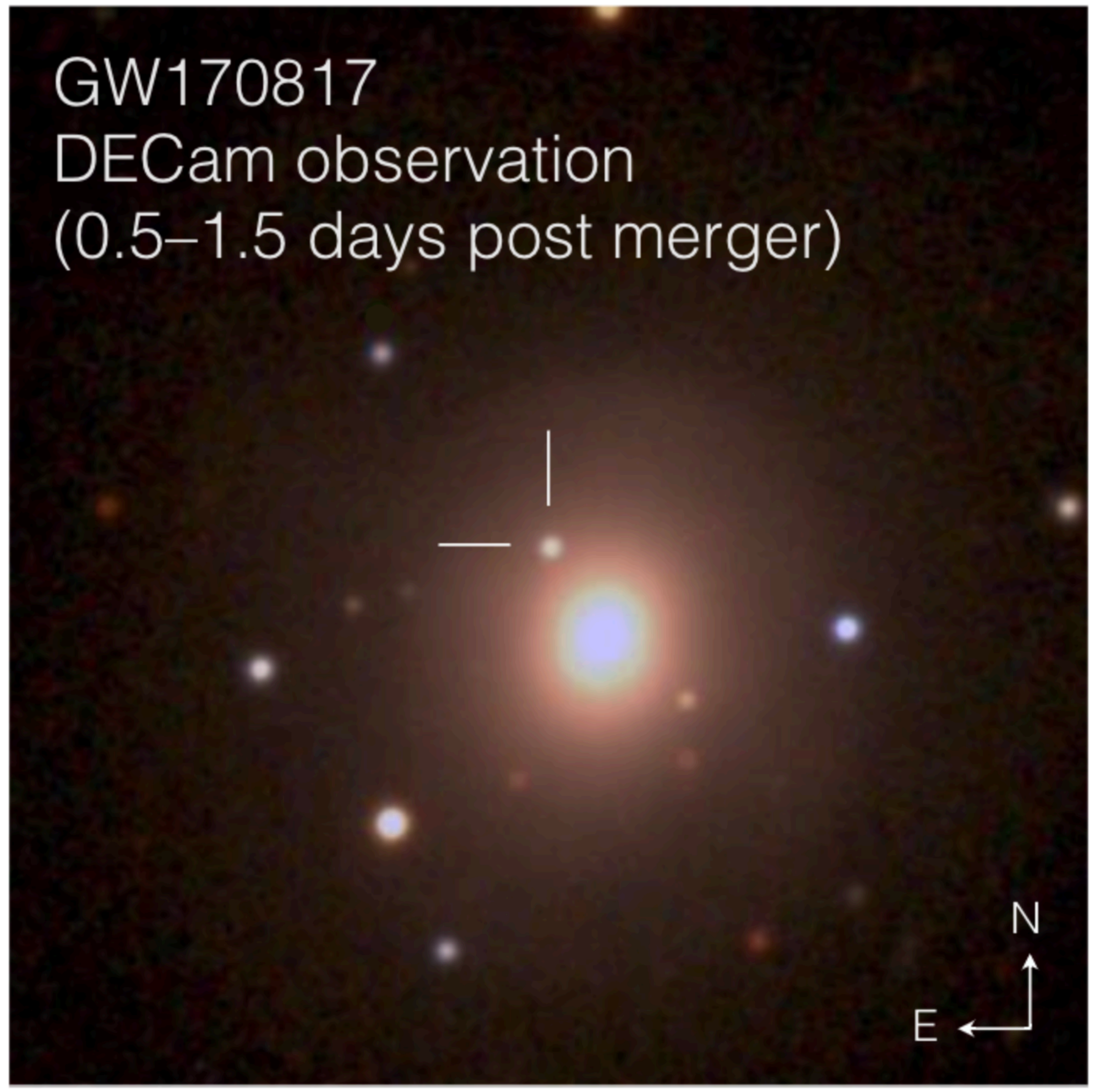
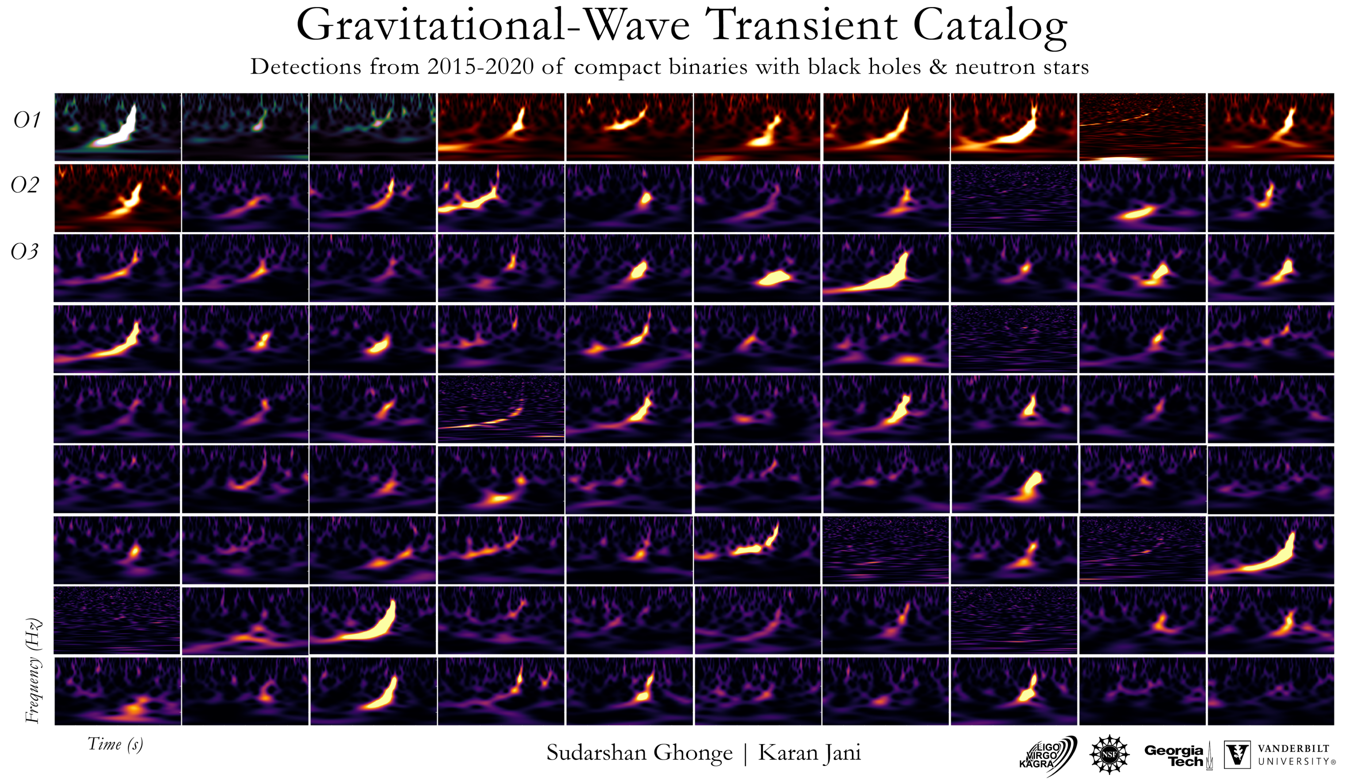


Overall compute



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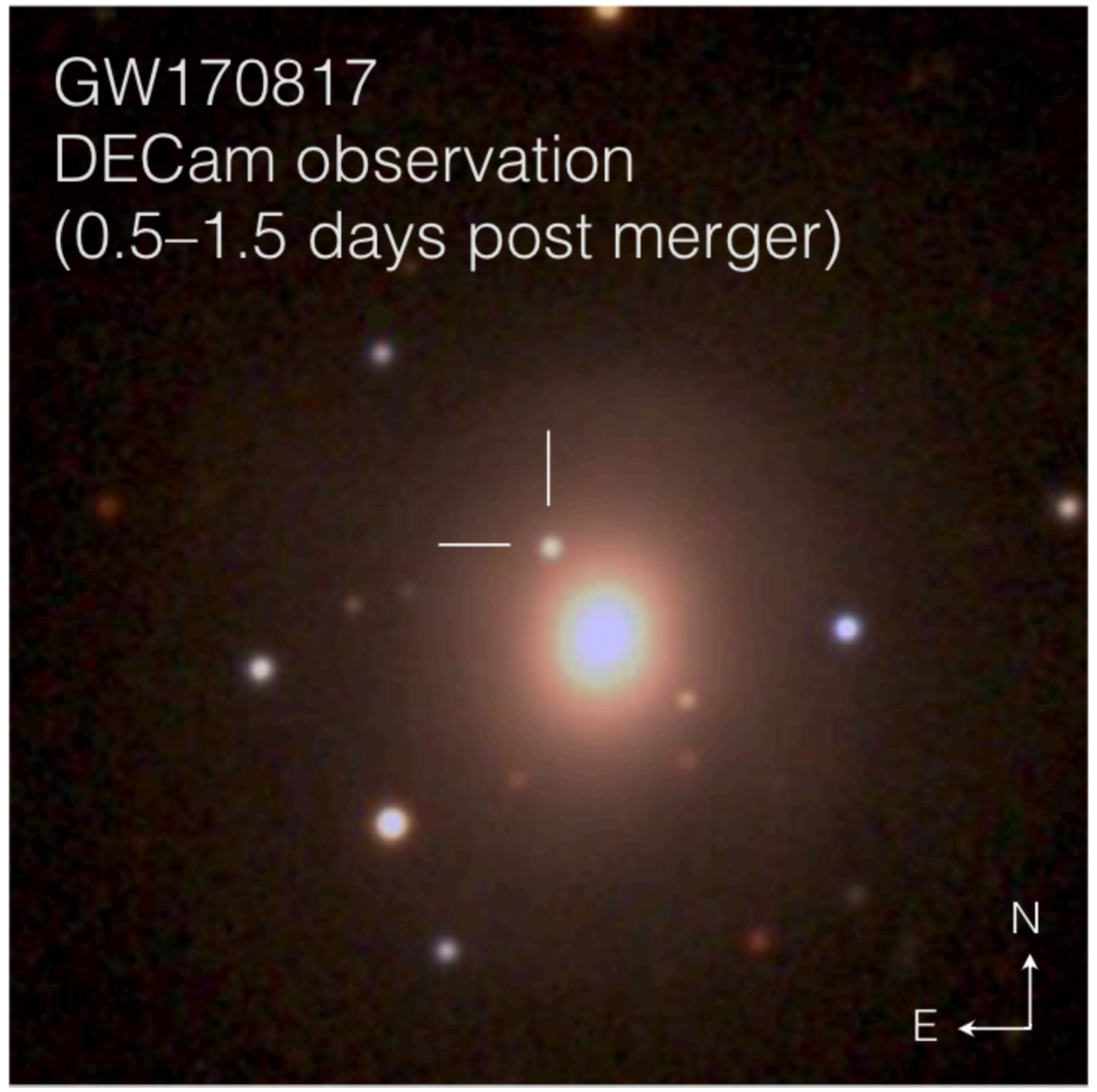

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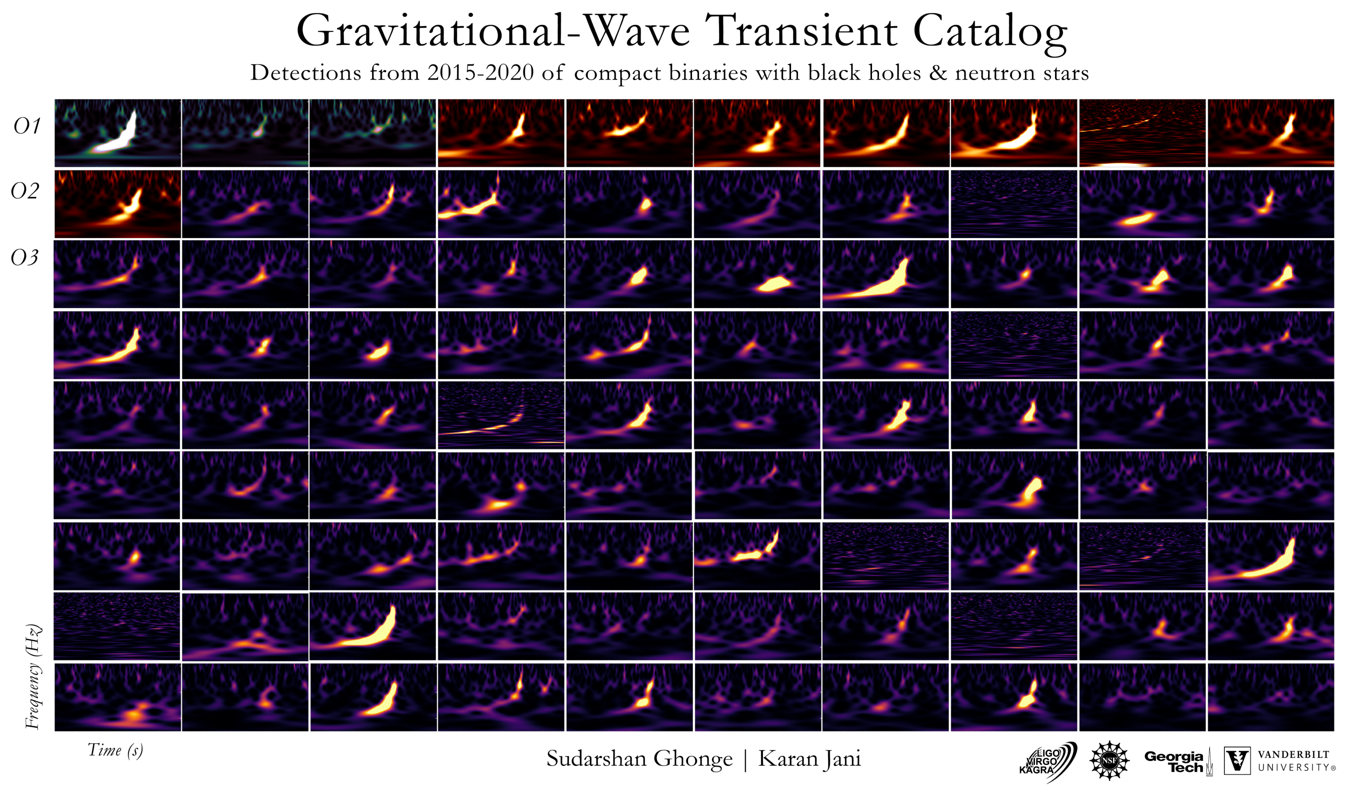
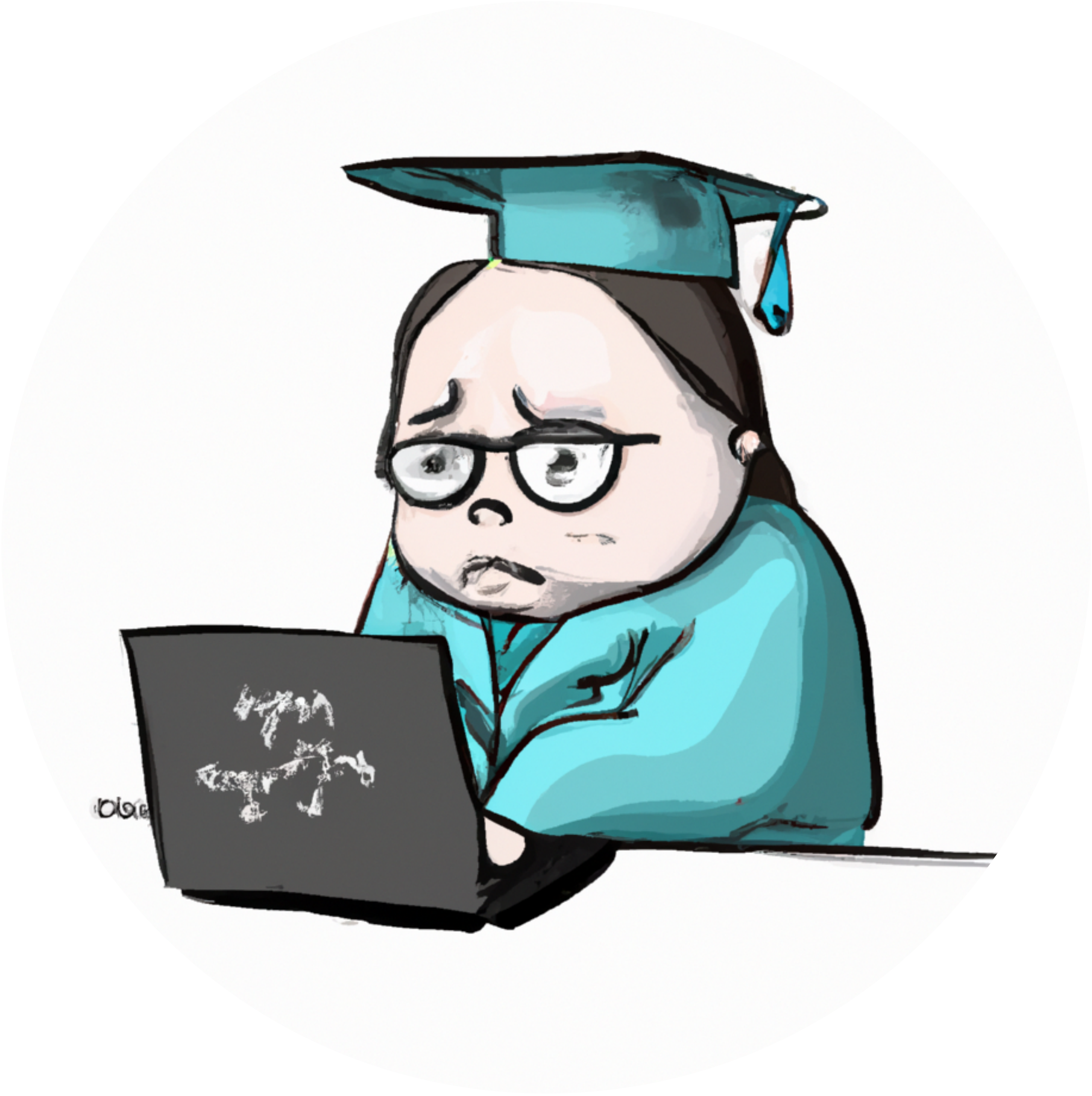


