

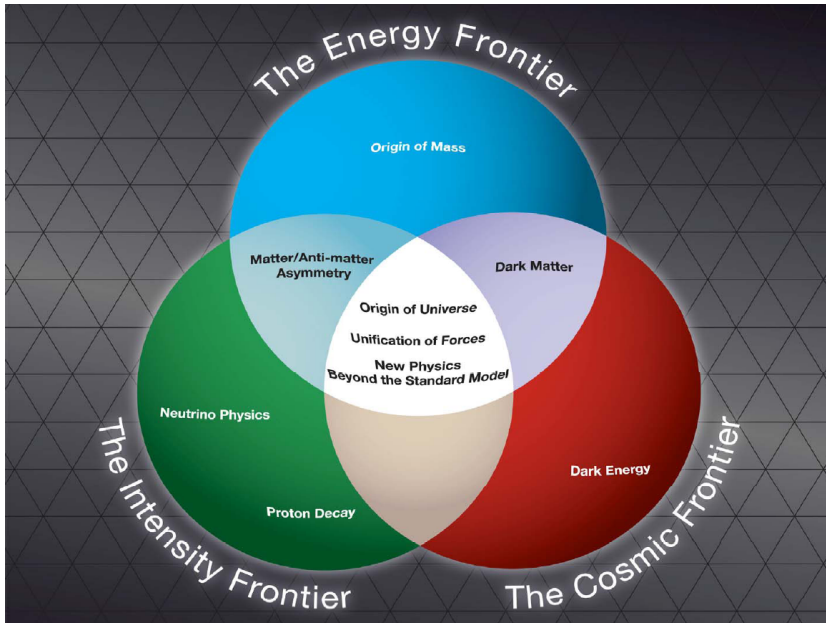
*The “old” and  
the “new”  
muon  $g-2$  puzzles*

**Antonio Masiero**

**Univ. of Padova and INFN, Padova**

Based on works on the muon  $g-2$  problem in  
collaboration with **Luca Di Luzio, Bill Marciano,  
Paride Paradisi and Massimo Passera**

## On the “old” muon g-2 puzzle



During the long sequel of restless attempts of finding experimental evidences or at least hints of **NEW PHYSICS** beyond the SM along the **traditional High-Energy (HE) and High-Intensity (HI) paths**, several 3 or even 4  $\sigma$  signals at variance w.r.t. the SM expectations **have shown up**, but they have also (rather sooner than later) **invariably faded away**.

A remarkable exception is represented by

**the anomalous magnetic moment of the muon**

which has been for **several years now** and **still** represents a **major observational evidence along the HI frontier of the possible presence of NEW PHYSICS**

The other more recent hint of NEW PHYSICS along these two roads is again in the HI frontier, namely the possible **violation of lepton flavour universality in some B-meson semileptonic decays**.

$$\vec{\mu}_\ell = \frac{e}{2m} \vec{\ell}$$

$$\vec{\mu}_s = g \frac{e}{2m} \vec{s}$$

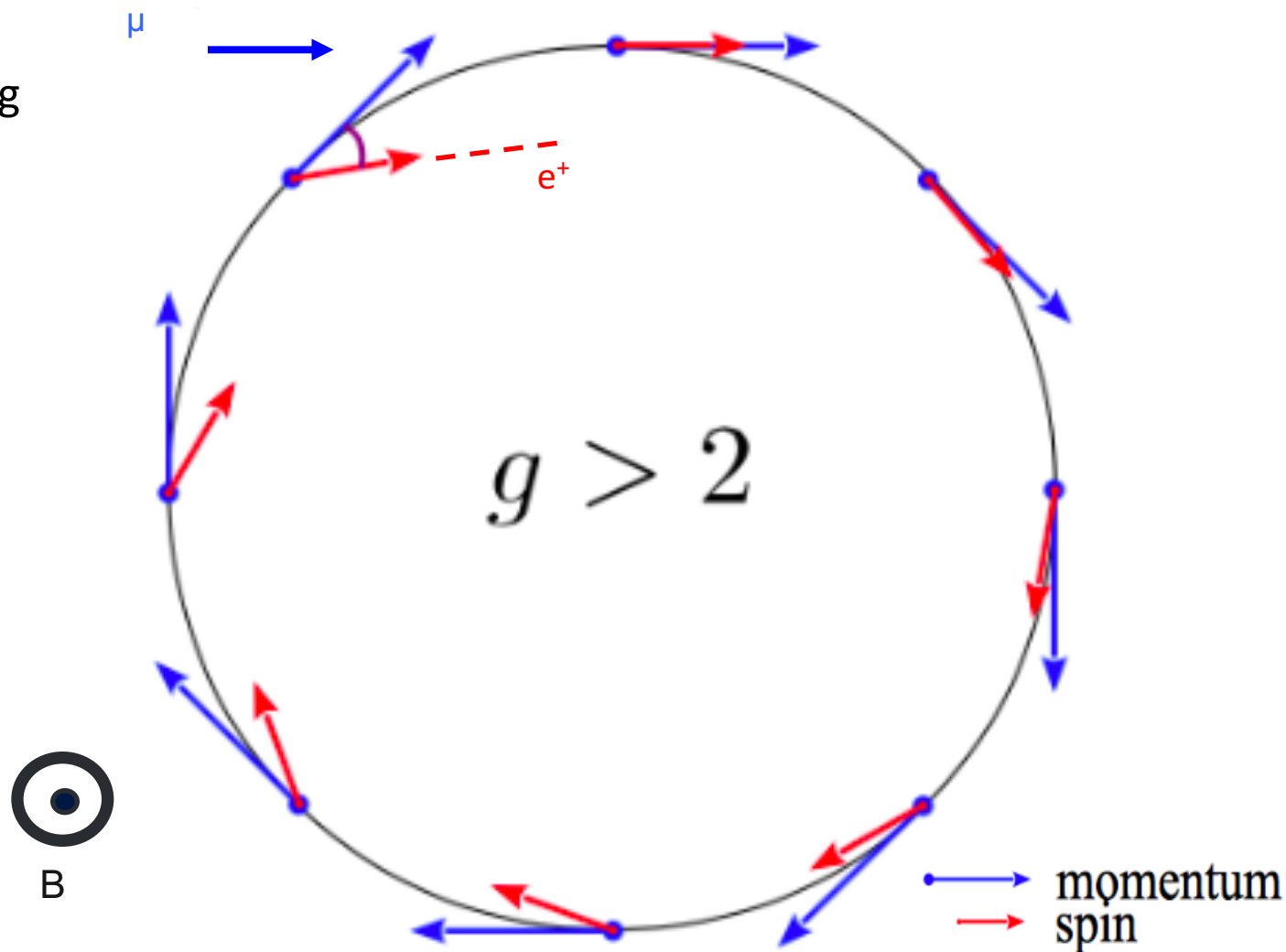
Put a beam of polarized muons into a storage ring

Both the muon spin and momentum precess

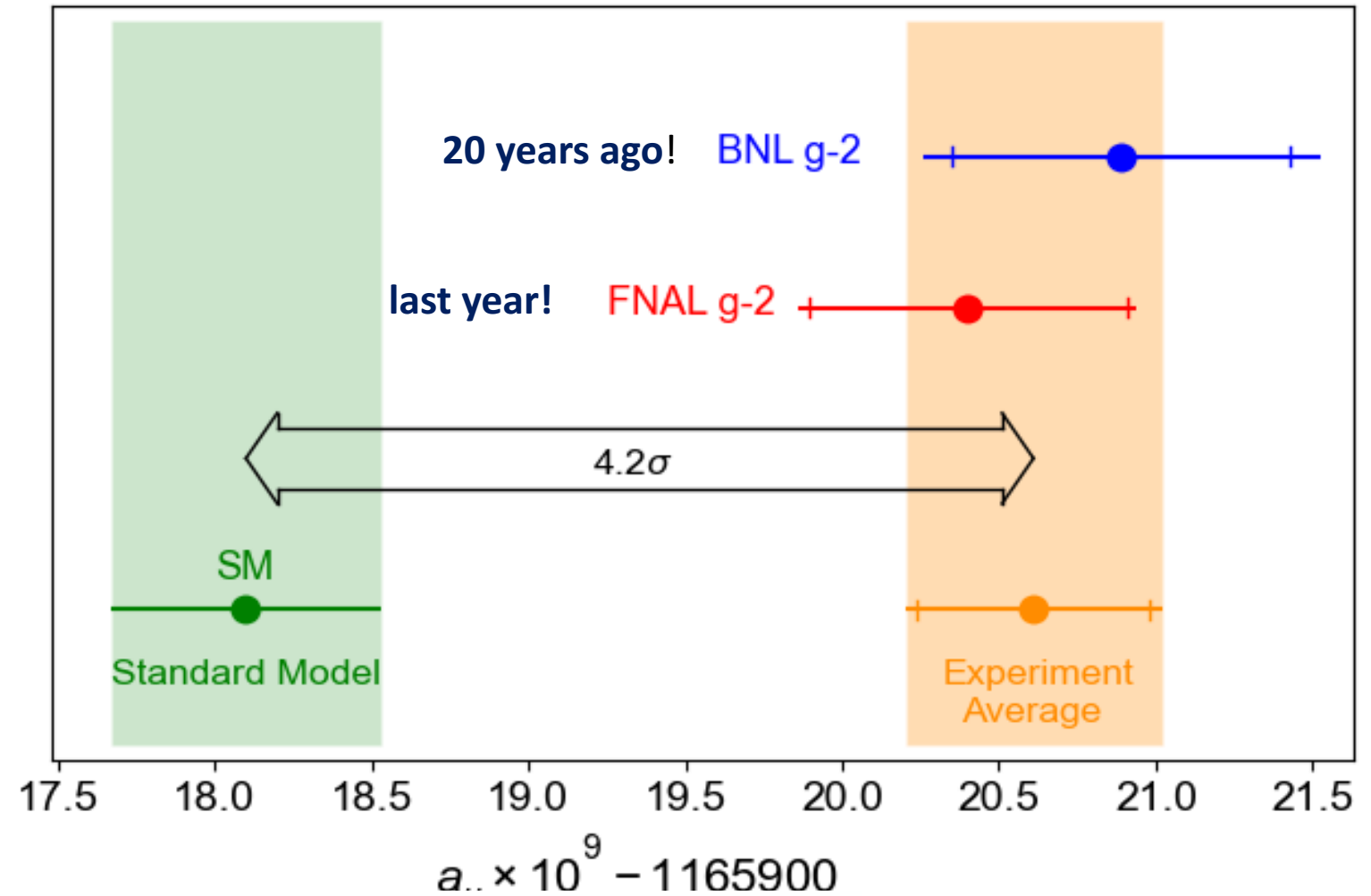
Because  $g$  is slightly greater than 2 the spin precesses faster than the momentum

$$a = (g-2)/2$$

$$a_\mu = \omega_a \frac{eB}{mc}$$



# The **EXP.** situation



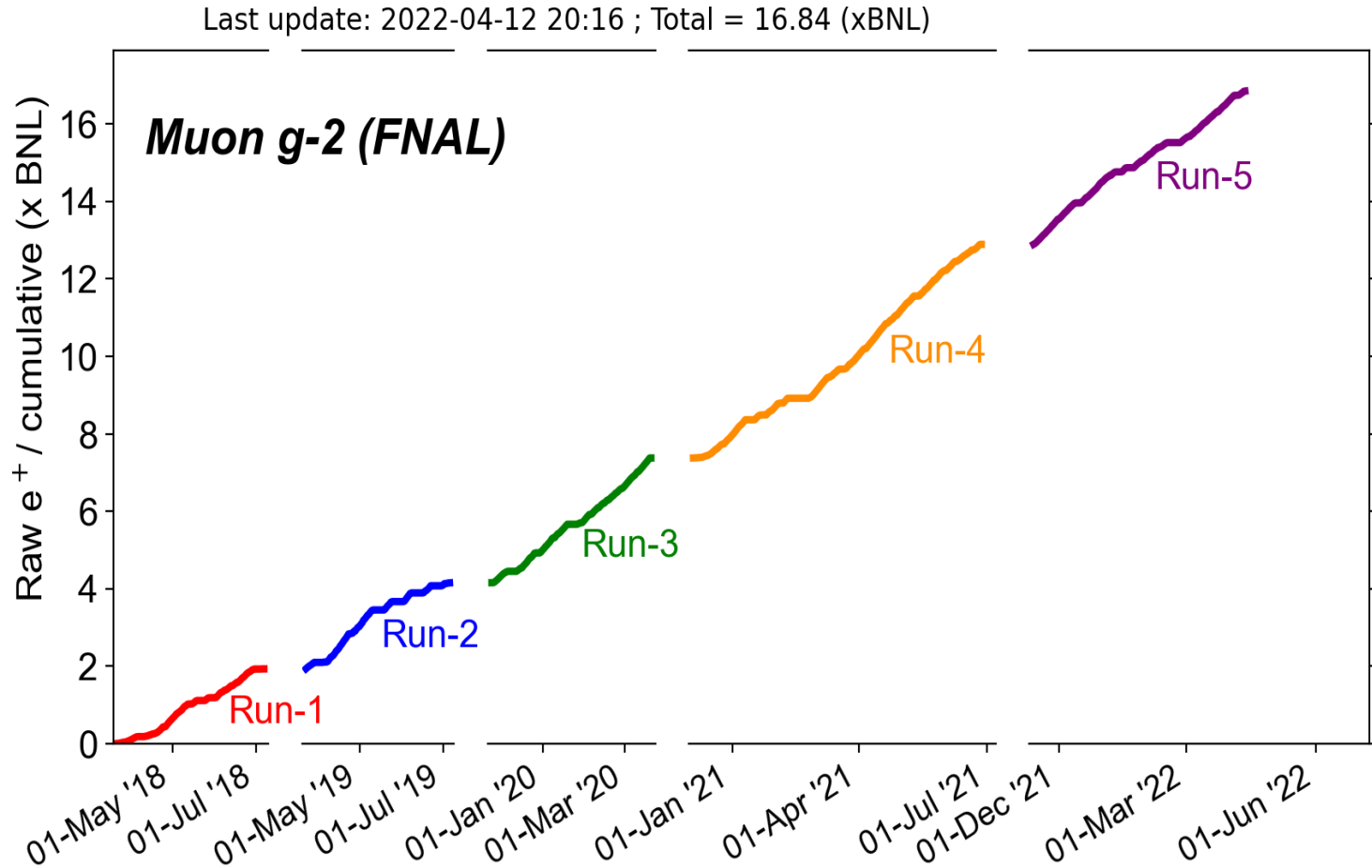
$$a_{\mu}^{\text{EXP}} = (116592089 \pm 63) \times 10^{-11} [0.54\text{ppm}] \quad \text{BNL E821}$$