Development cycle



flowMC

$$A\left(x^{\prime}, x\right)=\min \left(1, \frac{P\left(x^{\prime}\right)}{P(x)} \frac{g\left(x \mid x^{\prime}\right)}{g\left(x^{\prime} \mid x\right)}\right)$$

[github.com/kazewong/flowMC]

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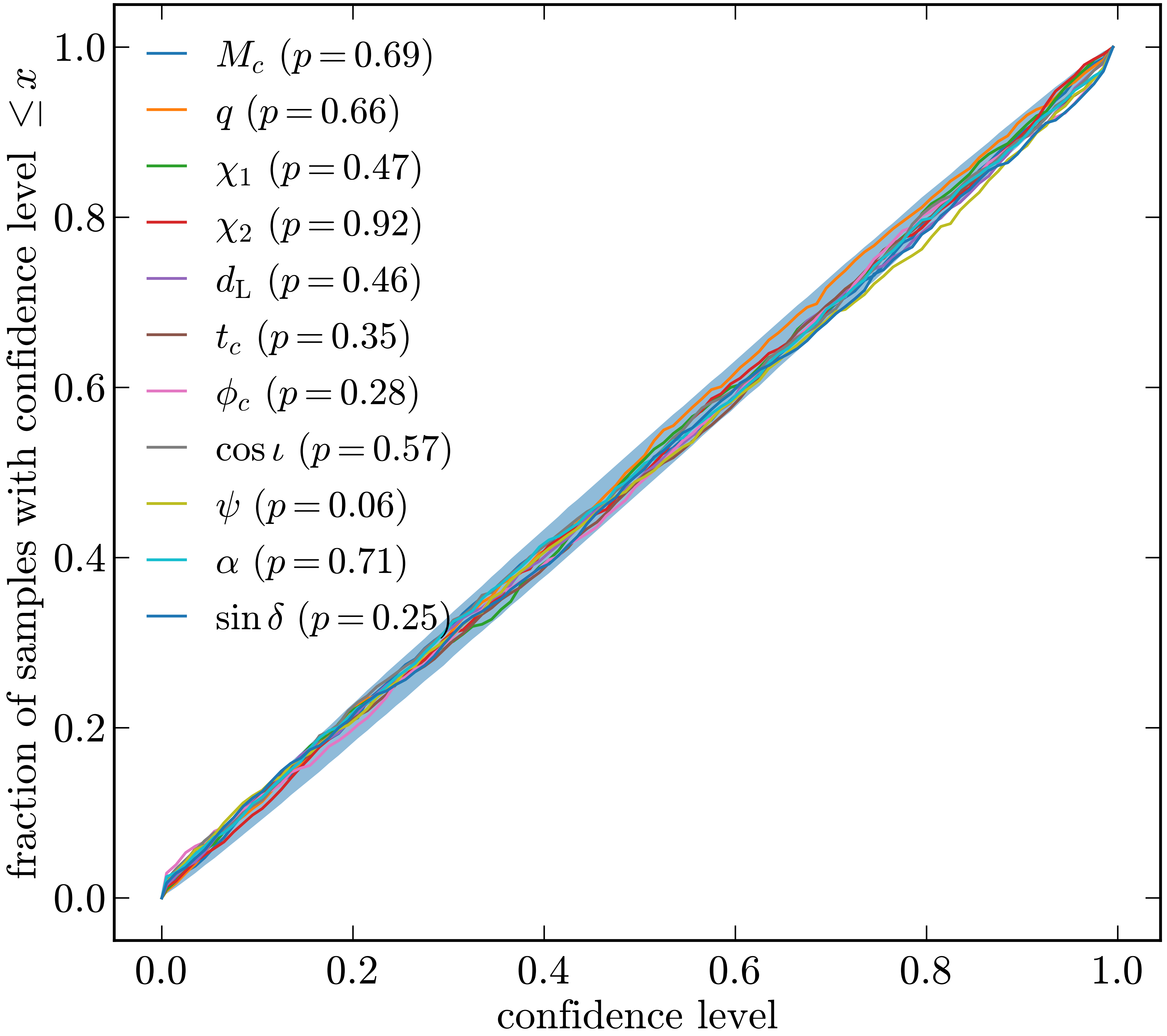
[github.com/kazewong/flowMC]

Accuracy and Speed

For both BNS and BBH, we achieve converged results in **~1 min** on an A100 GPU



[Kaze Wong, Max Isi, **TE**: 2302.05333]

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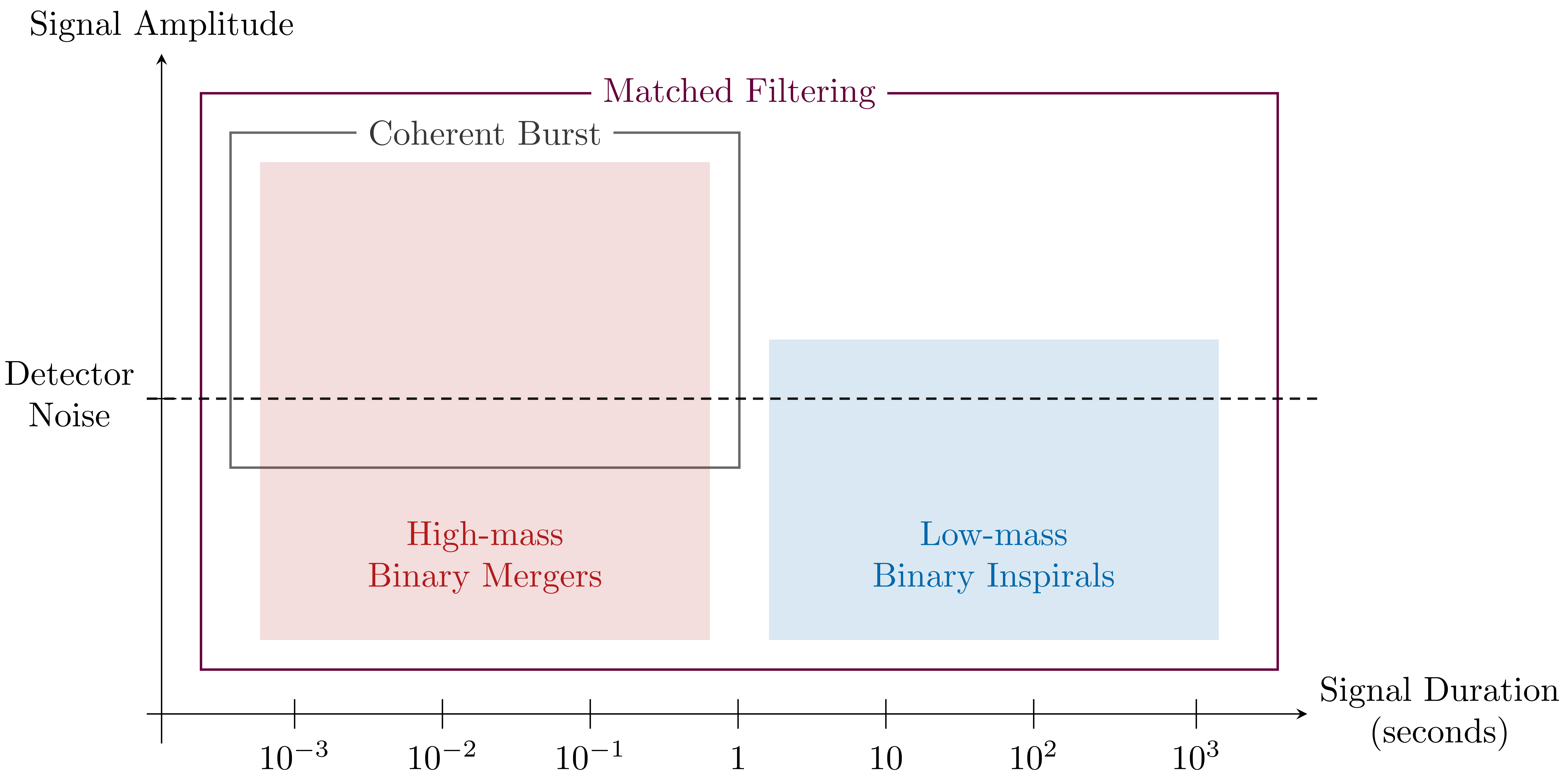
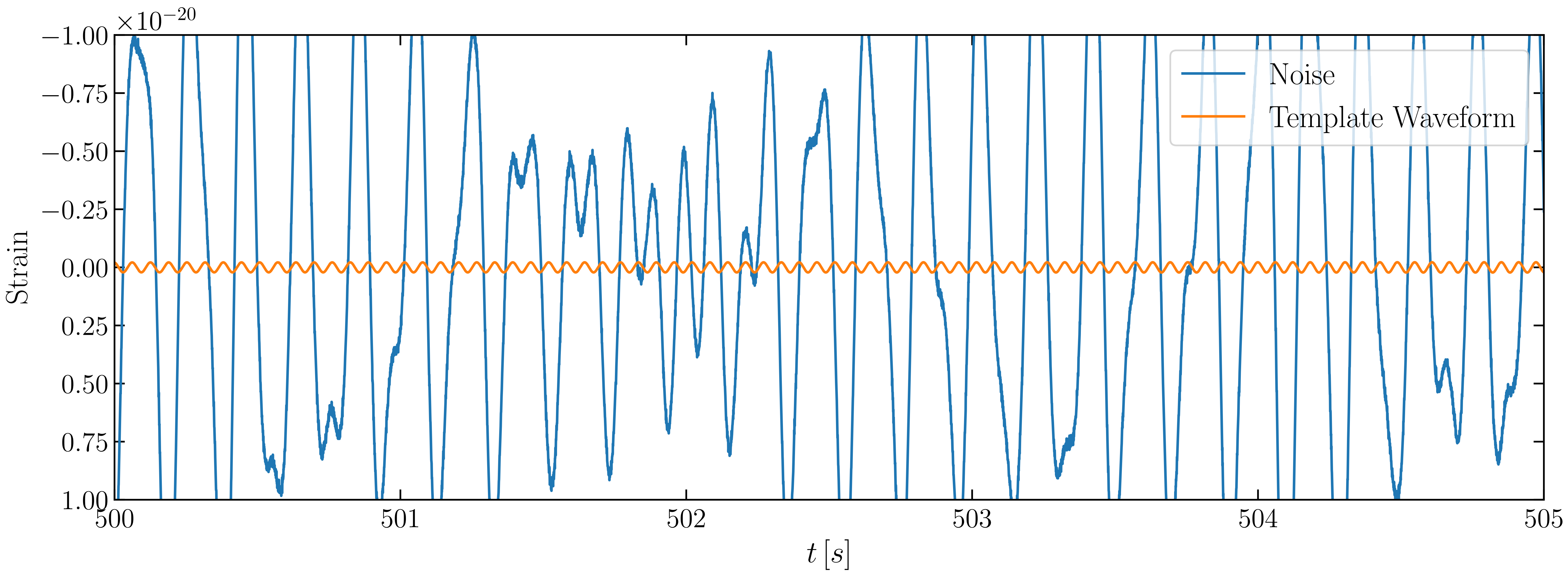


[**TE**, Chia (JCAP): 2004.06729]

Matched Filtering

Matched Filtering is the optimum technique for extracting known signals from **stationary Gaussian** noise