$$a_{\mu}^{\text{EXP}} = (116592040 \pm 54) \times 10^{-11} [0.46\text{ppm}] \quad \text{FNAL E989 Run 1}$$

$$a_{\mu}^{\text{EXP}} = (116592061 \pm 41) \times 10^{-11} [0.35\text{ppm}] \quad \text{WA}$$

**FNAL aims at  $16 \times 10^{-11}$**

## The **EXP.** prospects



- Run-1 result confirmed the BNL result with only 6% of our total statistics so far
- Run-2/3 result expected to be published early next year
  - ~ 2x improvement on the statistical error
  - Reduction in the systematic errors, closing in on the TDR goal
  - **Would be helpful to have a recommendation for what theory prediction(s) to compare to in the paper**
- There's still more data to analyse with runs 4 and 5 and we'll add more with run 6

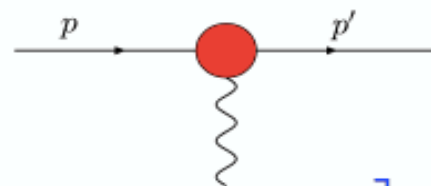
- **Kusch and Foley 1948:**

$$\left(\frac{g_e}{2}\right)^{\text{exp}} \equiv 1 + a_e^{\text{exp}} = 1.00119 \pm 0.00005$$

- **Schwinger 1948 (triumph of QED!):**

$$\left(\frac{g_e}{2}\right)^{\text{th}} \equiv 1 + a_e^{\text{th}} = 1.00116 \dots$$

- **We keep studying the lepton- $\gamma$  vertex:**



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \dots\right]u(p)$$

$$F_1(0) = 1 \quad F_2(0) = a_l$$

A pure "quantum correction" effect!

**"g – 2 is not an experiment: it is a way of life."**

[John Adams (Head of the Proton Synchrotron at CERN (1954-1961))]

This statement also applies to many theorists! [Nyffeler '16]

$$a_{\mu}^{\text{QED}} = (1/2) (\alpha/\pi) \text{ [Schwinger, 1948]}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

[Sommerfield; Petermann; Suura&Wichmann '57; Elend '66]

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

[Remiddi, Laporta, Barbieri...; Czarnecki, Skrzypek '99]

$$+ 130.8780 (60) (\alpha/\pi)^4$$

[Kinoshita et al. '81-'15; Steinhauser et al. '13-'16; Laporta '17]

$$+ 750.86 (88) (\alpha/\pi)^5 \text{ [Kinoshita et al. '90-'19]}$$

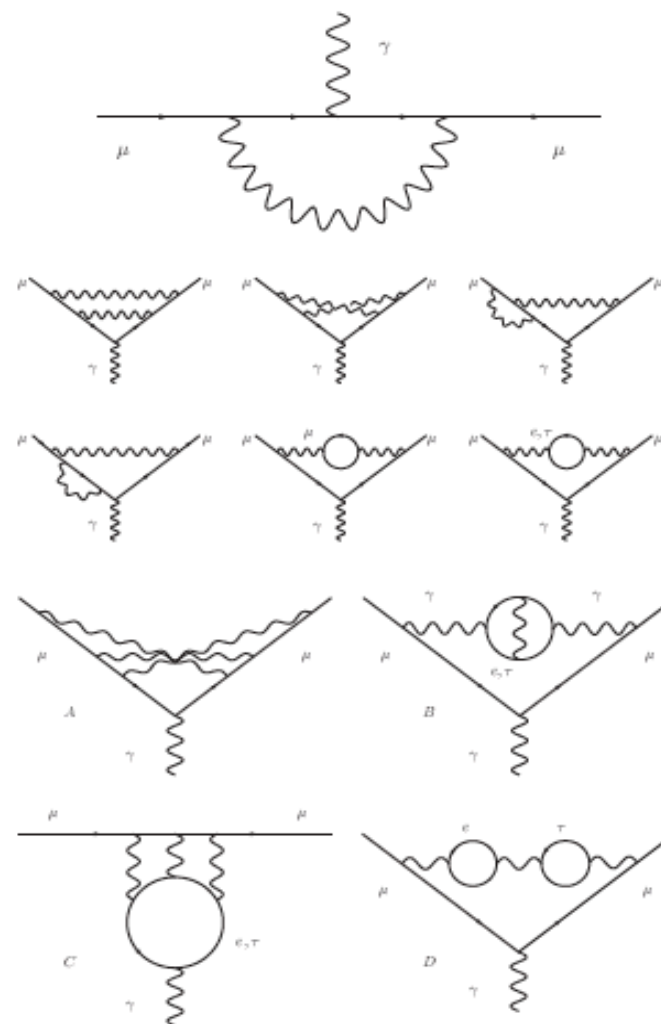
$$a_{\mu}^{\text{QED}} = 116584718.931 (19)(100)(23) \times 10^{-11}$$

mainly from 4-loop coeff. unc.  6-loop  from  $\alpha(\text{Cs})$

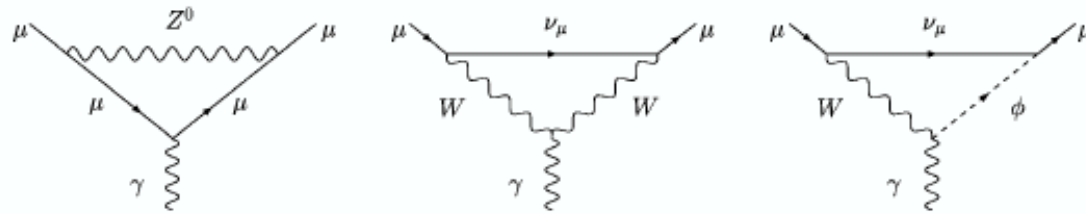
$\alpha = 1/137.035999046(27)$  [0.2ppb] Parker et al 2018

WP20 value

[WP20  $\equiv$  T. Aoyama *et al.*, Phys. Rept. '20]



## ● One-loop term:



$$a_{\mu}^{\text{EW}}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} (1 - 4\sin^2\theta_W)^2 + O\left(\frac{m_{\mu}^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

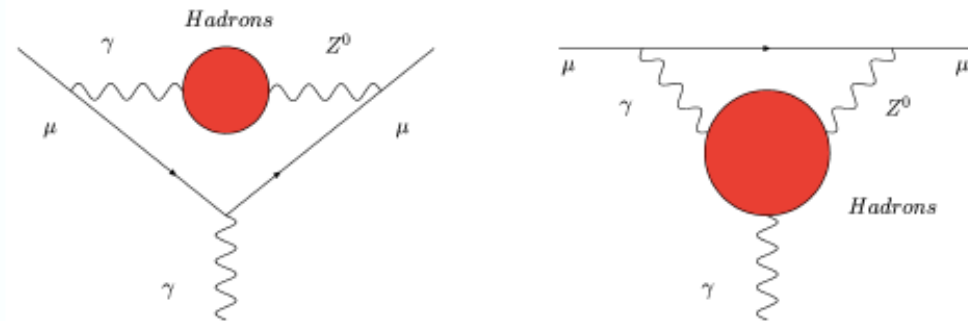
## ● One-loop plus higher-order terms:

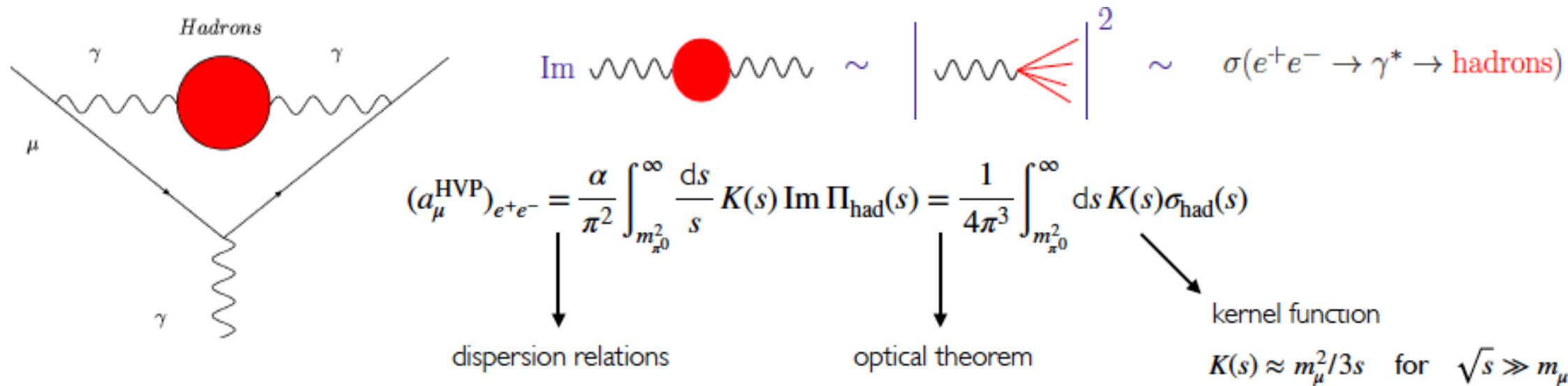
$$a_{\mu}^{\text{EW}} = 153.6 (1.0) \times 10^{-11}$$

Hadronic loop uncertainties (and 3-loop nonleading logs).

WP20 value

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degraasi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.





$$a_\mu^{\text{HLO}} = 6895 (33) \times 10^{-11}$$

F. Jegerlehner, arXiv:1711.06089

$$= 6939 (40) \times 10^{-11}$$

Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

$$= 6928 (24) \times 10^{-11}$$

Keshavarzi, Nomura, Teubner, arXiv:1911.00367

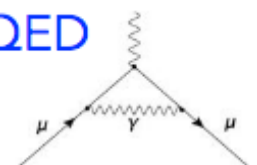
$$= 6931 (40) \times 10^{-11} (0.6\%)$$

WP20 value

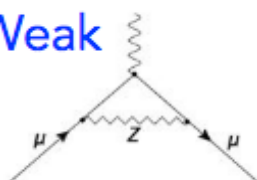
WP20 = White Paper of the Muon g-2 Theory Initiative: arXiv:2006.04822

The 4 classes of SM contributions: **uncertainty largely dominated** by the **hadronic contributions** in **Vacuum Polarization (HVP)** and **Light-by-Light (HLbL)**

$$a_{\mu}(\text{SM}) = a_{\mu}(\text{QED}) + a_{\mu}(\text{Weak}) + a_{\mu}(\text{Hadronic})$$

QED  + ...

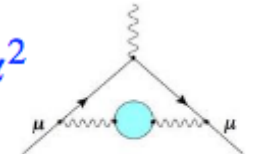
|  |                                      |           |
|--|--------------------------------------|-----------|
|  | $116\,584\,718.9(1) \times 10^{-11}$ | 0.001 ppm |
|--|--------------------------------------|-----------|

Weak  + ...

|  |                              |          |
|--|------------------------------|----------|
|  | $153.6(1.0) \times 10^{-11}$ | 0.01 ppm |
|--|------------------------------|----------|

Hadronic...

...Vacuum Polarization (HVP)

$\alpha^2$   + ...

|  |                            |          |
|--|----------------------------|----------|
|  | $6845(40) \times 10^{-11}$ | 0.37 ppm |
|  | [0.6%]                     |          |

...Light-by-Light (HLbL)

$\alpha^3$   + ...

|  |                          |          |
|--|--------------------------|----------|
|  | $92(18) \times 10^{-11}$ | 0.15 ppm |
|  | [20%]                    |          |

$$a_{\mu}^{\text{EXP}} = 116592061(41) \times 10^{-11} \text{ [BNL + FNAL]}$$

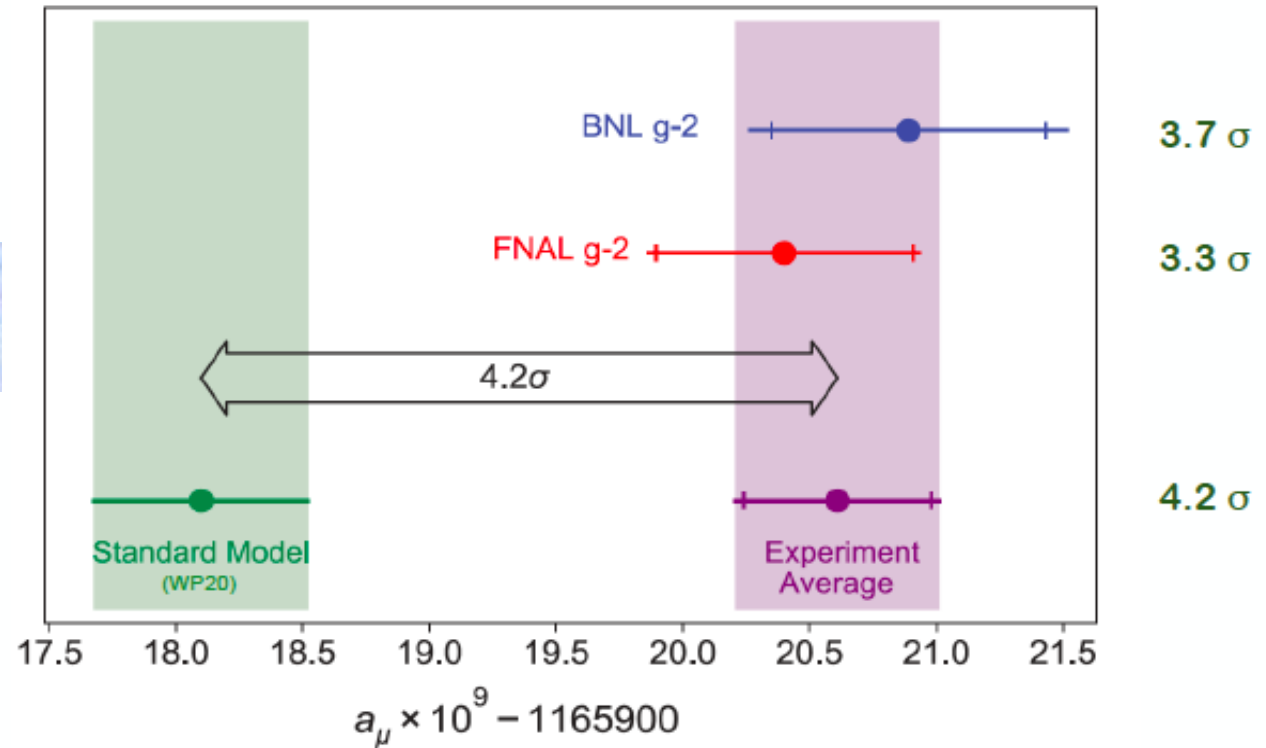
$$a_{\mu}^{\text{SM}} = 116591810(43) \times 10^{-11} \text{ [WP20]}$$

$$\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} \equiv a_{\mu}^{\text{NP}} = 251 (59) \times 10^{-11} \quad (4.2\sigma \text{ discrepancy!})$$

$$\underbrace{(0.1)_{\text{QED}}, \quad (1)_{\text{EW}}, \quad (18)_{\text{HLbL}}, \quad (40)_{\text{HVP}}, \quad (41)_{\delta a_{\mu}^{\text{EXP}}}}_{(43)_{\text{TH}}}$$

- ▶ Hadronic uncertainties (HLbL & HVP) are very hard to improve.
- ▶  $\delta a_{\mu}^{\text{EXP}} \approx 16 \times 10^{-11}$  by the E989 Muon  $g-2$  exp. in a few years.

# The OLD $(g-2)_\mu$ puzzle



$$a_\mu^{\text{EXP}} = (116592089 \pm 63) \times 10^{-11} [0.54\text{ppm}] \text{ BNL E821}$$

$$a_\mu^{\text{EXP}} = (116592040 \pm 54) \times 10^{-11} [0.46\text{ppm}] \text{ FNAL E989 Run 1}$$

$$a_\mu^{\text{EXP}} = (116592061 \pm 41) \times 10^{-11} [0.35\text{ppm}] \text{ WA}$$

- FNAL aims at  $16 \times 10^{-11}$ . First 4 runs completed, 5th in progress.
- Muon g-2 proposal at J-PARC: Phase-1 with  $\sim$  BNL precision.

$$a_{\mu}^{\text{EXP}} = 116592061 (41) \times 10^{-11}$$

BNL+FNAL

$$a_{\mu}^{\text{SM}} = 116591810 (43) \times 10^{-11}$$

WP20

$$a_{\mu, e^+e^-}^{\text{HLO}} = 6931(40) \times 10^{-11}$$

$$\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = 251 (59) \times 10^{-11}$$

4.2  $\sigma$

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha(M_Z) - \Delta\alpha_{\text{had}}^{(5)}(M_Z) - \Delta\alpha_{\text{top}}(M_Z)}$$

Can  $\Delta a_{\mu}$  be due to a missing contribution in  $\sigma_{\text{had}}$ ?

$$a_{\mu}^{\text{HLO}} \simeq \frac{m_{\mu}^2}{12\pi^3} \int_{4m_{\pi}^2}^{\infty} ds \frac{\sigma(s)}{s}, \quad \Delta\alpha_{\text{had}}^{(5)} = \frac{M_Z^2}{4\pi\alpha^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{\sigma(s)}{M_Z^2 - s}$$

$$\text{Im} \text{ (diagram with red circle)} \sim \left| \text{diagram with red lines} \right|^2 \sim \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$$

Shifts  $\Delta\sigma(s)$  to fix  $\Delta a_{\mu}$  are possible,

but conflict with the EW fit if they occur above  $\sim 1$  GeV

Shifts below  $\sim 1$  GeV conflict with the quoted exp. precision of  $\sigma(s)$

Crivellin, Hoferichter, Manzari, Montuli;  
de Rafael; Malaescu, Schott;  
Colangelo, Hoferichter, Stoffer

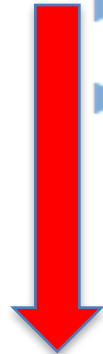
Keshavarzi, Marciano, Passera, Sirlin, PRD 2020 (updated 2021)

# NEW PHYSICS for the muon g-2: at which scale?

$$\Delta a_\mu \equiv a_\mu^{\text{NP}} \approx (a_\mu^{\text{SM}})_{\text{weak}} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$


- ▶ A weakly interacting NP at  $\Lambda \approx v$  can naturally explain  $\Delta a_\mu \approx 2 \times 10^{-9}$
- ▶  $\Lambda \approx v$  favoured by the *hierarchy problem* and by a WIMP DM candidate.

On the other hand, HE experiments (LEP, Tevatron, LHC) have NOT provided any clue for the presence of new (charged) particles at the ELW. scale

- 
- ▶ NP is very light ( $\Lambda \lesssim 1$  GeV) and feebly coupled to SM particles.
  - ▶ NP is very heavy ( $\Lambda \gg v$ ) and strongly coupled to SM particles.

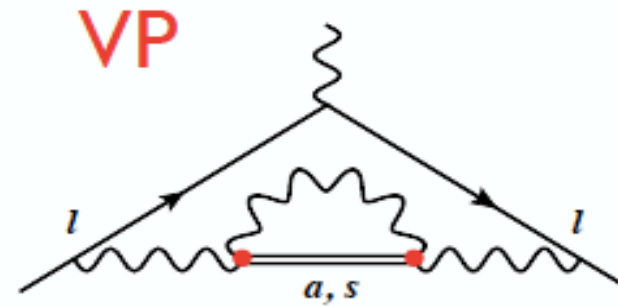
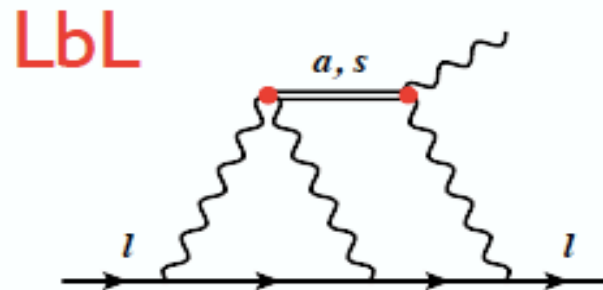
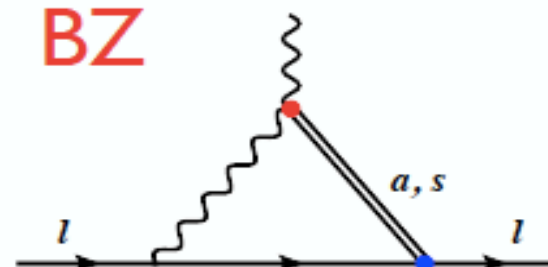
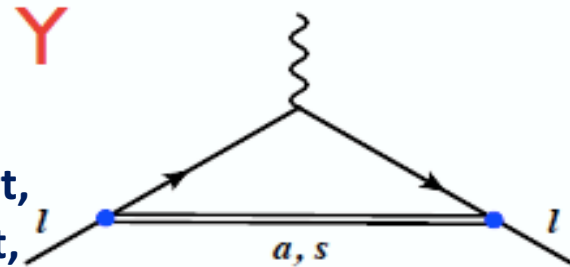
P. Paradisi, La Thuile 2021

**The case of AXION-LIKE PARTICLES (ALPs)**

# ALPs contributions to the muon g-2?

$\mu$

Marciano, AM, Paradisi,  
Passera '16; Bauer, Neubert,  
Thamm '17; Bauer, Neubert,  
Renner, Schnubel, Thamm '19;  
Cornella, Paradisi, Sumensari '19

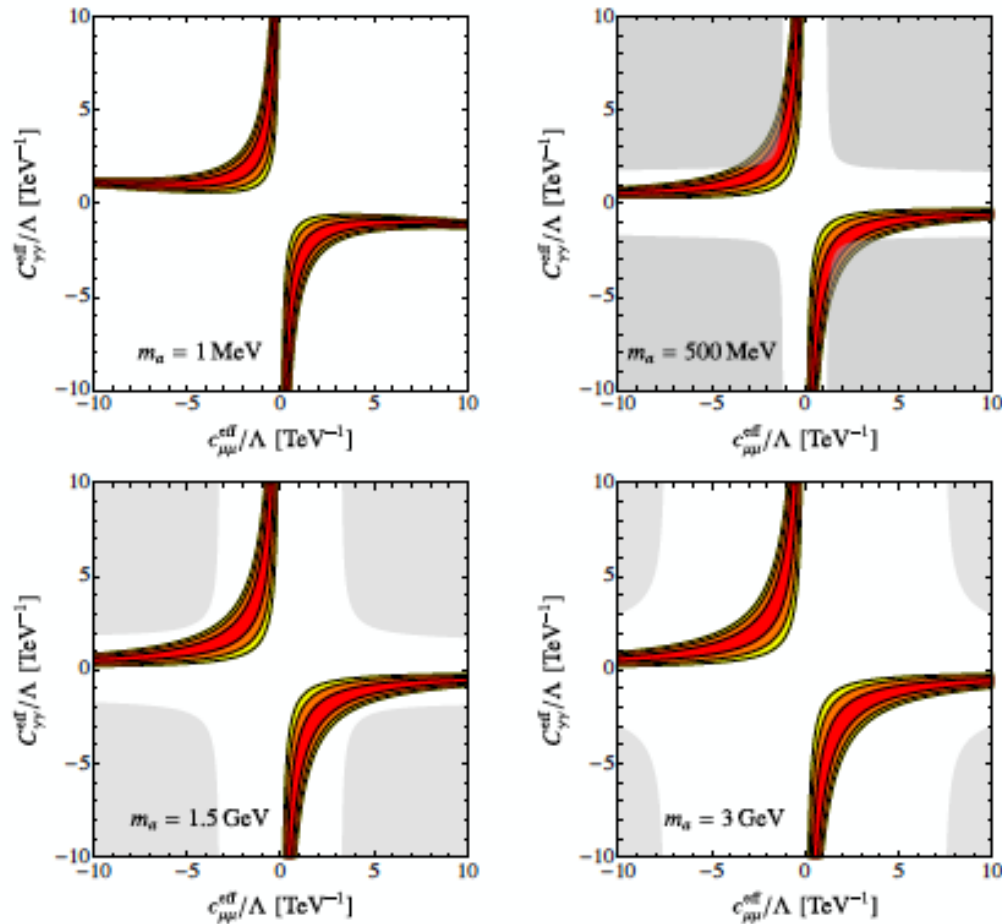


- Both scalar and pseudoscalar ALPs can solve  $\Delta a_\mu$  for masses  $\sim [100\text{MeV}-1\text{GeV}]$  and couplings allowed by current experimental constraints.
- They can be tested at present low-energy  $e^+e^-$  experiments, via dedicated  $e^+e^- \rightarrow e^+e^- + \text{ALP}$  &  $e^+e^- \rightarrow \gamma + \text{ALP}$  searches.