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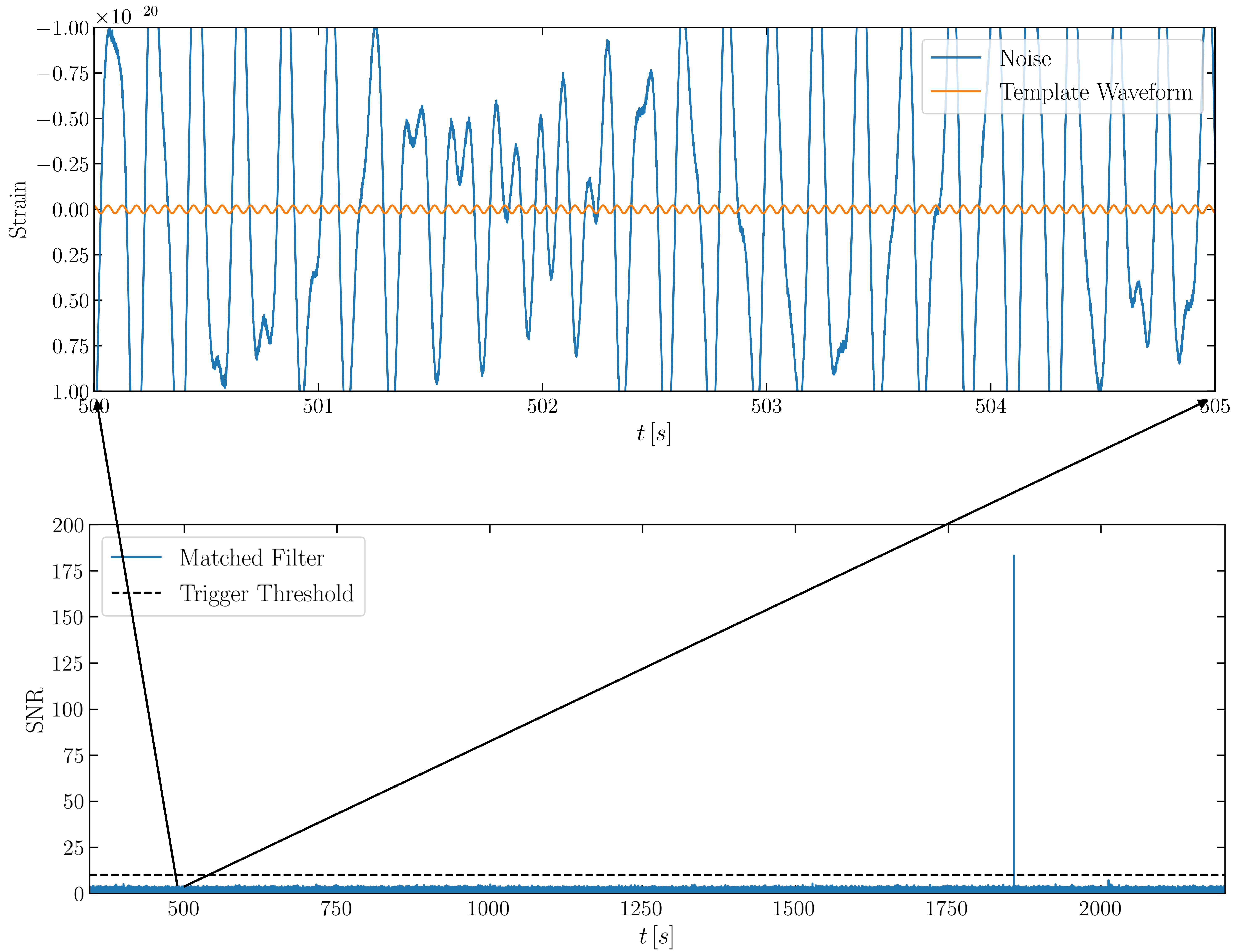
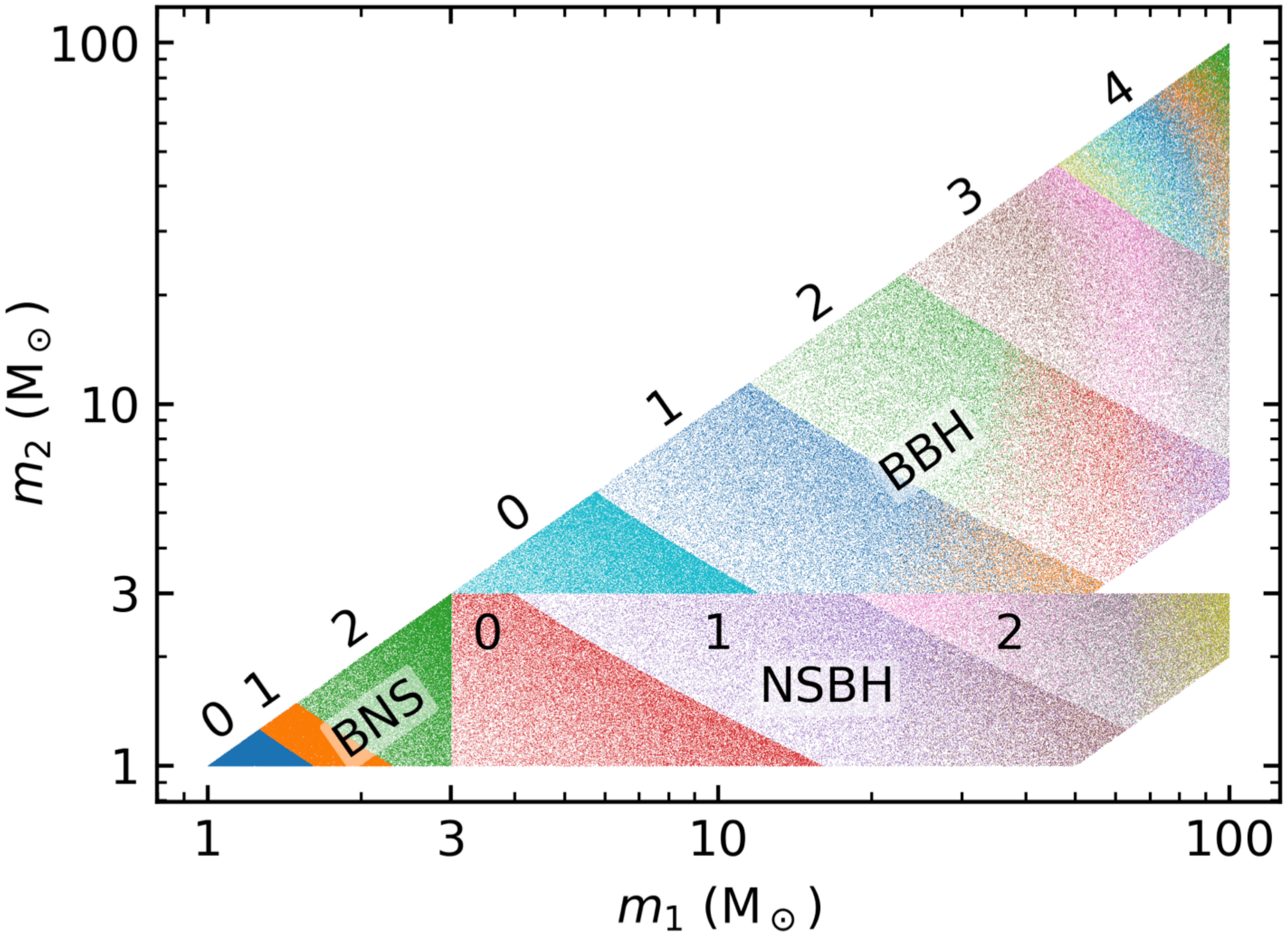
$$(h_1|h_2) \equiv 4\,\mathrm{Re}\int_0^\infty \mathrm{d}f\,\frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}$$



Effectualness/fitting-factor

The effectualness is the percentage of SNR retained by a template bank or model

[Roulet et al.: 1904.01683]



$$\varepsilon\left(h_{\text{finite-size}}\right) \equiv \max_{t_c, \phi_c, \boldsymbol{p}_{\text{bbh}}} \left[h_{\text{finite-size}} \mid h\left(\boldsymbol{p}_{\text{bbh}}\right)\right]$$

Finite-Size Effects

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$$\mathcal{A}(f;\boldsymbol{p},\boldsymbol{\lambda}) \propto \mathcal{A}_{\rm PN}(\boldsymbol{p}) + \mathcal{A}_{\rm Int}(\boldsymbol{\lambda}) + \mathcal{A}_{\rm MR}(\boldsymbol{\lambda})$$

$$\phi(f;\boldsymbol{p},\boldsymbol{\lambda}) \propto \phi_{\rm PN}(\boldsymbol{p}) + \phi_{\rm Int}(\boldsymbol{\lambda}) + \phi_{\rm MR}(\boldsymbol{\lambda})$$

$$\phi(f;\boldsymbol{p}) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\nu v^5}\left(\phi_{\rm point-particle} + \phi_{\rm finite-size}\right)$$

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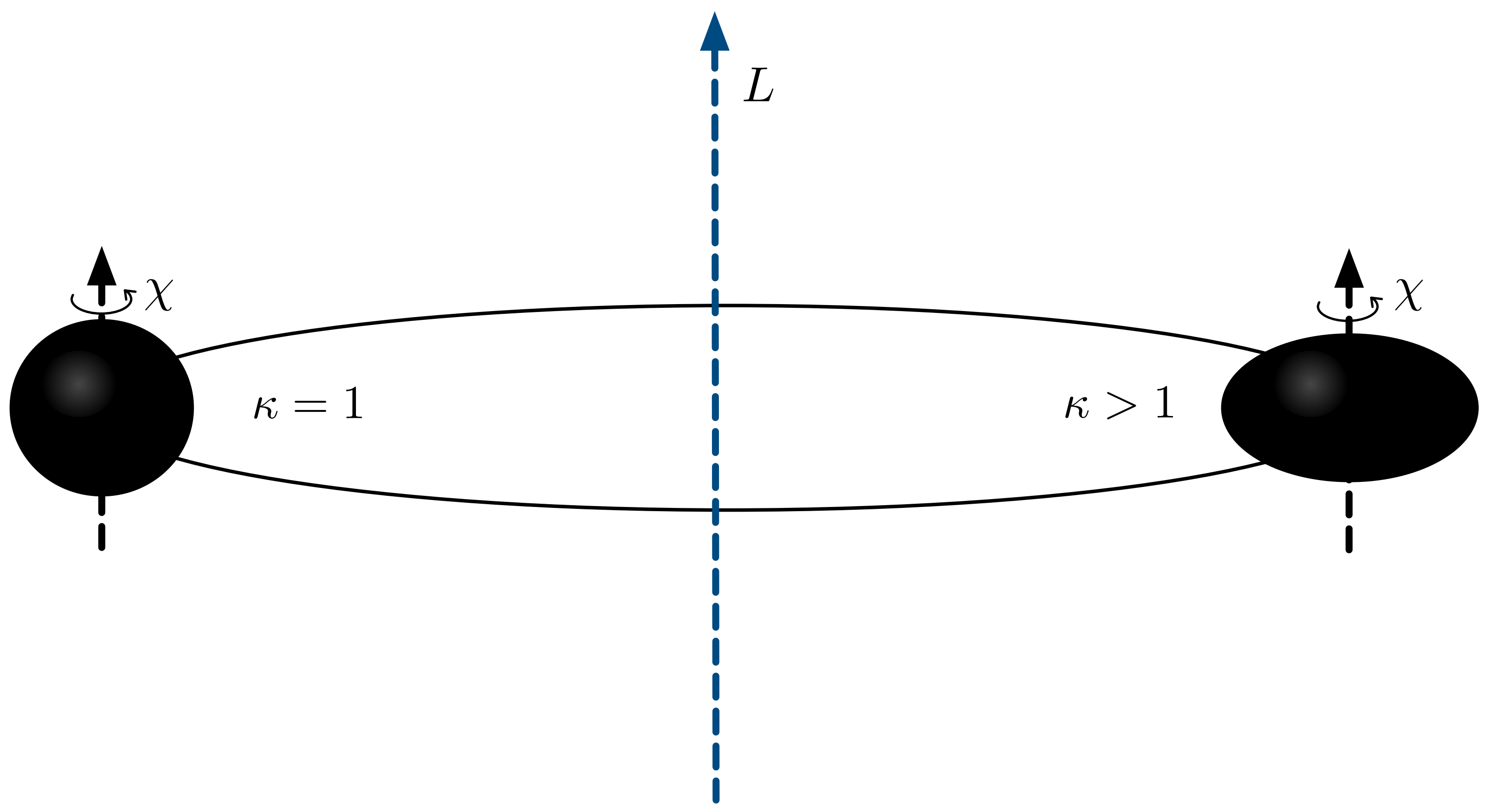
Spin Induced Quadruple