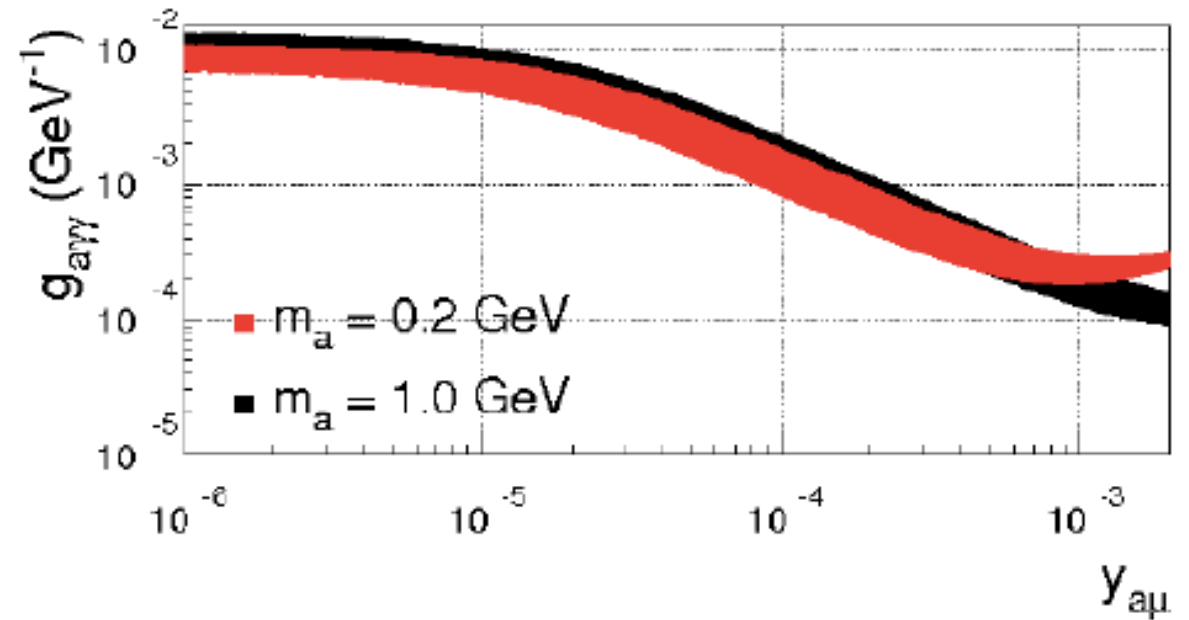
$$\mathcal{L} = e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{c_{\mu\mu}}{2} \frac{\partial^\nu a}{\Lambda} \bar{\mu} \gamma_\nu \gamma_5 \mu$$

$$\mathcal{L} = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} + i y_{a\psi} a \bar{\psi} \gamma_5 \psi$$

$$g_{a\gamma\gamma} \equiv \frac{2\sqrt{2}\alpha}{\Lambda} c_{a\gamma\gamma}$$



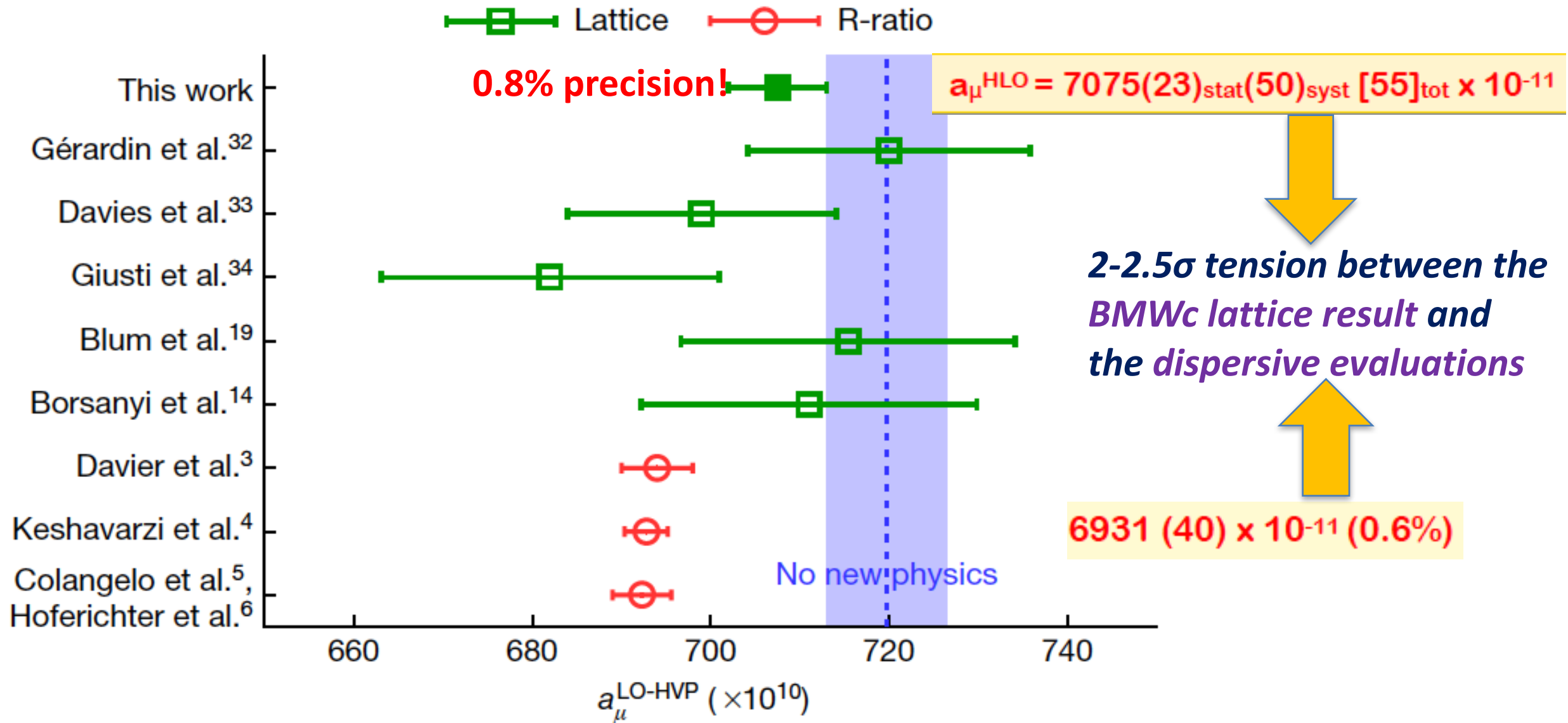
**Figure:**  $\Delta a_\mu$  regions favoured at 68% (red), 95% (orange) and 99% (yellow) CL. Gray regions are excluded by the BaBar search  $e^+e^- \rightarrow \mu^+\mu^- + \mu^+\mu^-$  [Bauer, Neubert, Thamm, '17]



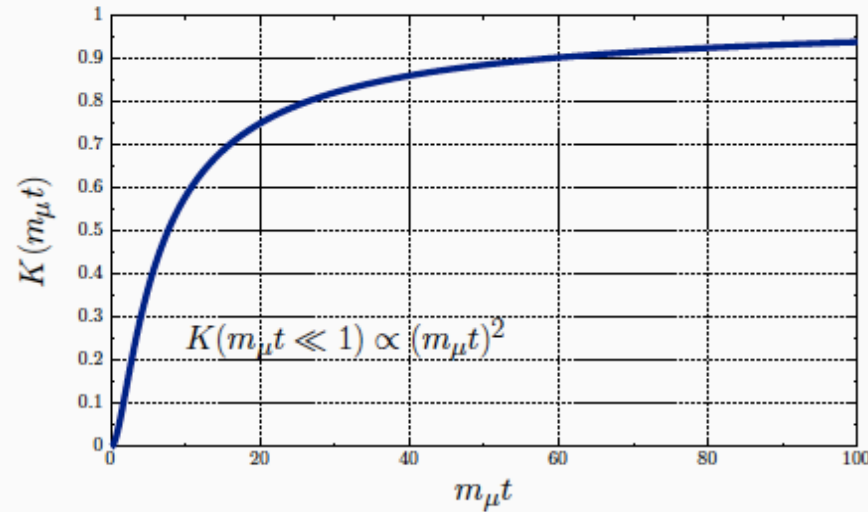
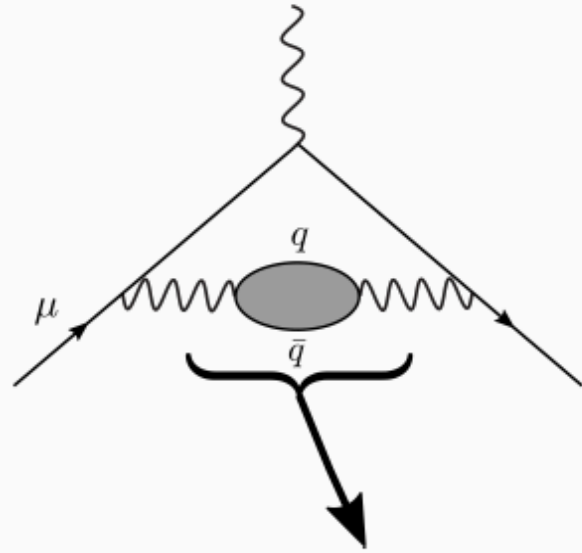
Pseudoscalar  $1\sigma$  solution bands  
to the  $g-2$  muon anomaly taking  
 $\Lambda = 1 \text{ TeV}$

Marciano, A.M.,  
Paradisi, Passera '16

BMWc20: S. Borsanyi et al. 2022.12347, published on Nature, April 7, 2021  
first published lattice result with **sub-percent precision!**



# LO-HVP from Lattice QCD



$$\Pi_{\mu\nu}(Q) = \int d^4x e^{iQ \cdot x} \langle J_\mu(x) J_\nu(0) \rangle = (\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu) \Pi(Q^2)$$

$$a_\mu^{\text{LO-HVP}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \frac{1}{m_\mu^2} f\left(\frac{Q^2}{m_\mu^2}\right) \cdot (\Pi(Q^2) - \Pi(0)).$$

**Time-Momentum representation (Bernecker & Meyer, 2011)**

$$a_\mu^{\text{LO-HVP}} = 2\alpha_{em}^2 \int_0^\infty dt t^2 K(m_\mu t) V(t), \quad V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \langle J_i(\vec{x}, t) J_i(0) \rangle_i$$

Colangelo, El-Khadra, Hoferichter, Keshavarzi, Lehner, Stoffer, Teubner, arXiv:2205.12963v2 (2022)

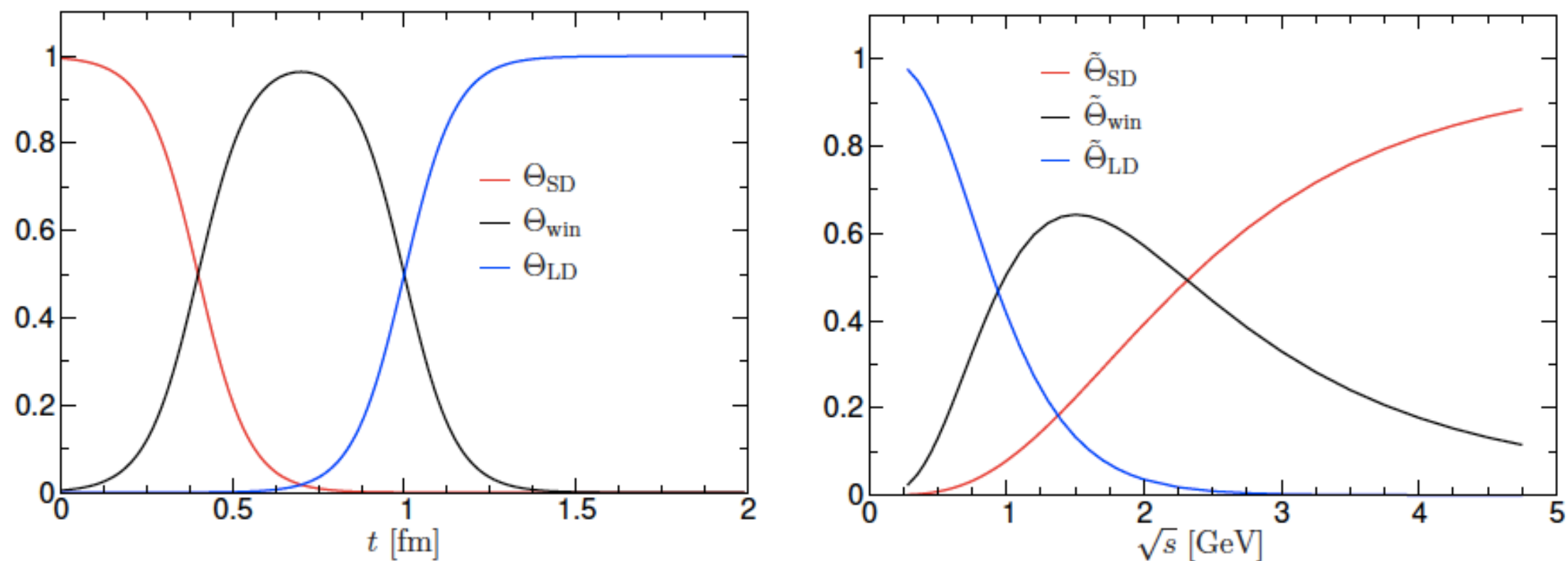
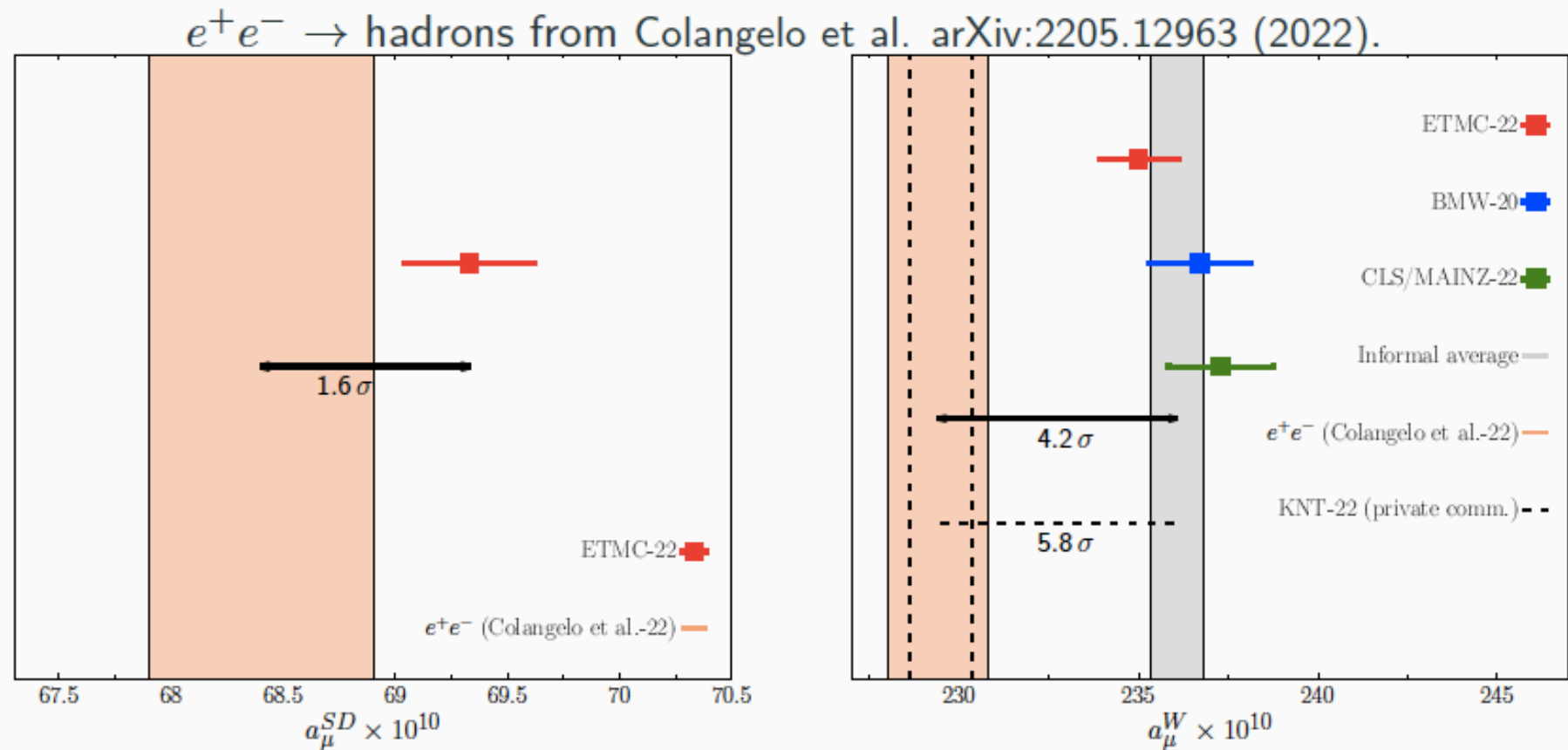


Figure 1: Short-distance, intermediate, and long-distance weight functions in Euclidean time (left), and their correspondence in center-of-mass energy (right).

# Comparison with $e^+e^- \rightarrow \text{hadrons}$ results

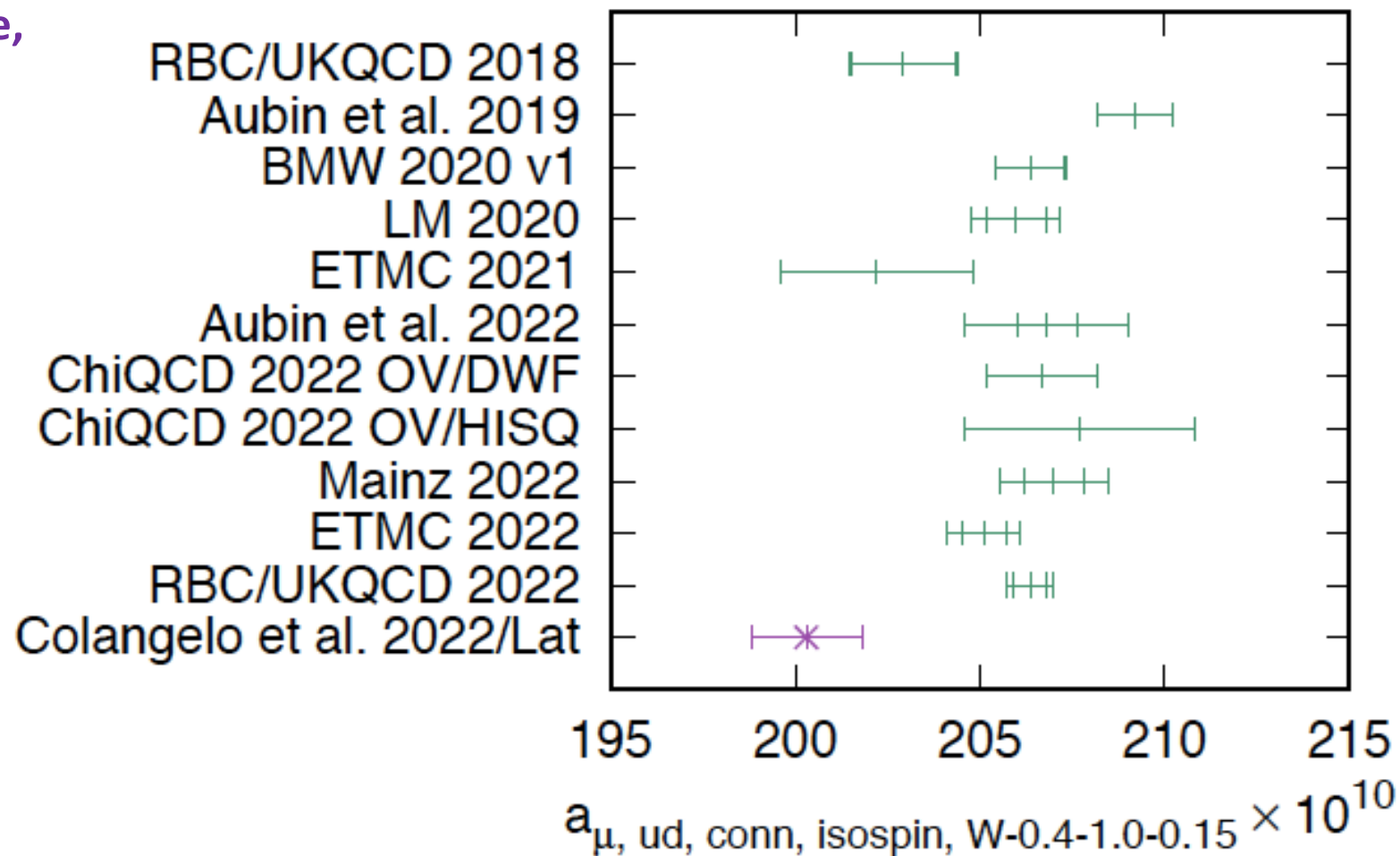
G. Gagliardi, Edinburgh  
2022, on behalf of the  
ETM Collaboration



- Tension in  $a_\mu^W$  rises to  $4.2\sigma$  if we combine ETMC '22, BMW '20 and CLS/Mainz '22 (informal average  $\rightarrow$  next WP).
- Deviation of  $e^+e^- \rightarrow \text{hadrons}$  data w.r.t. the SM in the low and (possibly) intermediate energy regions, but not in the high energy region.

# The RBC/UKQCD22 result in context

C. Lehner, Workshop of the  
Muon g-2 Theory Initiative,  
Edinburgh, Sept. 2022



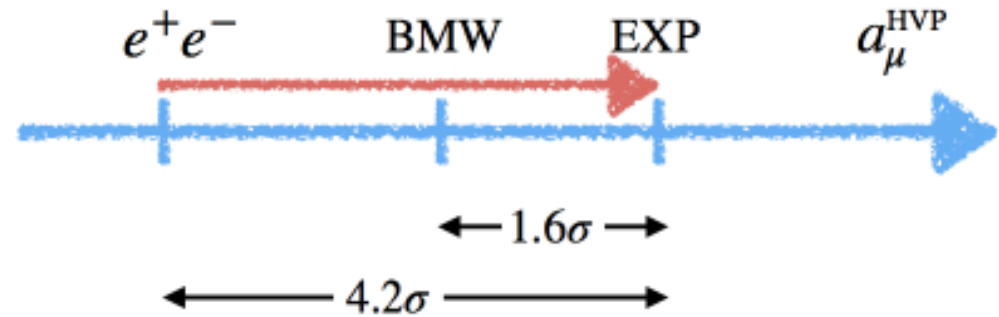
- 3.9 $\sigma$  tension of RBC/UKQCD22 with Colangelo et al. 22/Lattice

# The NEW g-2 puzzle

$$(a_\mu^{\text{HVP}})_{\text{EXP}} = a_\mu^{\text{EXP}} - a_\mu^{\text{SM, rest}}$$

$$(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}} = 6931(40) \times 10^{-11}$$

$$(a_\mu^{\text{HVP}})_{\text{BMW}} = 7075(55) \times 10^{-11}$$



If the new lattice results \* – i.e., **BMWc** & (only for the (SD) + W windows, but not for the relevant LD window) **Mainz 2022+ETMC 2022 + RBC/UKQCD 2022** are correct (and will be confirmed also for the LD window!), then:

i) The “old” g-2 discrepancy would be basically gone, but

ii) A new significant discrepancy between the  $e^+e^-$  data- driven and lattice QCD evaluations of  $a_\mu^{\text{HVP}}$  becomes quite significant

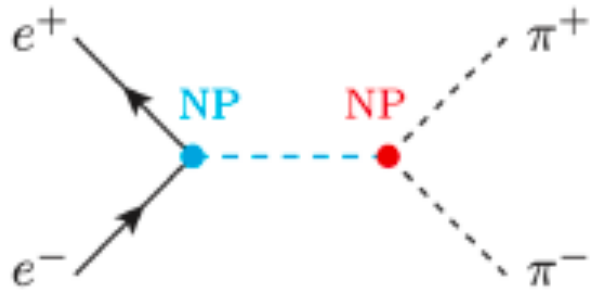
\* The lattice FNAL/QCDMILC collaboration is going to unblind its data soon

# New Physics to solve the new muon $g-2$ puzzle ?

NP in  $\sigma_{\text{had}}(e^+e^- \rightarrow \text{hadrons})$  such that

1.  $(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}} \approx (a_\mu^{\text{HVP}})_{\text{EXP}}$
2. the approximate agreement between BMW and EXP is not spoiled
3. w/o a direct contribution  $a_\mu^{\text{NP}}$  (i.e. NP not in muons)

**L. Di Luzio, A.M., P. Paradisi, M. Passera, PLB 2022 (arXiv 2112.08312)**



NP coupled both to **hadrons** and **electrons**  
but **not** directly to the **muons**

$$\text{Im} \left[ \text{wavy line} \right] \sim \left| \text{wavy line} \rightarrow \text{hadrons} \right|^2 \sim \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$$

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^{\infty} \frac{ds}{s} K(s) \text{Im} \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds K(s) \sigma_{\text{had}}(s) \quad \sigma_{\text{had}} = \sigma_{\text{had}}^{\text{SM}} + \Delta\sigma_{\text{had}}^{\text{NP}}$$

**SUBTRACTION** since NP does **NOT** contribute to the HVP at the LO, but it **DOES** contribute to the cross-section at the LO

$$\sigma_{\text{had}} - \Delta\sigma_{\text{had}}^{\text{NP}}$$


a **POSITIVE** SHIFT on

$(a_\mu^{\text{HVP}})_{e^+e^-}$  requires  $\Delta\sigma_{\text{had}}^{\text{NP}} < 0$  (negative interference)