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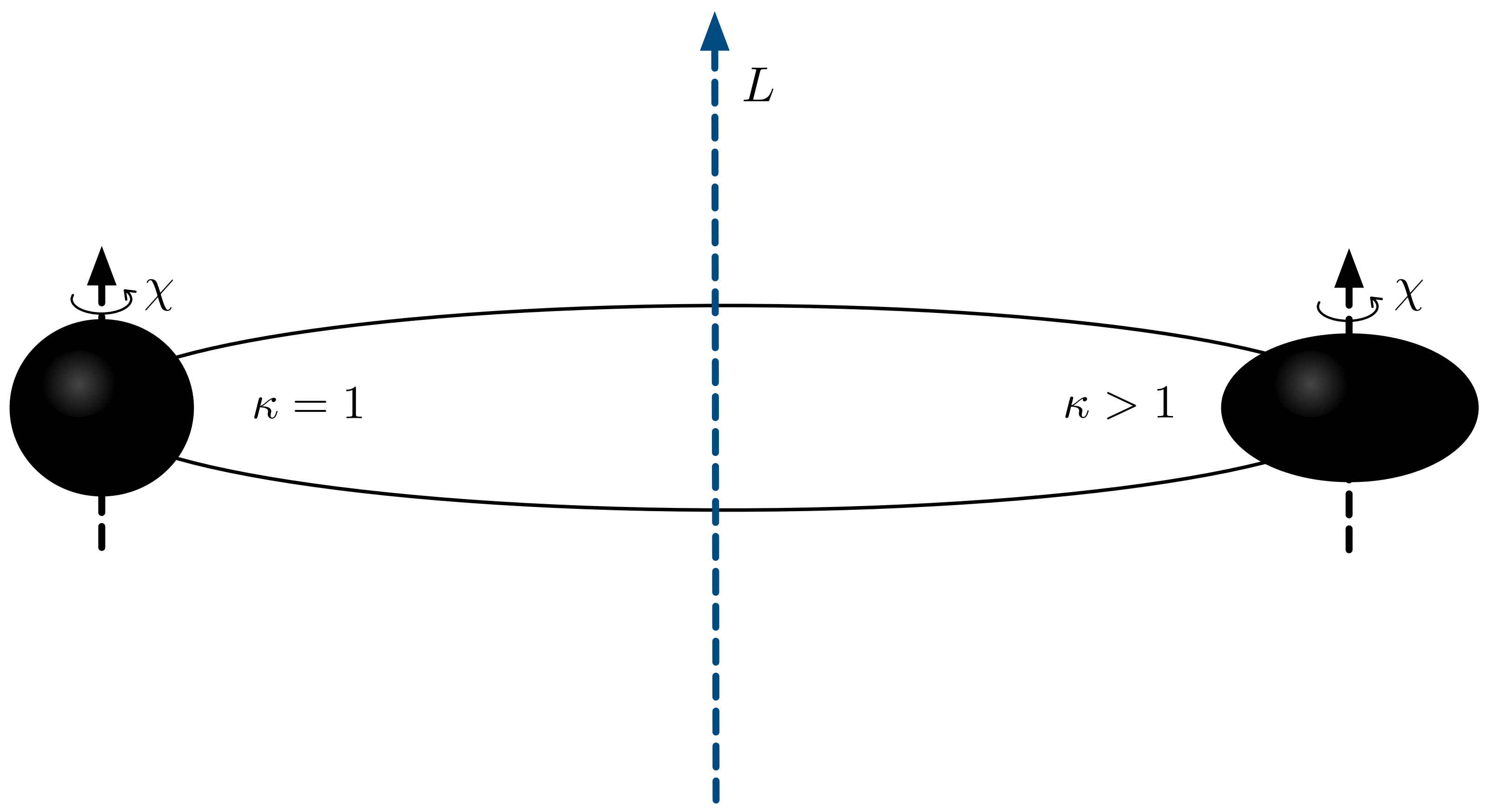
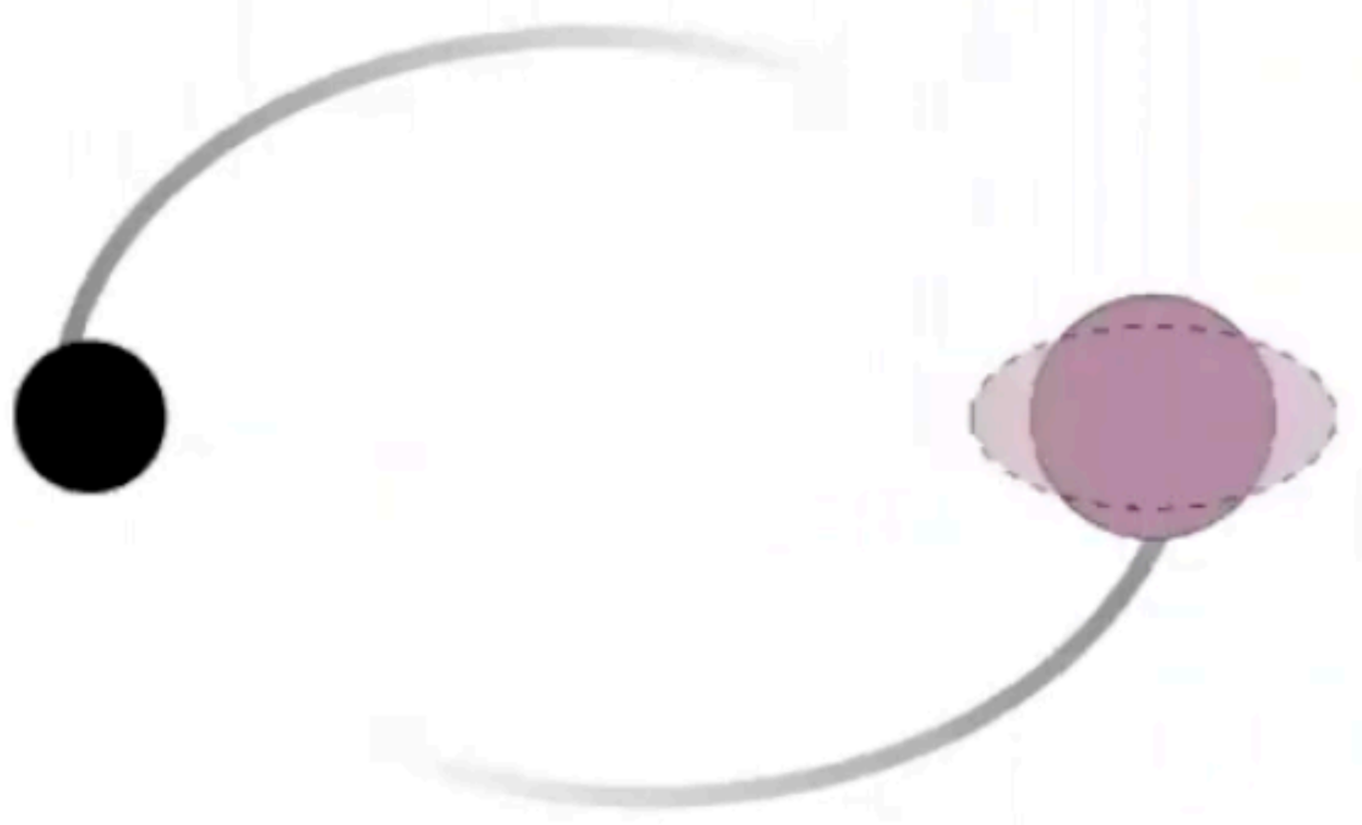


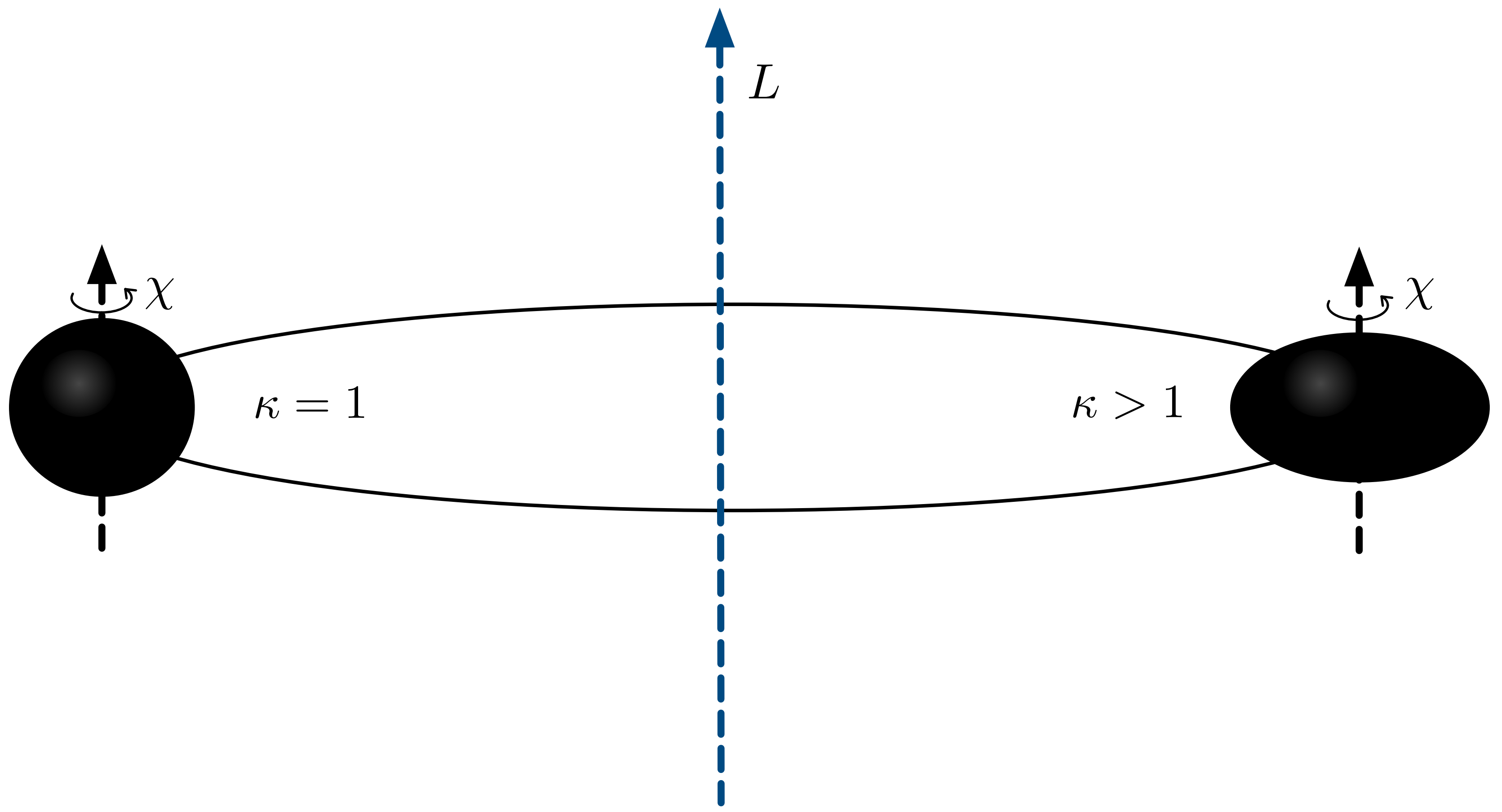
Tidal Love Number

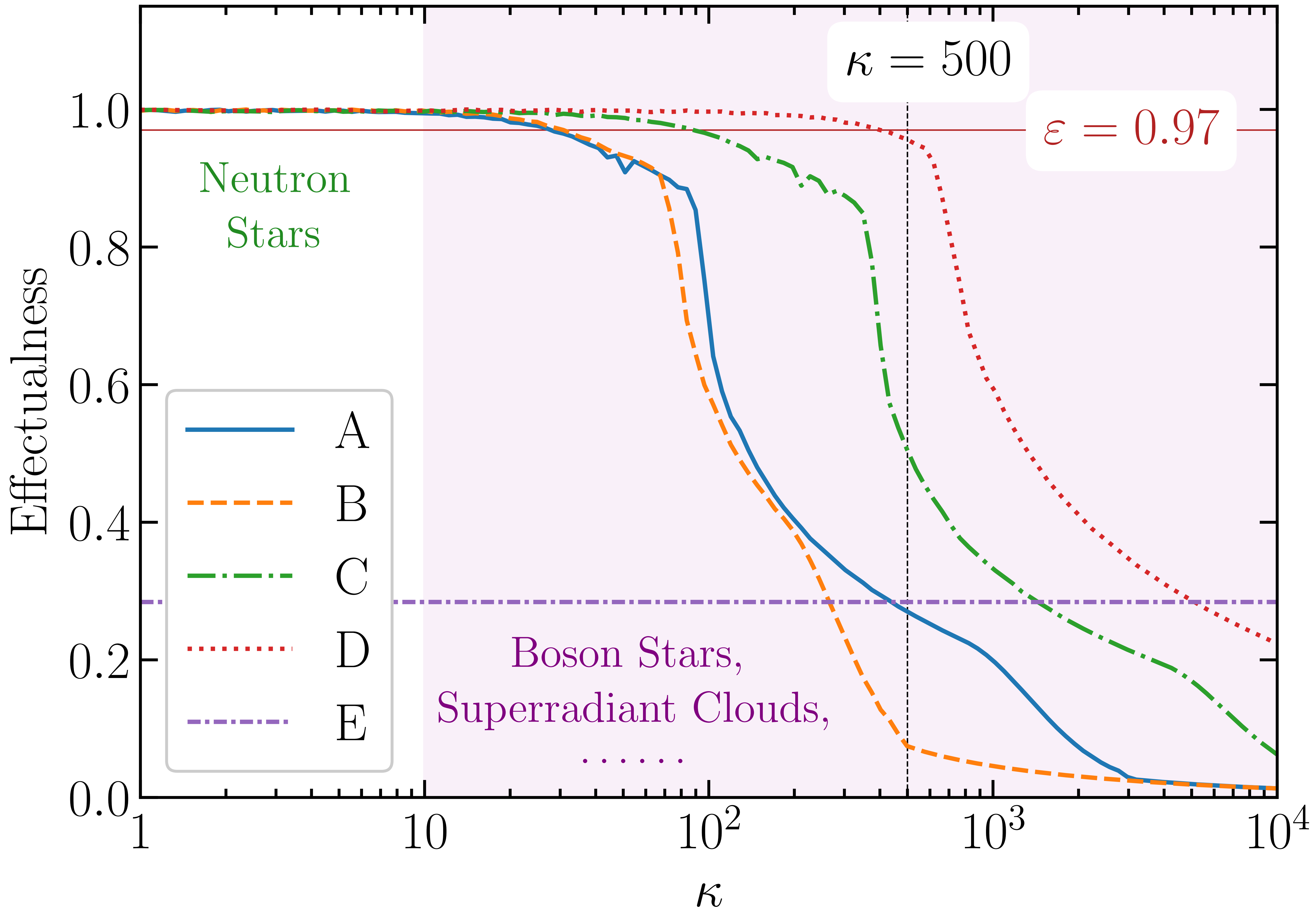


Finite-Size Effects

Spin-Induced Quadrupoles

Kappa controls the axisymmetric response of an object to spin *i.e.*, how oblate it becomes





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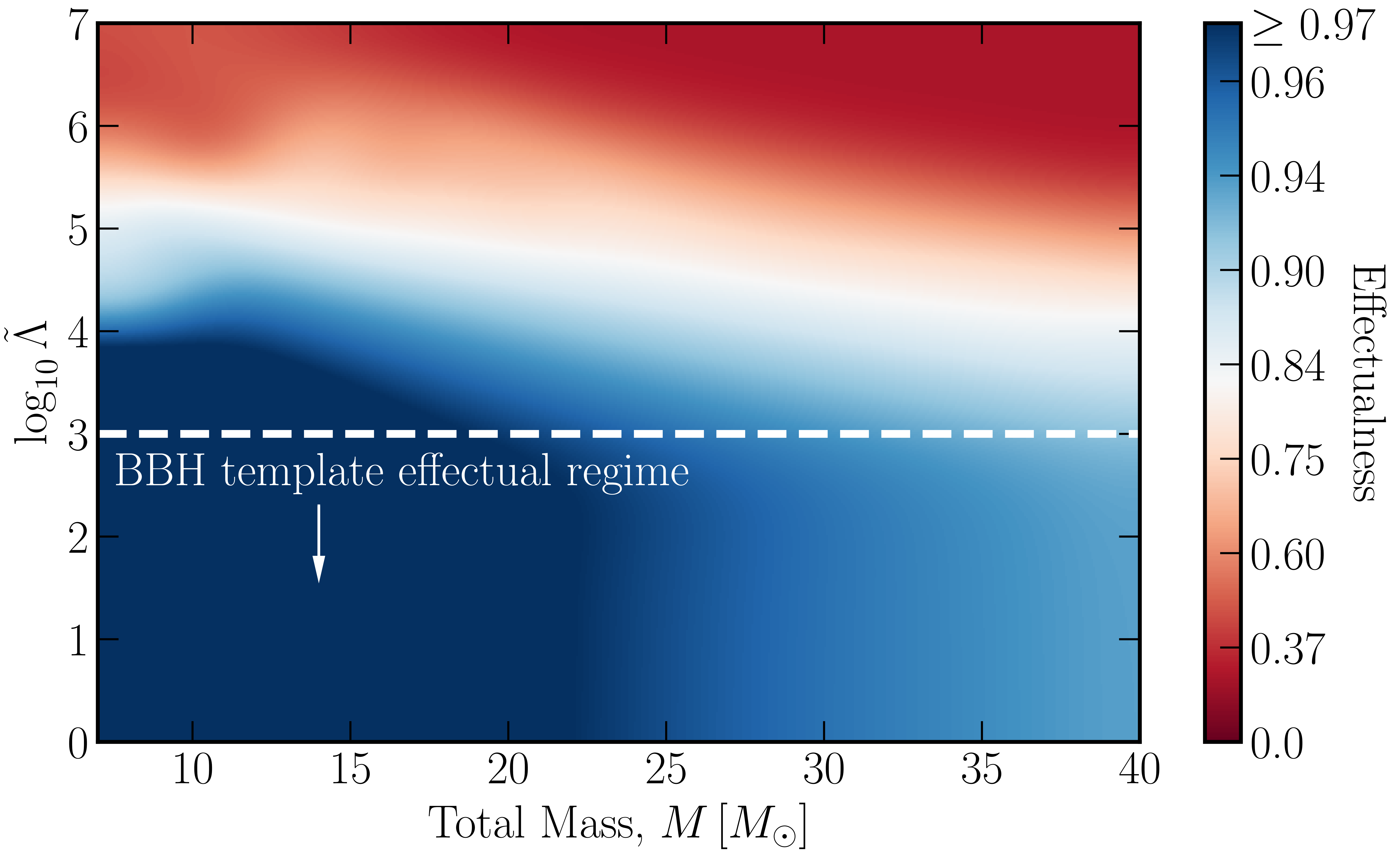