The unique scenario to obtain such a **SIZEABLE NEGATIVE interference**

- **SIZEABLE**  $\rightarrow$  **TREE-LEVEL** contribution to modify  $\sigma_{\text{had}}$  at  $\sqrt{s} < 1 \text{ GeV}$  (hence, **sub-GeV mediator** coupling to the hadronic and electron currents at tree-level)
- **NEGATIVE INTERF.**  $\rightarrow$  NP particle couples via a **VECTOR** current to the u, d quarks (given the dominance of the  $\pi^+\pi^-$  channel)

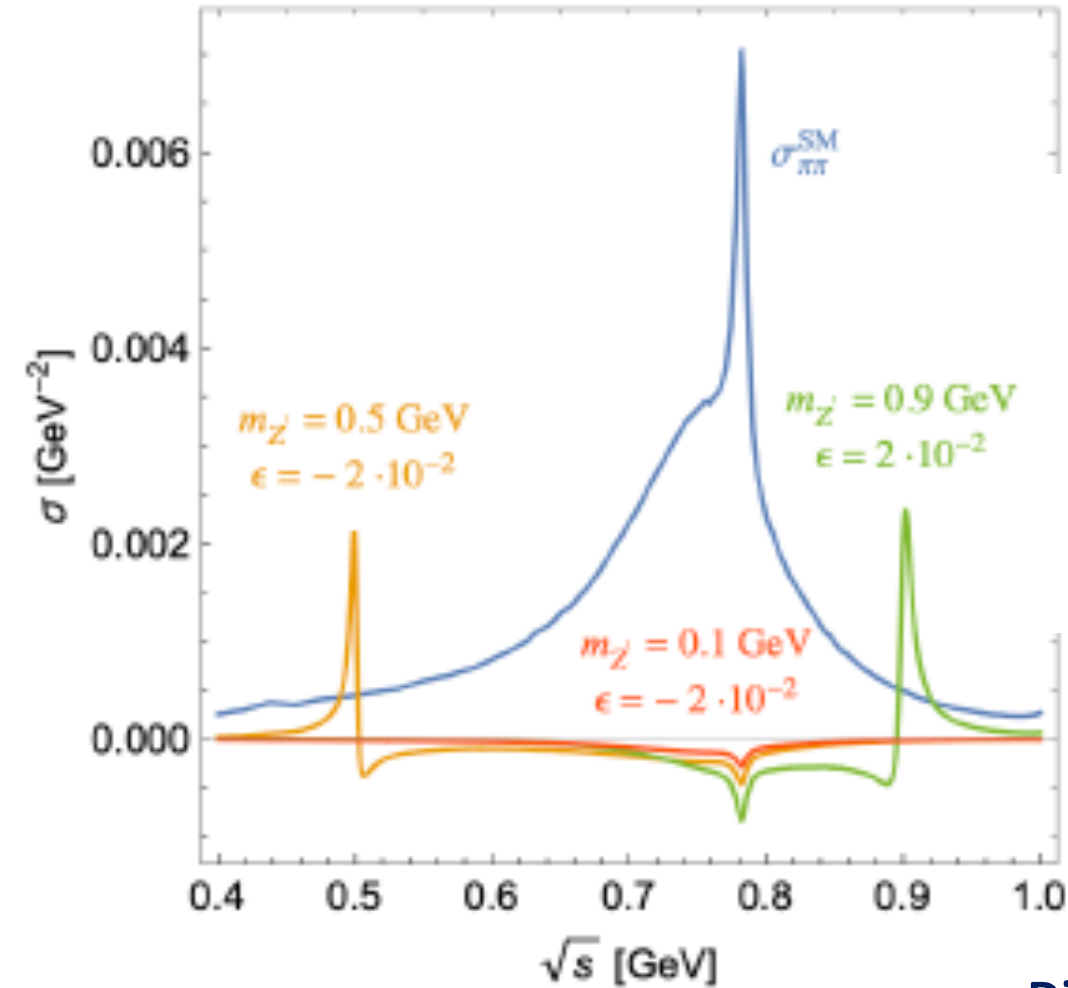
$$\mathcal{L}_{Z'} \supset (g_V^e \bar{e} \gamma^\mu e + g_V^q \bar{q} \gamma^\mu q) Z'_\mu \quad q = u, d \quad m_{Z'} \lesssim 1 \text{ GeV}$$

 a light spin-1 mediator with vector couplings to first generation SM fermions

$$\frac{\sigma_{\pi\pi}^{\text{SM+NP}}}{\sigma_{\pi\pi}^{\text{SM}}} = \left| 1 + \frac{g_V^e (g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}} \right|^2$$

Examples of **benchmark values** for  $m_{Z'}$  and  $Z'$  couplings to electrons and up- and down-quarks suitable to **solve the g-2 discrepancy**

$$\gamma = 10^{-2}$$



$$\Delta a_\mu = \frac{1}{4\pi^3} \int_{s_{\text{exp}}}^{\infty} ds K(s) (-\Delta\sigma_{\text{had}}^{\text{NP}}(s))$$

$\sqrt{s_{\text{exp}}} \approx 0.3 \text{ GeV}$   
for  $\pi^+\pi^-$  channel

$$\Delta\sigma_{\text{had}}^{\text{NP}}(s) \approx \sigma_{\pi\pi}^{\text{SM}}(s) \times \frac{2\epsilon s(s - m_{Z'}^2) + \epsilon^2 s^2}{(s - m_{Z'}^2)^2 + m_{Z'}^4 \gamma^2}$$

$$\epsilon \equiv g_V^e (g_V^u - g_V^d) / e^2$$

$$\gamma \equiv \Gamma_{Z'} / m_{Z'}$$

However, **severe constraints on the  $Z'$  couplings** to electrons and to hadrons

- for  $m_{Z'} \lesssim 0.3 \text{ GeV}$  ( $Z' \rightarrow e^+e^-$  is the main decay mode)

$$e^+e^- \rightarrow \gamma Z' \text{ @ BaBar} \longrightarrow g_V^e \lesssim 2 \cdot 10^{-4}$$

- for  $m_{Z'} \gtrsim \text{MeV}$

$$\text{electron g-2} \longrightarrow |g_V^e| \lesssim 10^{-2} (m_{Z'}/0.5 \text{ GeV})$$

$e^+e^- \rightarrow q\bar{q}$  has been measured with per-cent accuracy at LEP-II

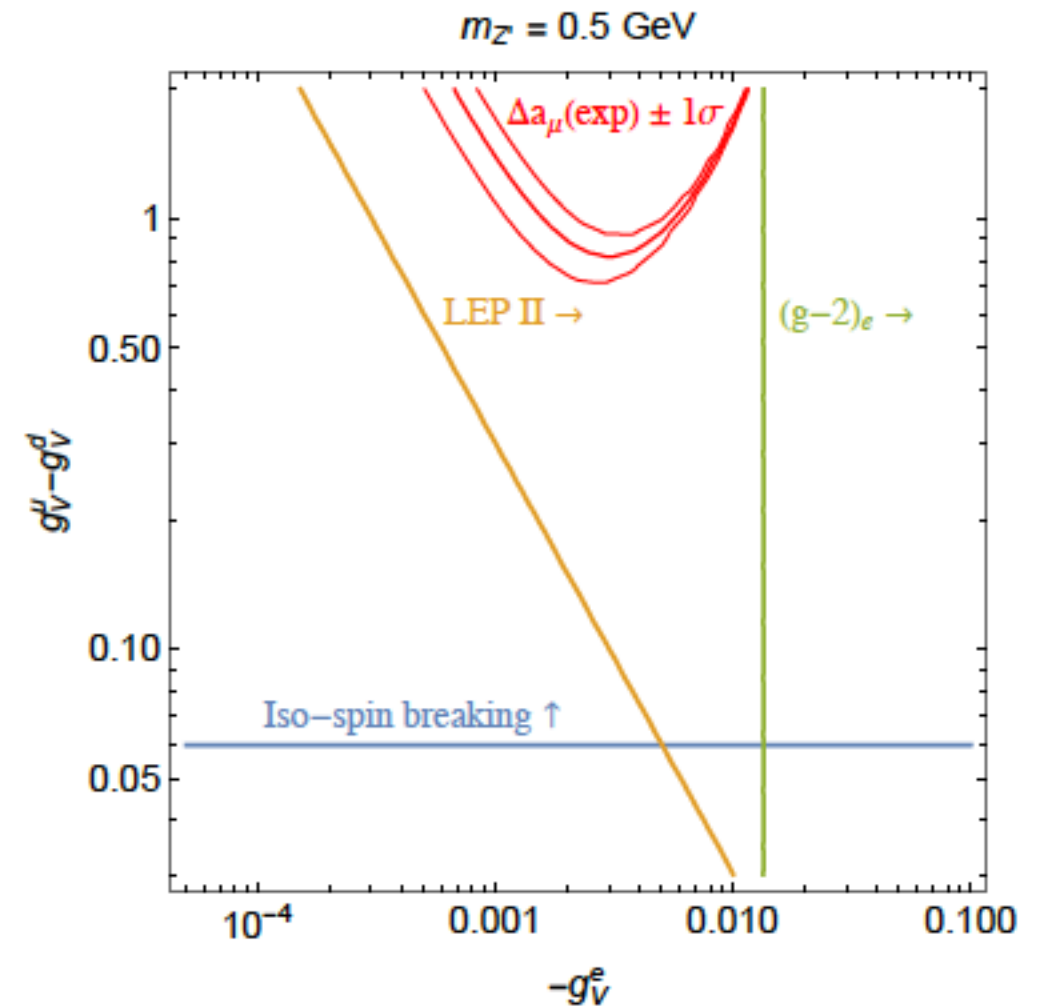
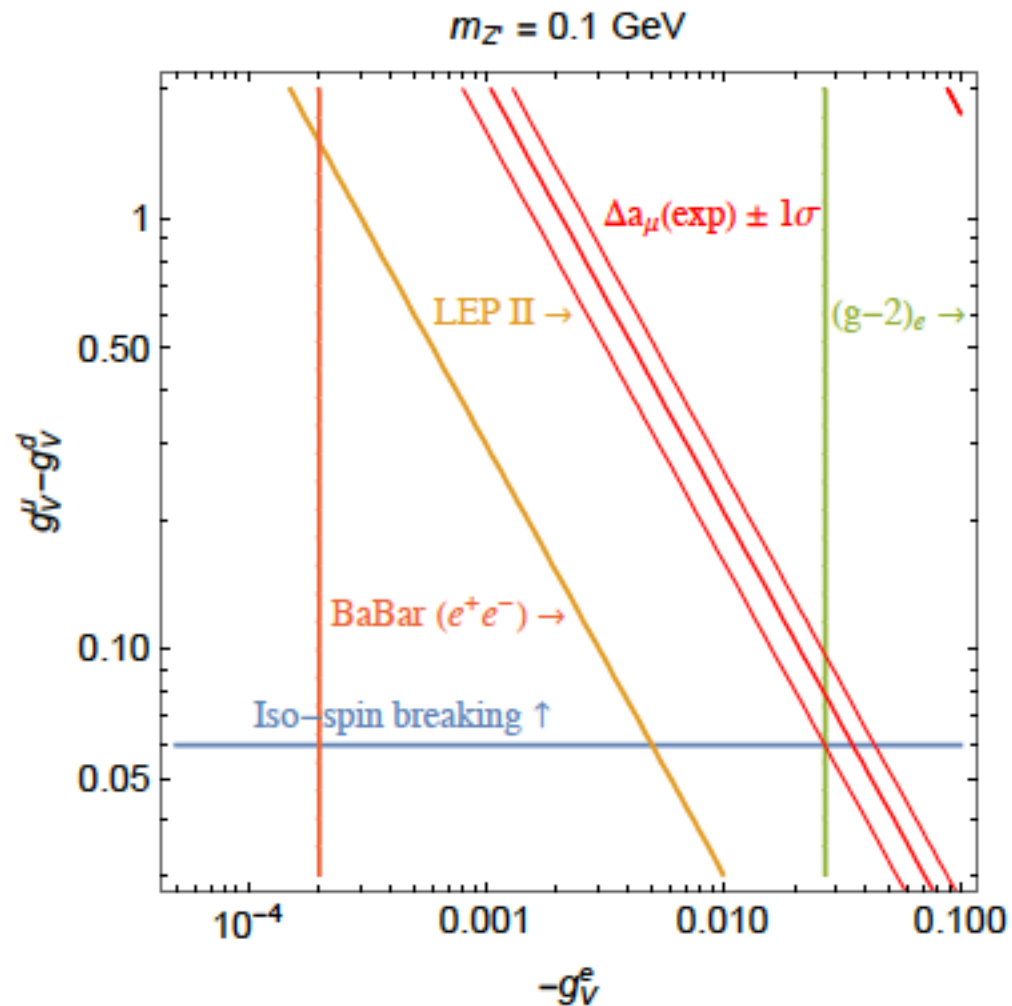
$$\frac{\sigma_{qq}^{\text{SM+NP}}}{\sigma_{qq}^{\text{SM}}} \approx 1 + 2 \frac{g_V^e g_V^q}{e^2 Q_q} \longrightarrow |g_V^e g_V^q| \lesssim 4.6 \cdot 10^{-4} |Q_q| \quad (\epsilon \lesssim 3.3 \cdot 10^{-3})$$

Iso-spin breaking observables

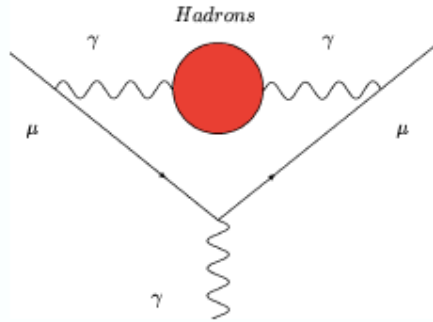
$$\longrightarrow |g_V^u - g_V^d| \lesssim 0.06$$

charged vs. neutral pion mass<sup>2</sup> difference  $\Delta m^2 = m_{\pi^+}^2 - m_{\pi^0}^2$  (rescaling the lattice QCD calculation of Frezzotti, Gagliardi, Lubicz, Martinelli, Sanfilippo and Simula 2112.01066)

At least **TWO independent bounds prevent** to get a sizeable contribution to  $\Delta a_\mu$  modifying  $\sigma_{\text{had}}$  via  $Z'$  exchange to **solve** the “**new**”  $\mu$  g-2 puzzle



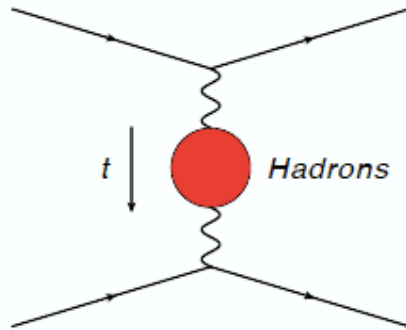
- At present, the leading hadronic contribution  $a_\mu^{\text{HLO}}$  is computed via the **timelike** formula:



$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma_{\text{had}}^0(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)}$$

- Alternatively, exchanging the  $x$  and  $s$  integrations in  $a_\mu^{\text{HLO}}$



$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

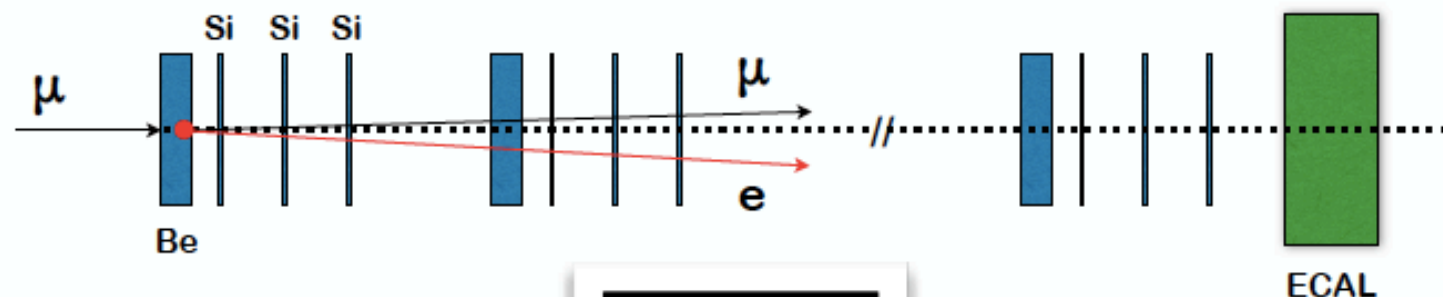
Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{\text{had}}(t)$  is the hadronic contribution to the running of  $\alpha$  in the **spacelike** region:  $a_\mu^{\text{HLO}}$  can be extracted from scattering data!

## MUonE: Muon-electron scattering @ CERN



- $\Delta\alpha_{\text{had}}(t)$  can be measured via the **elastic scattering**  $\mu e \rightarrow \mu e$ .
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.

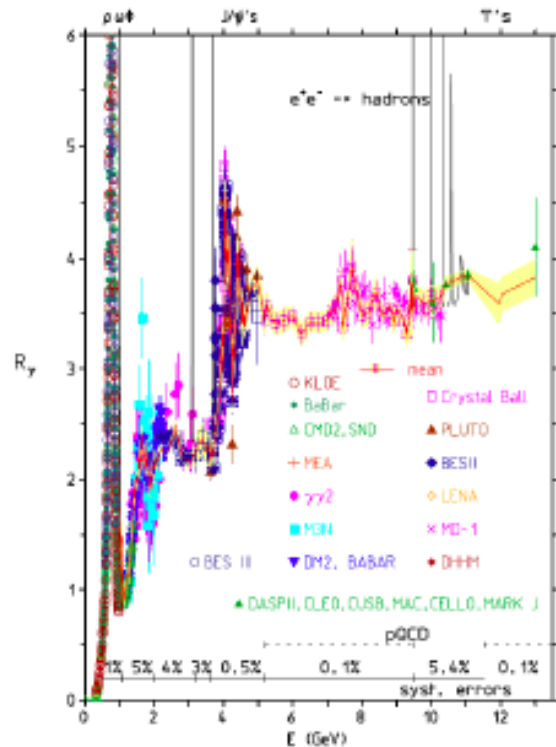


Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,  
Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni  
EPJC 2017 - arXiv:1609.08987

[Courtesy by M. Passera]

- Letter of Intent submitted to CERN SPSC in 2019: **Test run approved for 2021**

Timelike

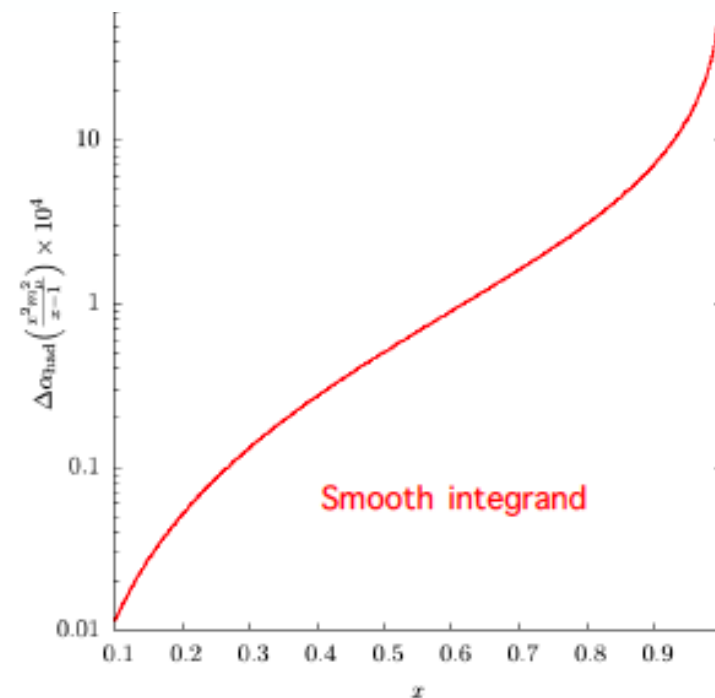


F. Jegerlehner, arXiv:1511.04473



Spacelike

$\Delta\alpha_{\text{had}}(t)$  can be measured via the elastic scattering  $\mu e \rightarrow \mu e$ .



Carlson Calame, Passera, Trentadue, Venanzoni, PLB 2015

- ✓ Inclusive measurement
- ✓ Smooth integrand
- ✓ Direct interplay with lattice QCD

# The **ELECTRON** magnetic moment

- Status of  $\Delta a_e$  as of 2012

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -9.2(8.1) \times 10^{-13},$$

$$\delta a_e \times 10^{13} : (0.6)_{\text{QED4}}, (0.4)_{\text{QED5}}, (0.2)_{\text{HAD}}, (7.6)_{\delta\alpha}, (2.8)_{\delta a_e^{\text{EXP}}}.$$

- ▶ The errors from QED4 and QED5 will be reduced soon to  $0.1 \times 10^{-13}$  [Kinoshita]
- ▶ We expect a reduction of  $\delta a_e^{\text{EXP}}$  to a part in  $10^{-13}$  (or better). [Gabrielse]
- ▶ Work is also in progress for a significant reduction of  $\delta\alpha$ . [Nez]

- Status of  $\Delta a_e$  as of 2018: **2.4 $\sigma$  discrepancy** [Parker et al., Science, '18]

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}}(\alpha_{\text{Berkeley}}) = -8.8(3.6) \times 10^{-13}$$

$$\delta a_e \times 10^{13} : (0.1)_{\text{QED5}}, (0.1)_{\text{HAD}}, (2.3)_{\delta\alpha}, (2.8)_{\delta a_e^{\text{EXP}}}.$$

- Status of  $\Delta a_e$  as of 2020: **1.6 $\sigma$  discrepancy** [Morel et al., Nature, '20]

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}}(\alpha_{\text{LKB2020}}) = 4.8(3.0) \times 10^{-13}$$

$$\delta a_e \times 10^{13} : (0.1)_{\text{QED5}}, (0.1)_{\text{HAD}}, (0.9)_{\delta\alpha}, (2.8)_{\delta a_e^{\text{EXP}}}.$$

# NEW Measurement of the Electron Magnetic Moment

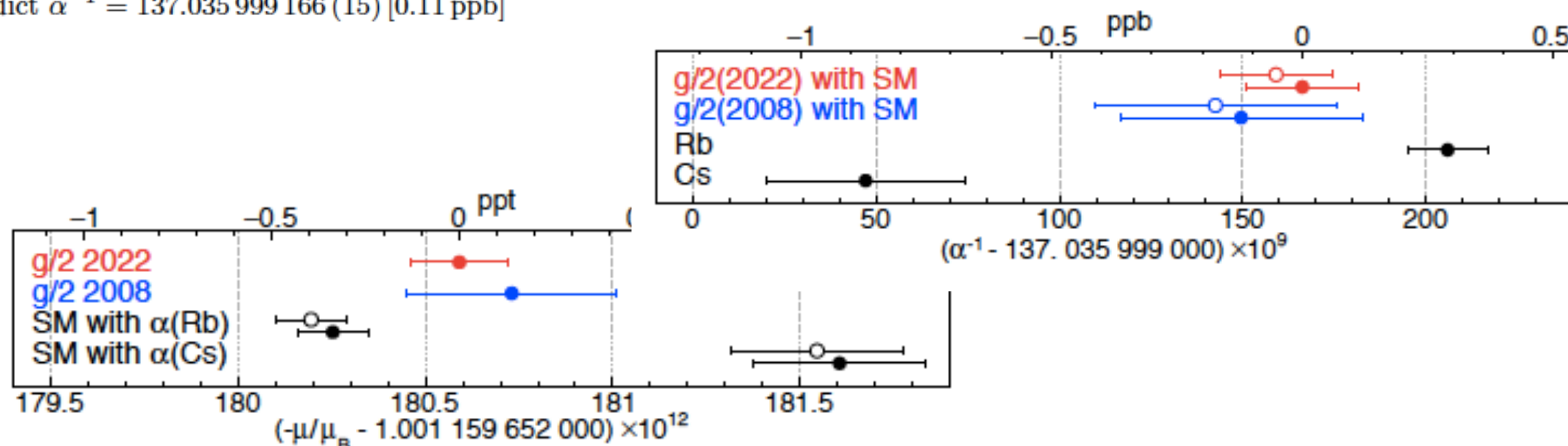
X. Fan,<sup>1,2,\*</sup> T. G. Myers,<sup>2</sup> B. A. D. Sukra,<sup>2</sup> and G. Gabrielse<sup>2,†</sup>

<sup>1</sup>*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*

<sup>2</sup>*Center for Fundamental Physics, Northwestern University, Evanston, Illinois 60208, USA*

(Dated: September 28, 2022)

The electron magnetic moment in Bohr magnetons,  $-\mu/\mu_B = 1.001\,159\,652\,180\,59(13)$  [0.13 ppt], is consistent with a 2008 measurement and is 2.2 times more precise. The most precisely measured property of an elementary particle agrees with the most precise prediction of the Standard Model (SM) to 1 part in  $10^{12}$ , the most precise confrontation of all theory and experiment. The SM test will improve further when discrepant measurements of the fine structure constant  $\alpha$  are resolved, since the prediction is a function of  $\alpha$ . The magnetic moment measurement and SM theory together predict  $\alpha^{-1} = 137.035\,999\,166(15)$  [0.11 ppb]



# LFV, $(g - 2)_{\text{lept}}$ and $(\text{EDM})_{\text{lept}}$ correlations in Effective Theories

- $\text{BR}(\ell_I \rightarrow \ell_J \gamma)$  vs.  $(g - 2)_\mu$

$$\text{BR}(\mu \rightarrow e \gamma) \approx 3 \times 10^{-13} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{e\mu}}{10^{-5}} \right)^2$$

$$\text{BR}(\tau \rightarrow \mu \gamma) \approx 4 \times 10^{-8} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{\mu\tau}}{10^{-2}} \right)^2$$