[Horng Sheng Chia, **TE**, Digvijay Wadekar, Aaron Zimmerman et al.: 2306.00050]

Why would Love Numbers be Large?

$$\Lambda = \frac{2}{3}k\left(\frac{r}{m}\right)^5$$

[Kent Yagi, Nicolas Yunes: 1303.1528]

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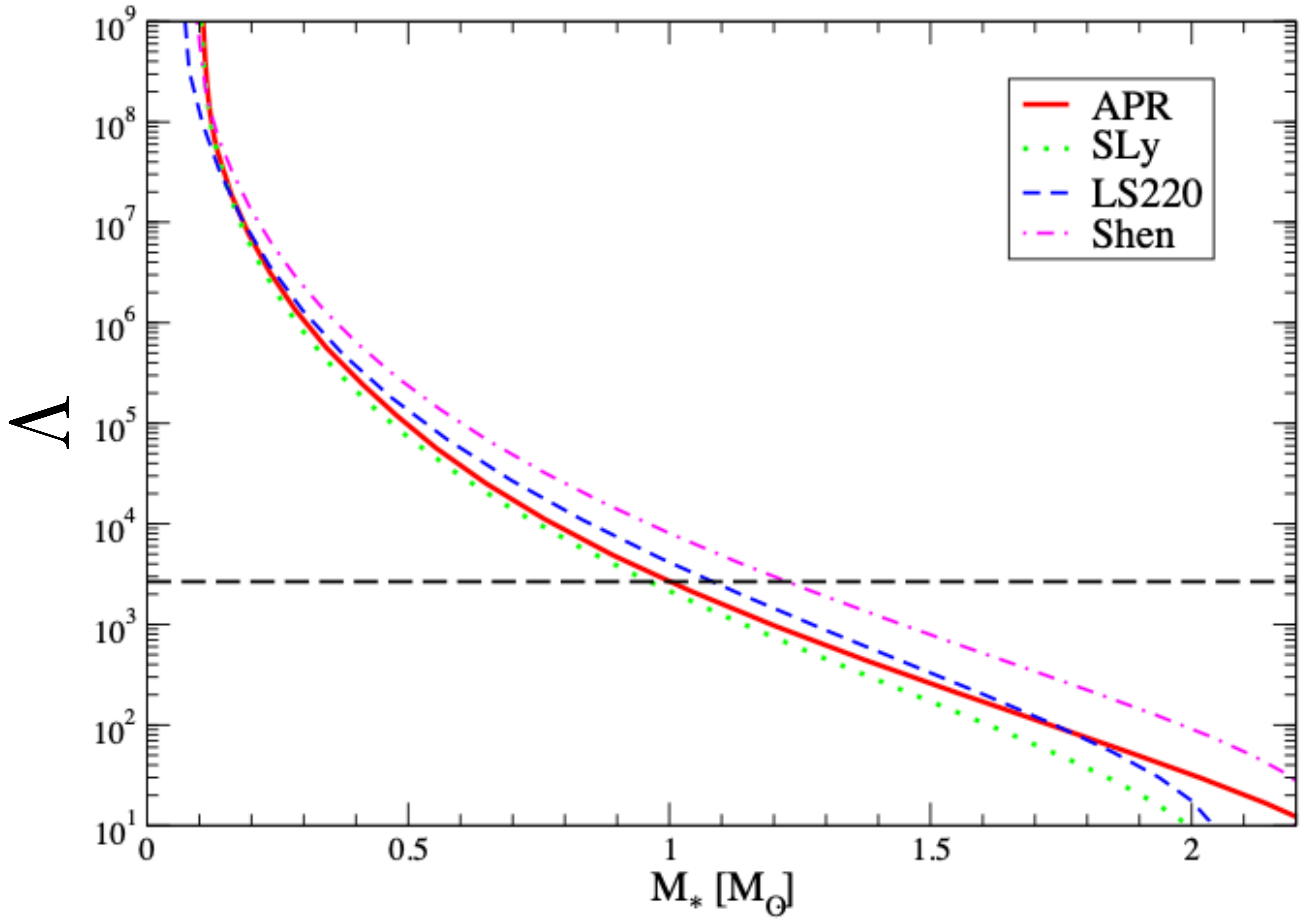
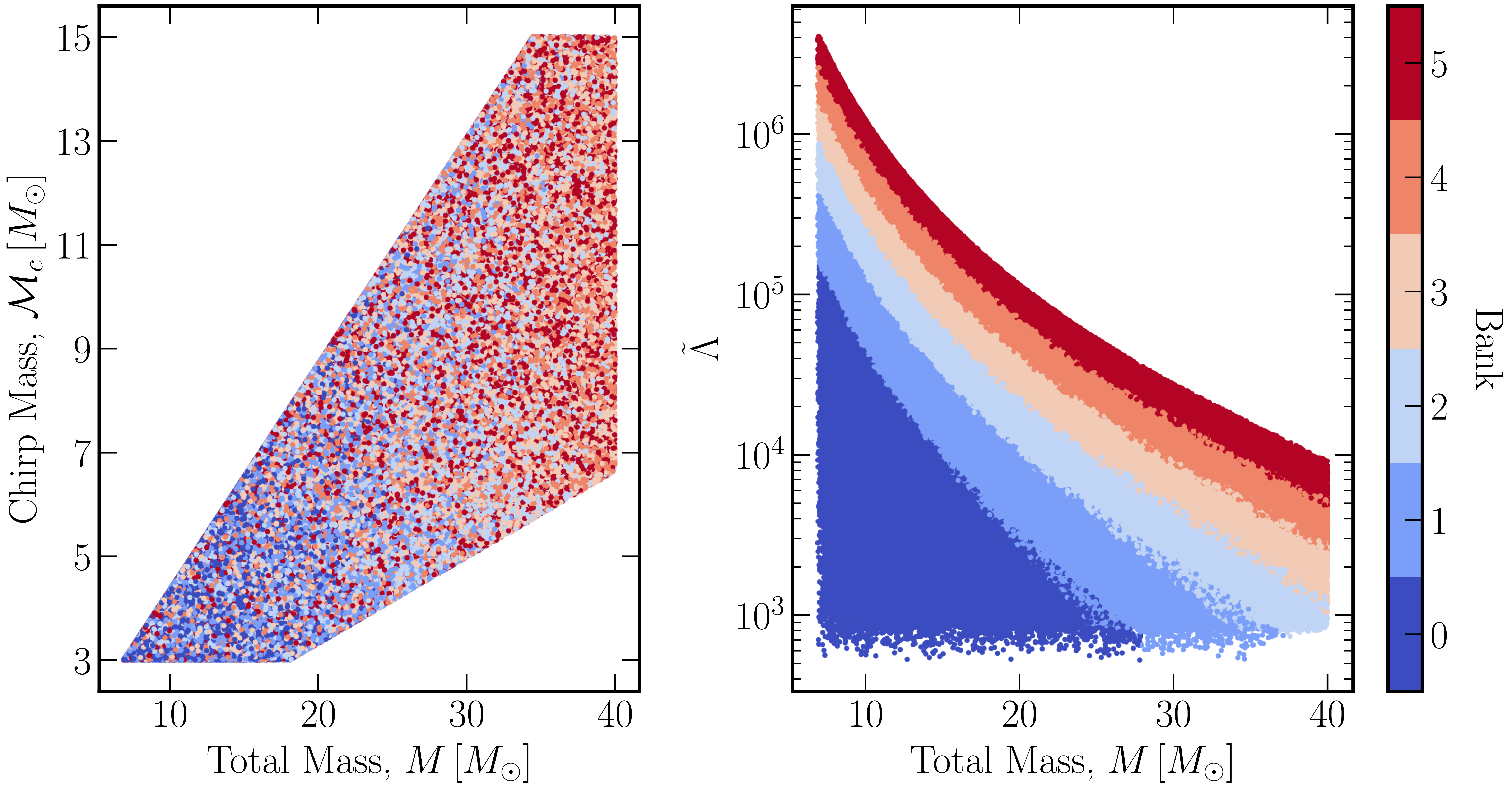


Template Bank



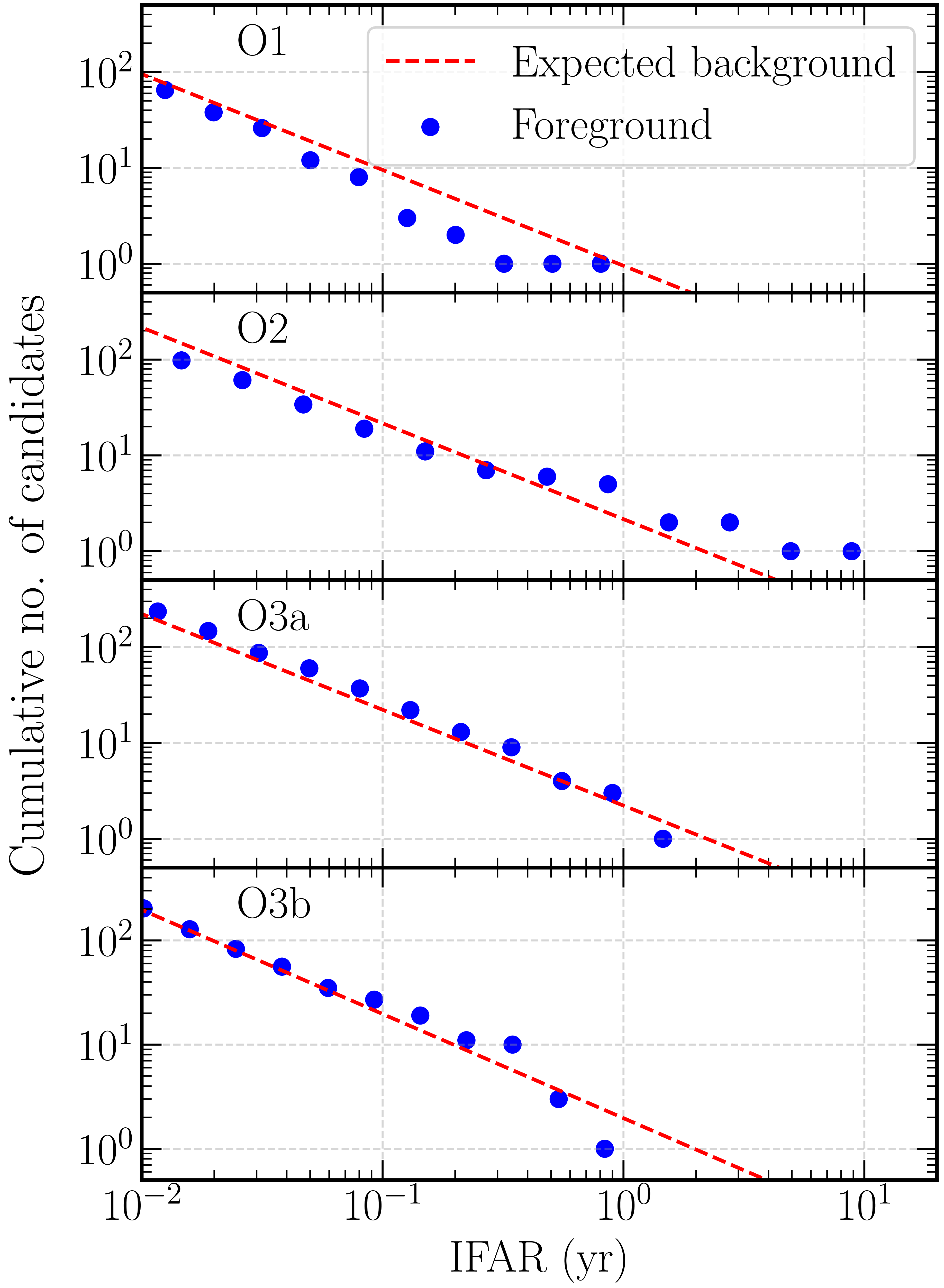
Anything outside this space we cannot guarantee coverage

[Roulet et al.: 1904.01683]

Null Results

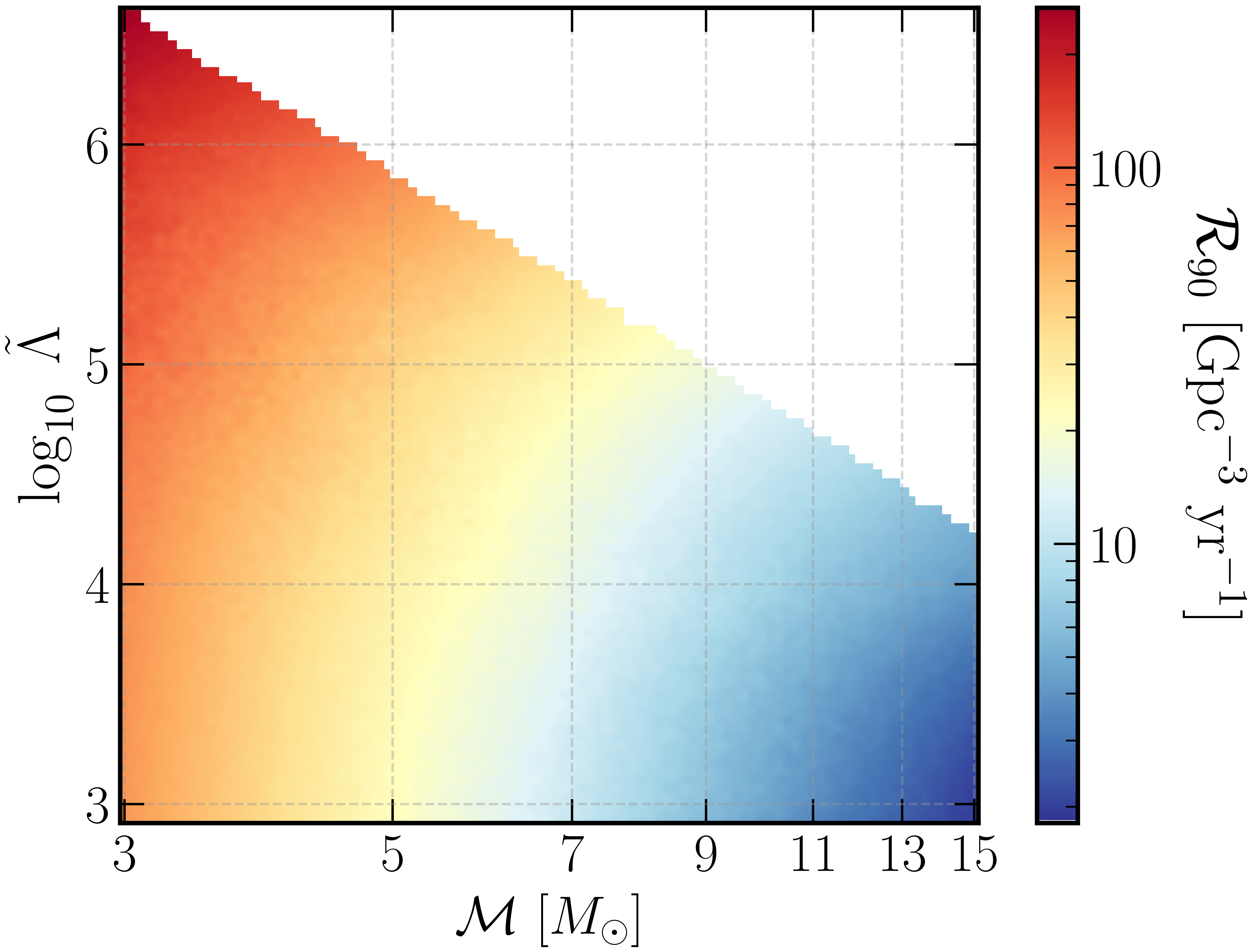
We performed the first inspiral-only search for signals that are **not** aligned-spin BBHs

No significant candidates found



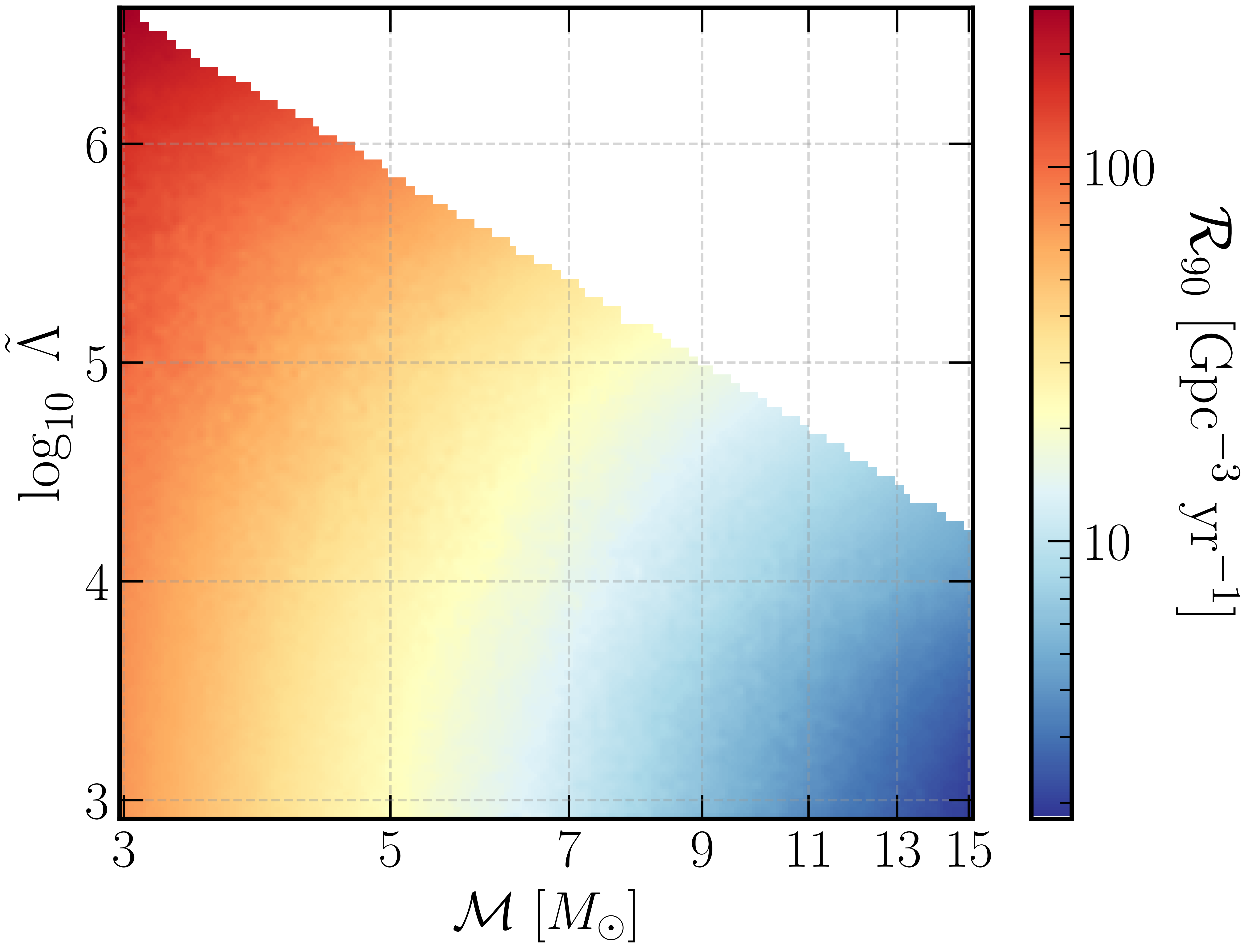
[Horng Sheng Chia, **TE**, Digvijay Wadekar, Aaron Zimmerman et al.: 2306.00050]

Rate Constraints



Rate Constraints

Constraints get weaker, due to shorter signal



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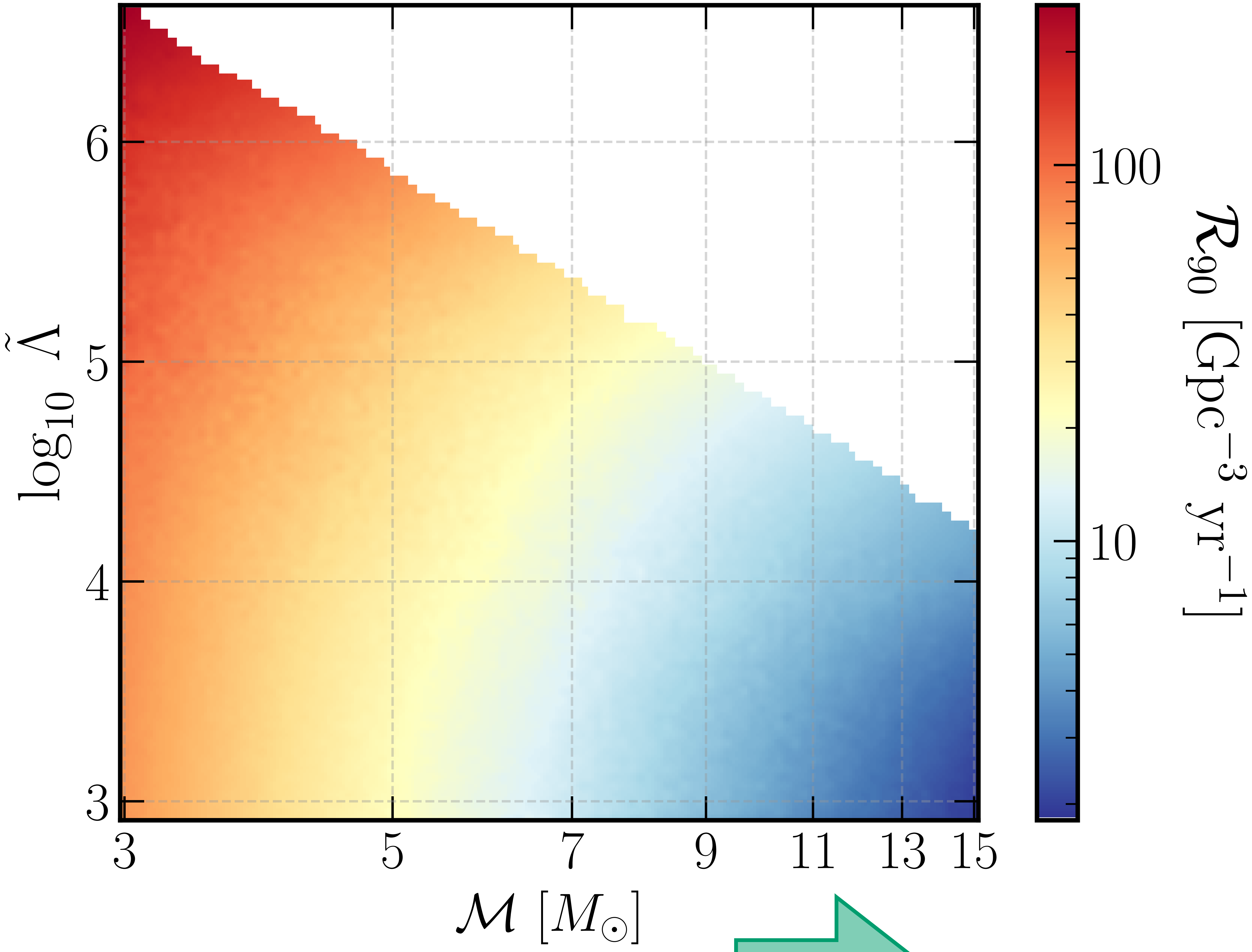
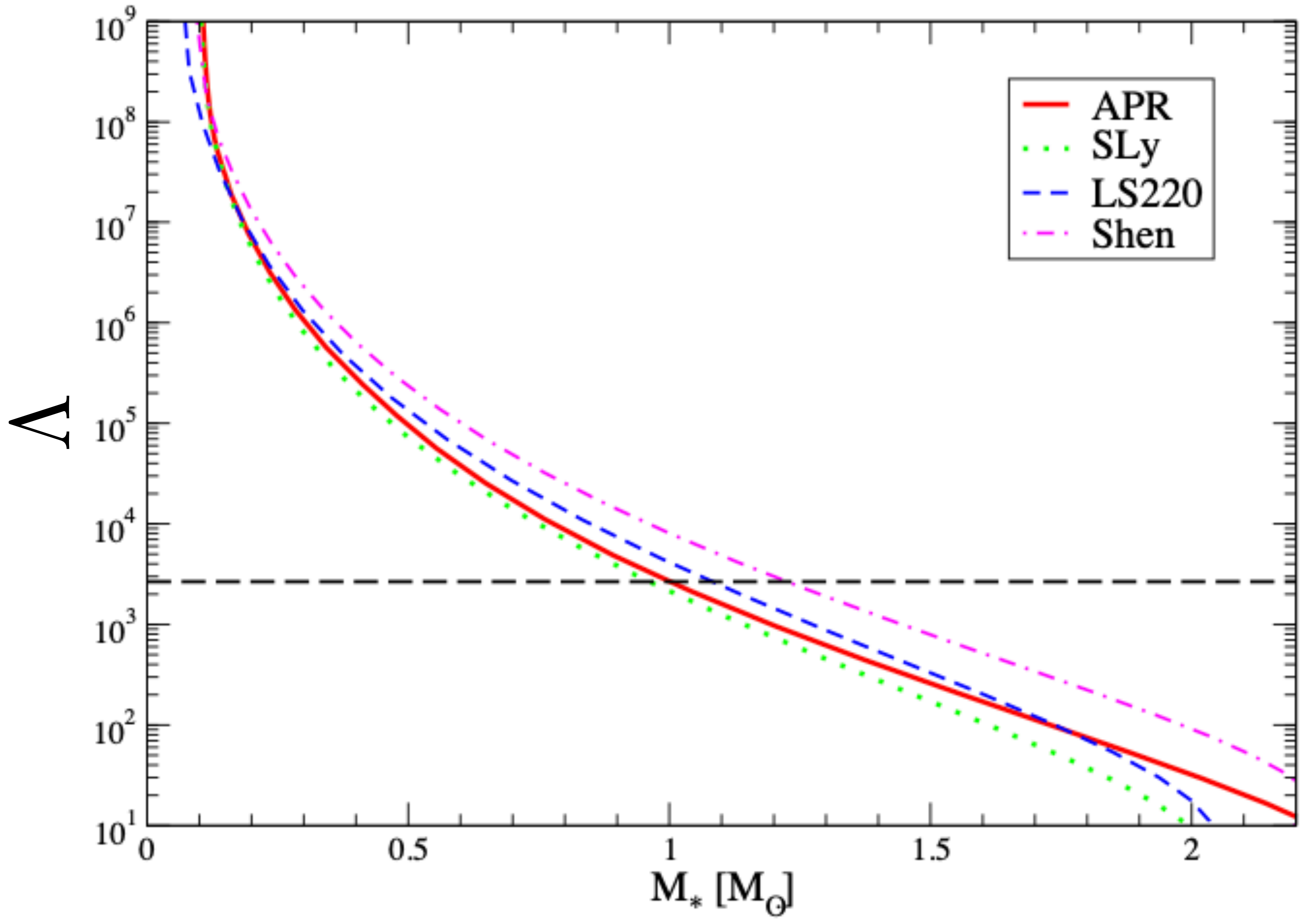


Constraints get stronger as higher chirp masses produce higher SNR signals

Future Searches



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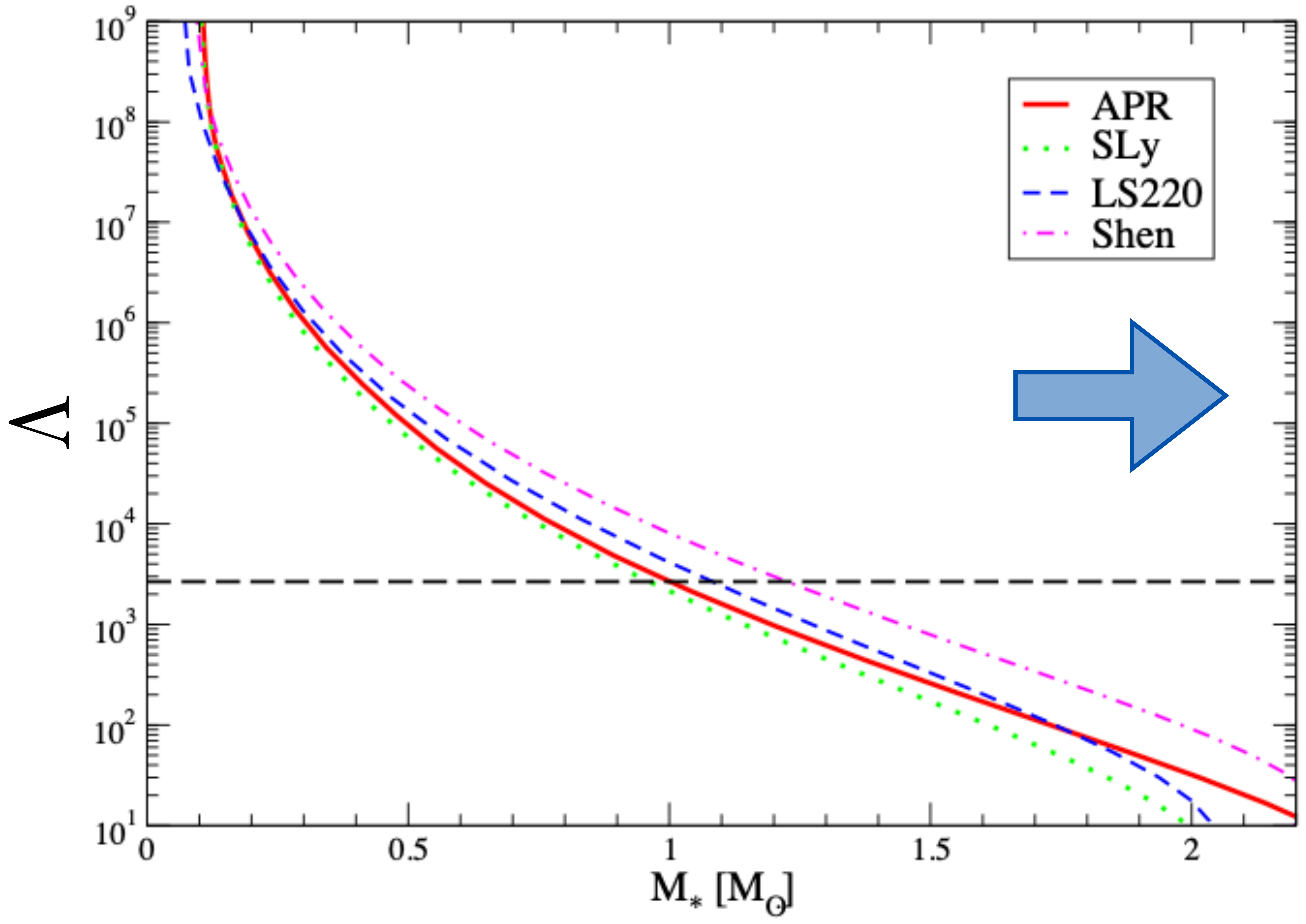
Future Searches

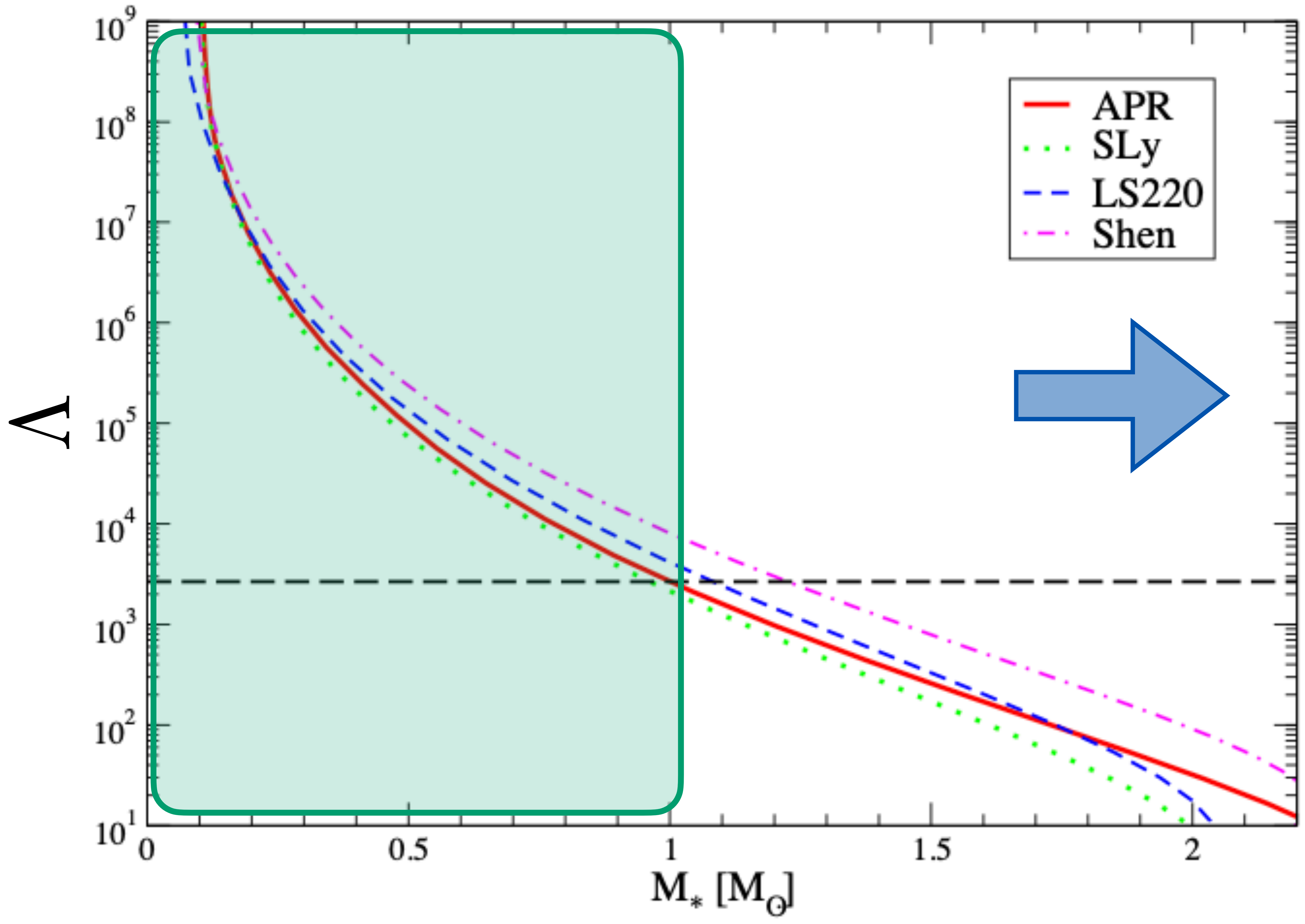
We have currently search in the high-mass region of parameter space due to computational limitations



Future Searches

We have currently search in the high-mass region of parameter space due to computational limitations

Future searchers should focus on the low-mass region, where neutron stars may be hiding



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Differentiable models can contribute to many aspects of the GW data analysis pipeline

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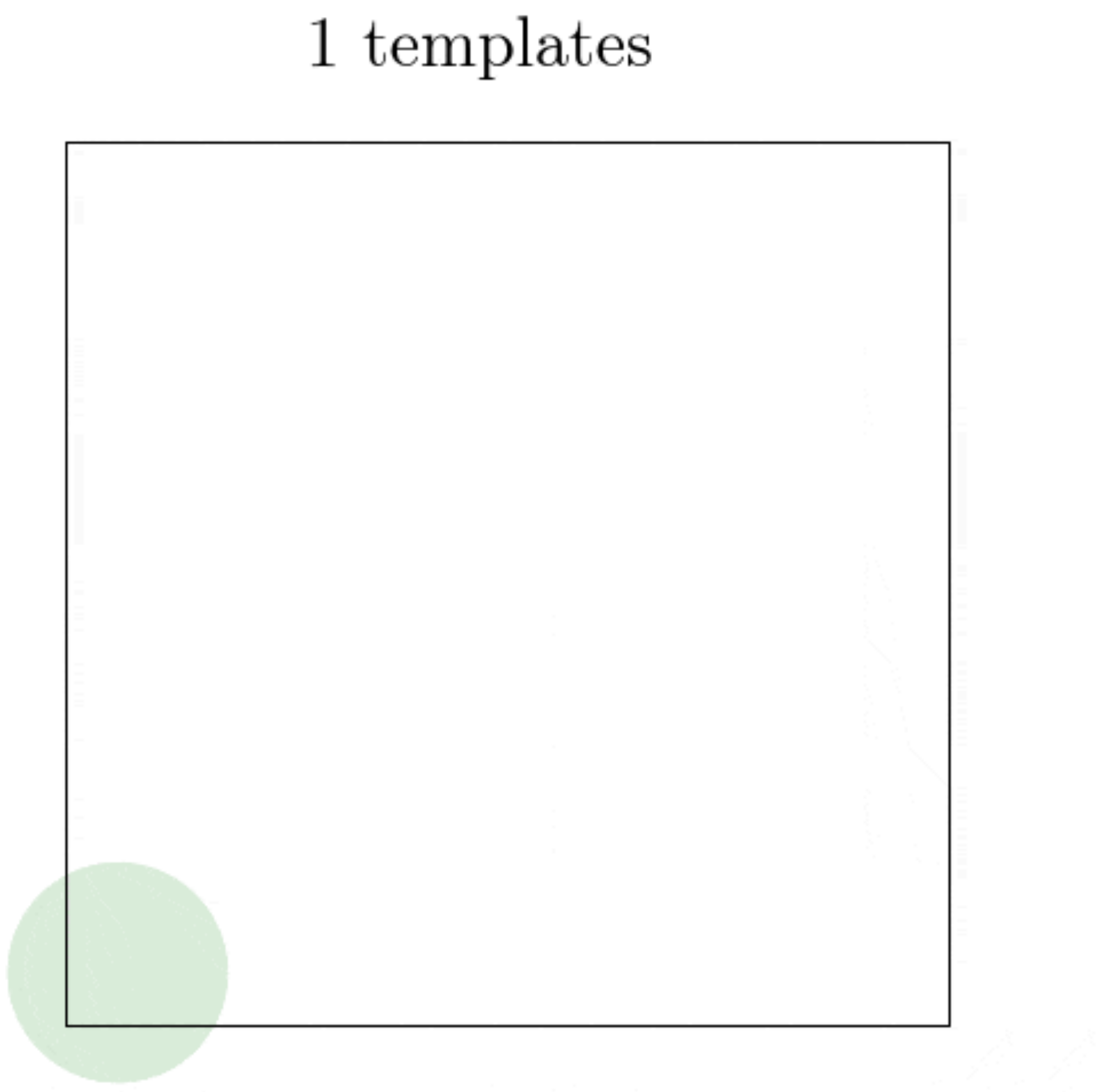
- Real time PE processing could potentially replace matched filtering search pipelines

Searches, especially at low masses, are currently limited by their template bank coverage

- Many more searches to be done in order to fully utilize the GW data we have
- Low-mass neutron stars are a great target for a future Love numbers search

Backup - Template Banks



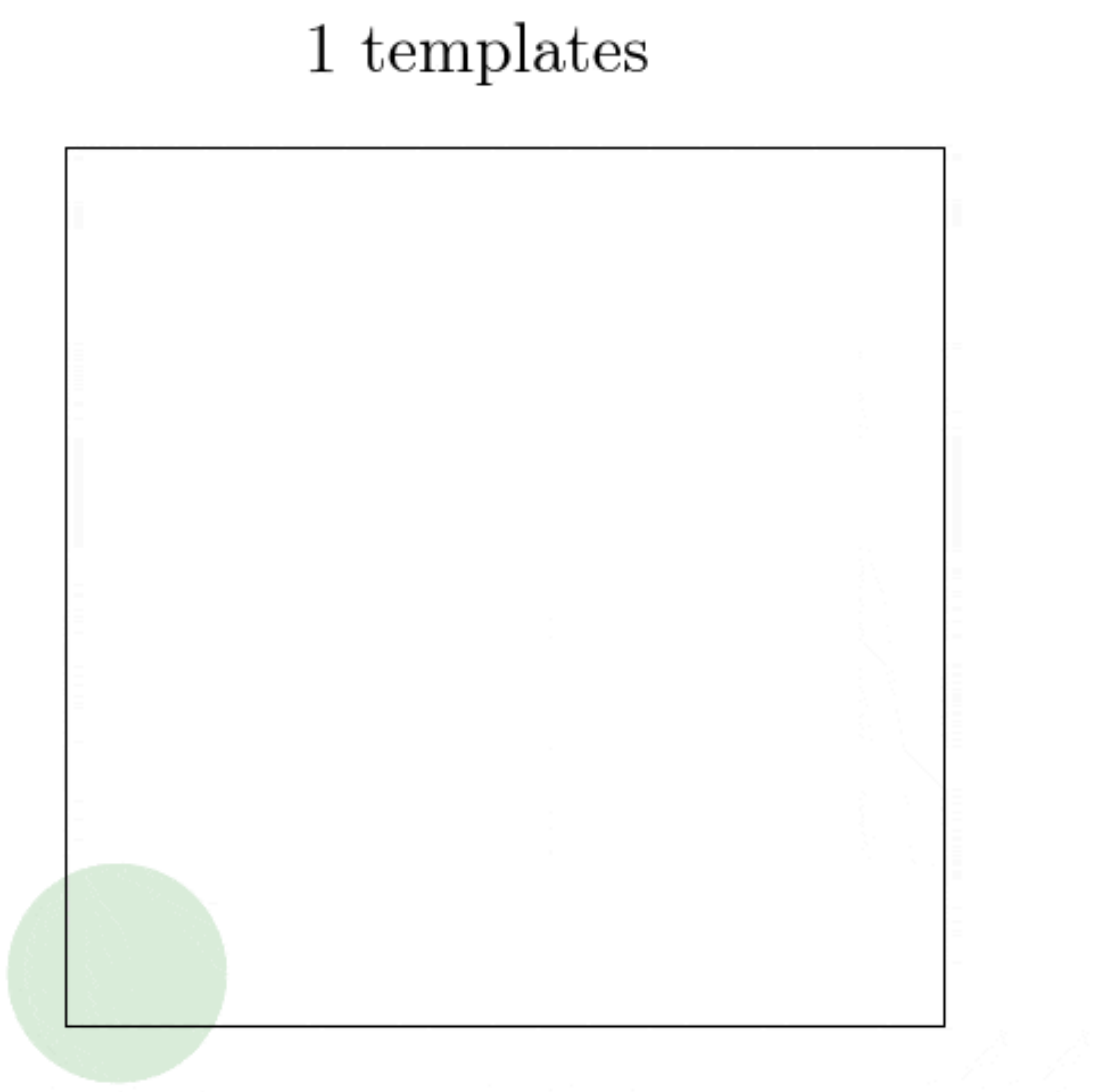
- Quick to generate
- Can achieve ~100% coverage
- Placement of points in highly curved spaces unknown or difficult
- Require a known *metric*

[Cokelaer: 0706.4437]

[Owen: 9511032]

[Owen and Sathyaprakash: 9808076]

Backup - Template Banks



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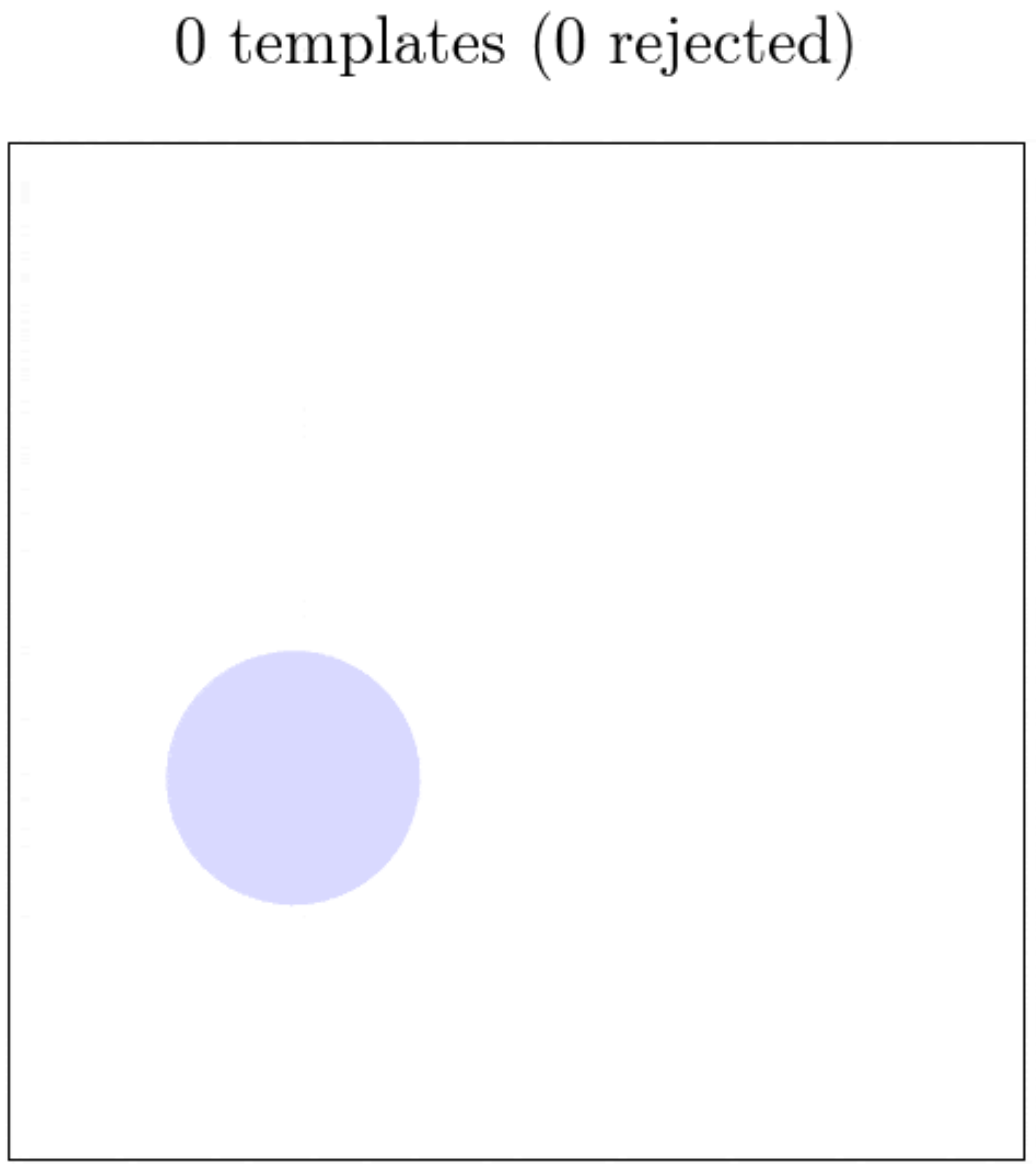

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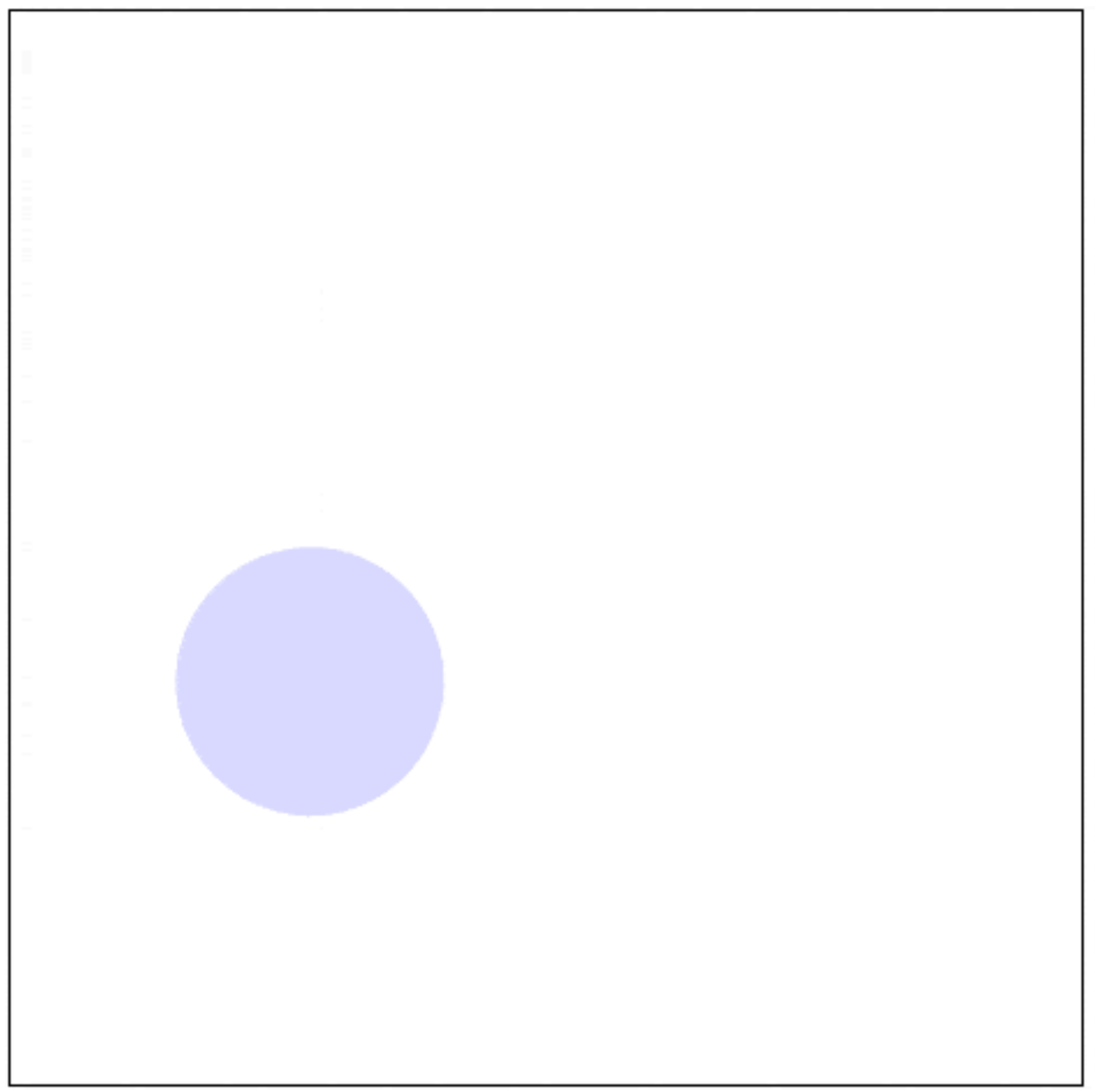

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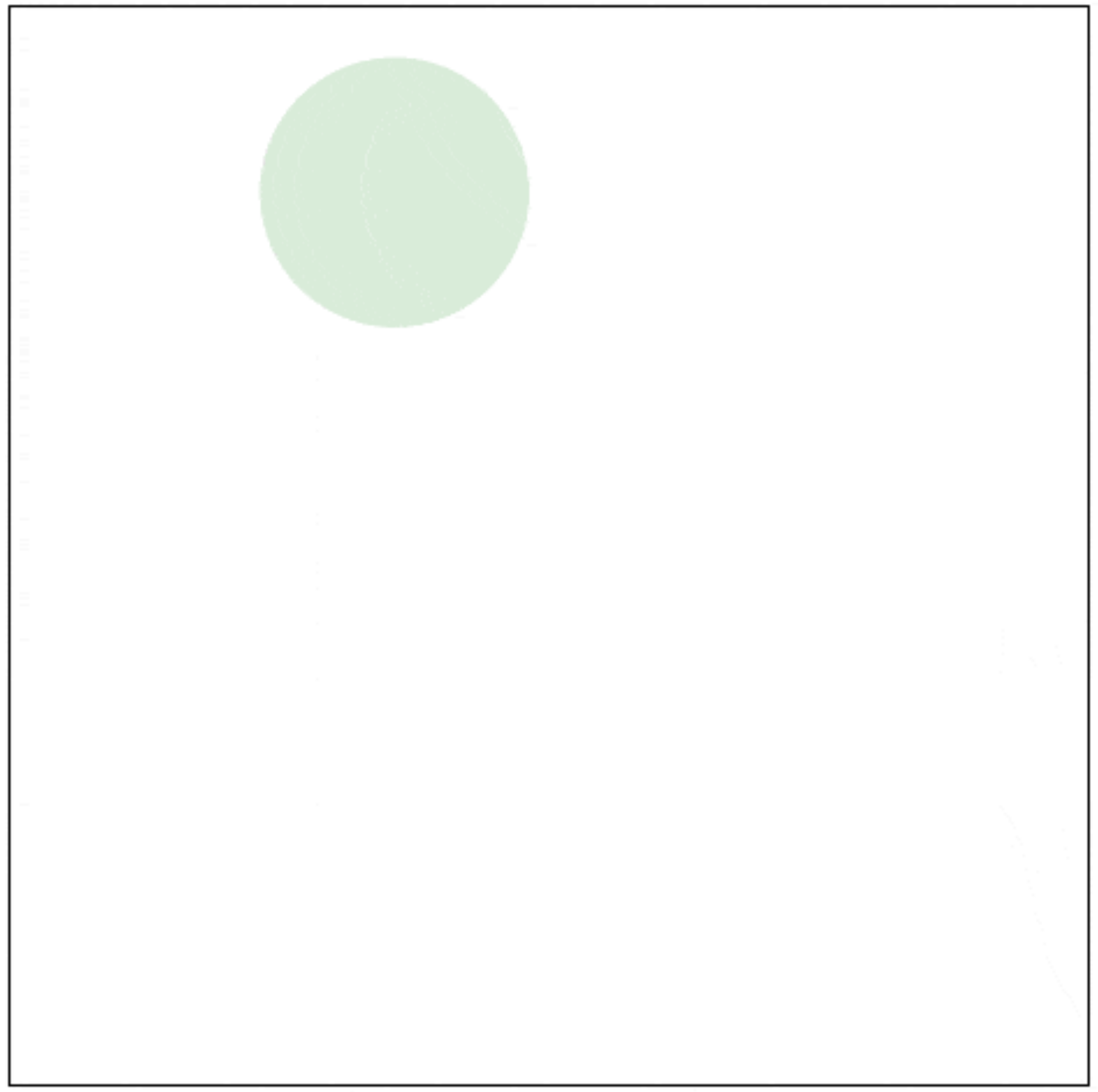
[Harry, Allen, and Sathyaprakash: 0908.2090]

Backup - Template Banks

0 templates



Random Placement:

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- Over covers the parameter space (in low dimensions)
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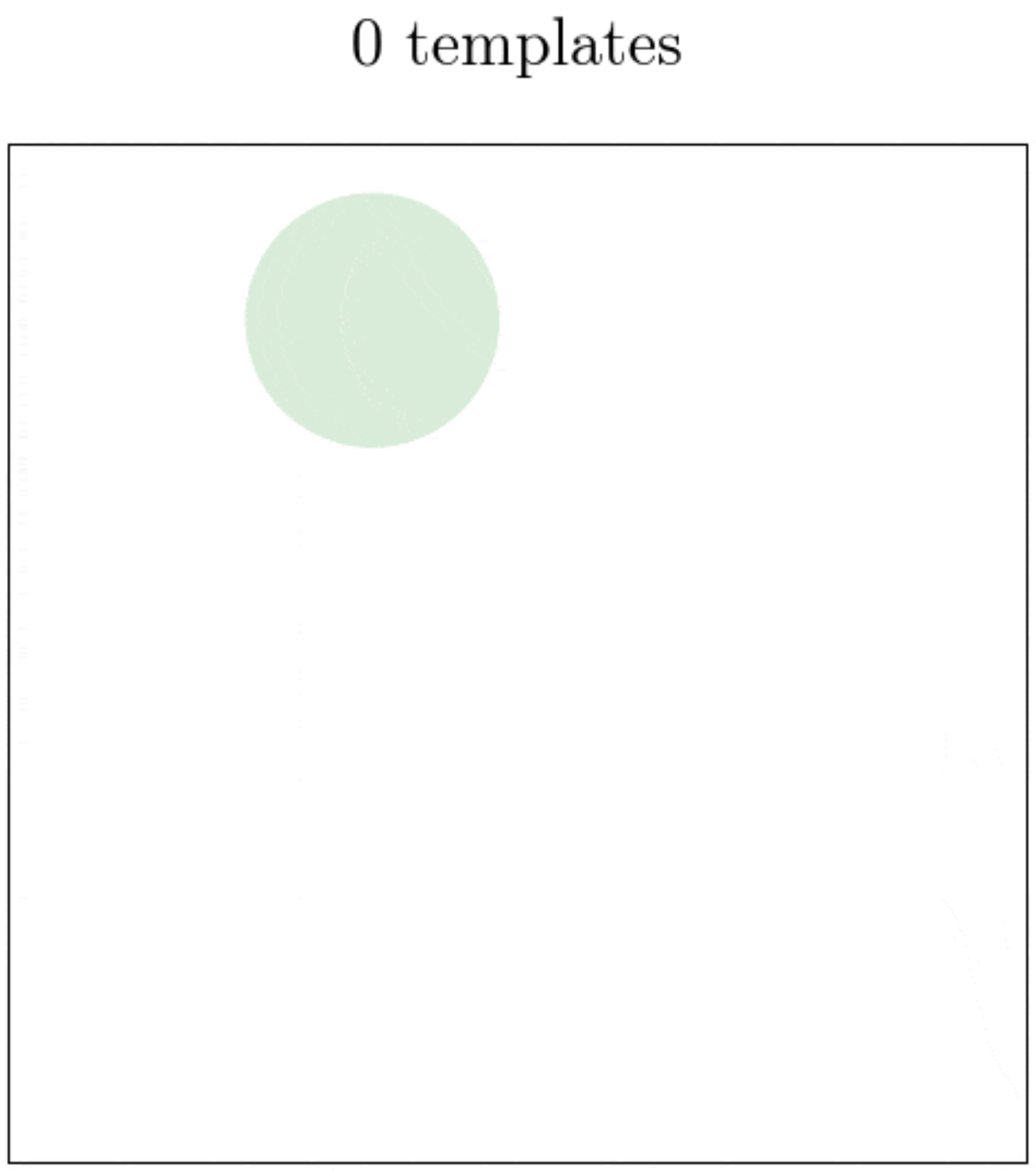

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[Messenger: 0809.5223]

Backup