- EDMs vs.  $(g - 2)_\mu$

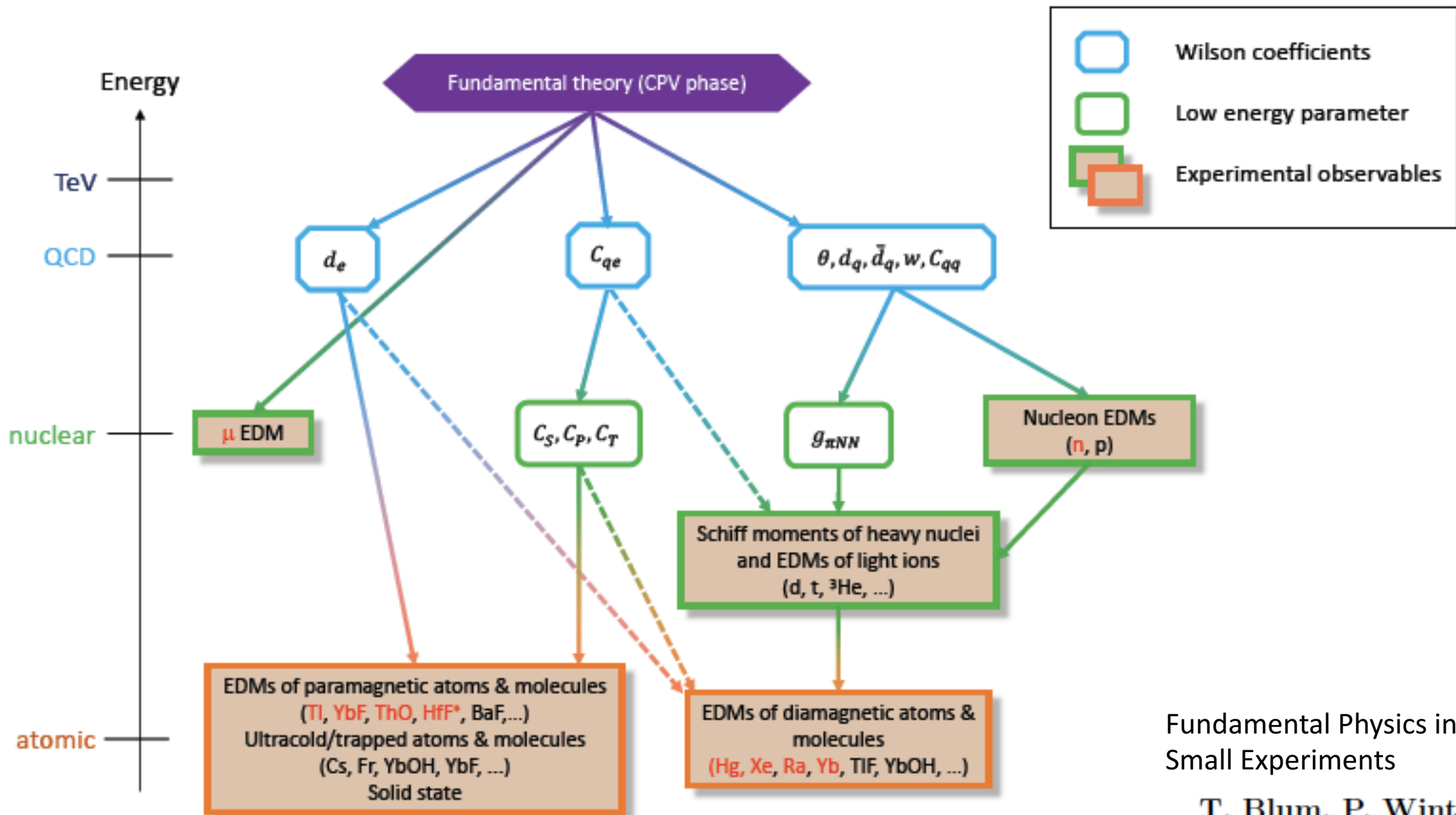
$$d_e \simeq \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 10^{-29} \left( \frac{\phi_e^{\text{CPV}}}{10^{-5}} \right) e \text{ cm},$$

$$d_\mu \simeq \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22} \phi_\mu^{\text{CPV}} e \text{ cm},$$

- Main messages:

- ▶  $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$  requires a nearly flavor and CP conserving NP
- ▶ Large effects in the muon EDM  $d_\mu \sim 10^{-22} e \text{ cm}$  are still allowed!

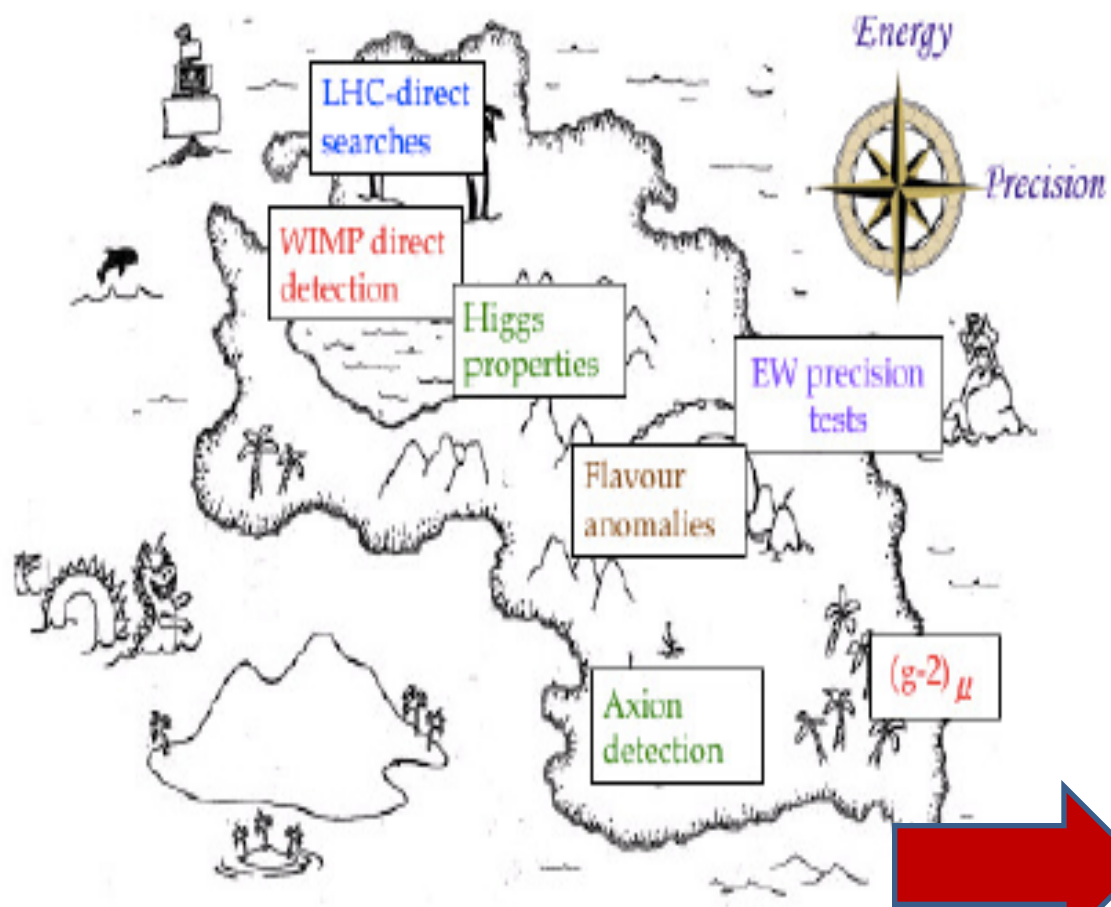
$$\frac{\Delta a_e}{\Delta a_\mu} = \frac{m_e^2}{m_\mu^2} \iff \Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}$$



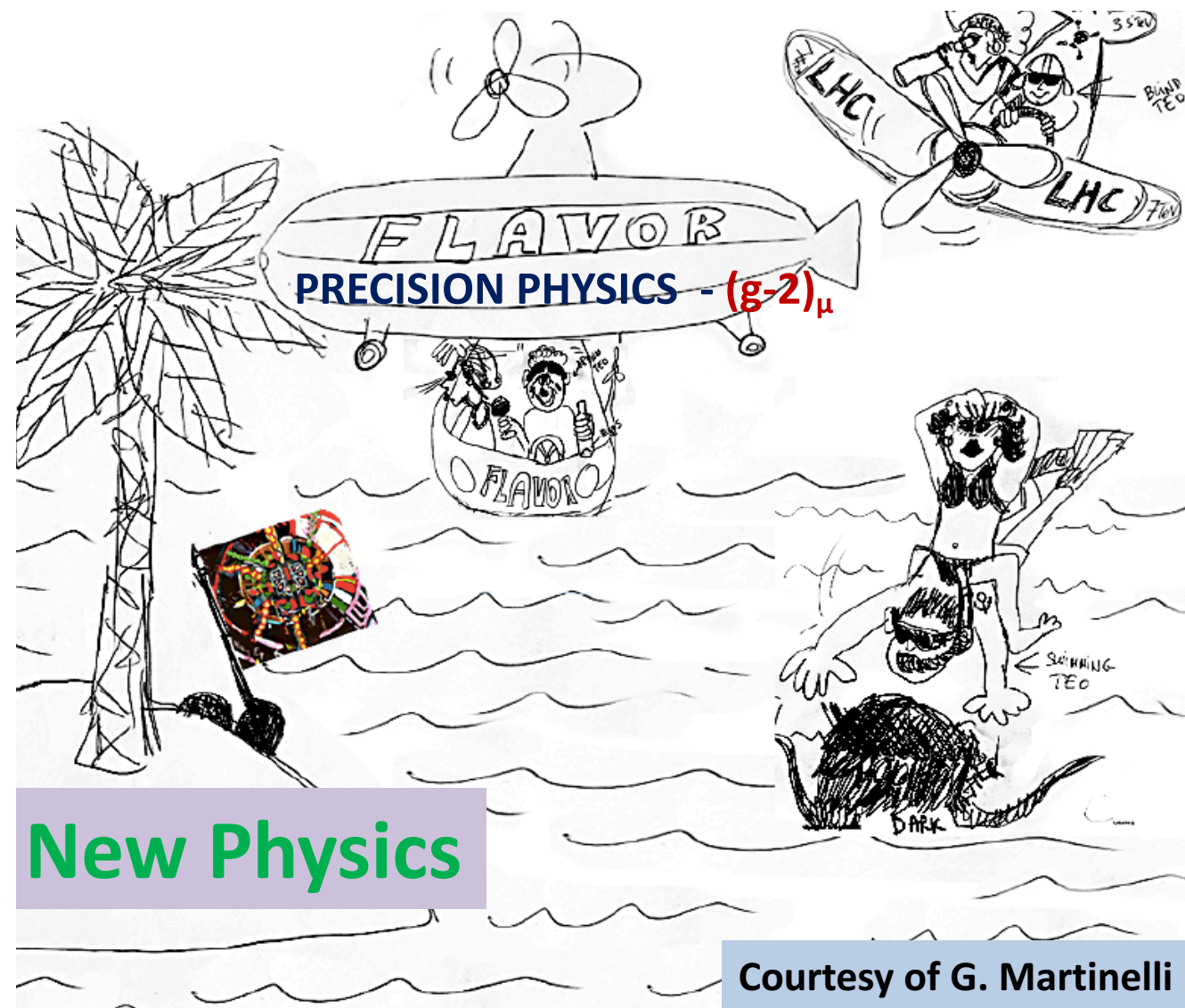
Fundamental Physics in  
Small Experiments

T. Blum, P. Winter

# TERRA INCOGNITA



[Casas @ Moriond 2017]

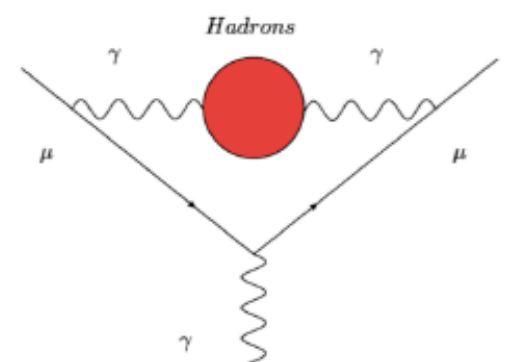


Courtesy of G. Martinelli

**BACK-UP SLIDES**

- dominated by  $e^+e^- \rightarrow \pi^+\pi^-$  channel (70% of the full hadronic)

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^{\infty} \frac{ds}{s} K(s) \text{Im} \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$



- what is  $\sigma_{\text{had}}(s)$  ?
  - Includes Final State Radiation (FSR)
  - Initial State Radiation (ISR) and FSR/ISR interference are subtracted
  - Vacuum polarization also subtracted (by rescaling exp. cross-section by  $|\alpha/\alpha(s)|^2$ )

➡ part of higher-order HVP

[WP20, 2006.04822]

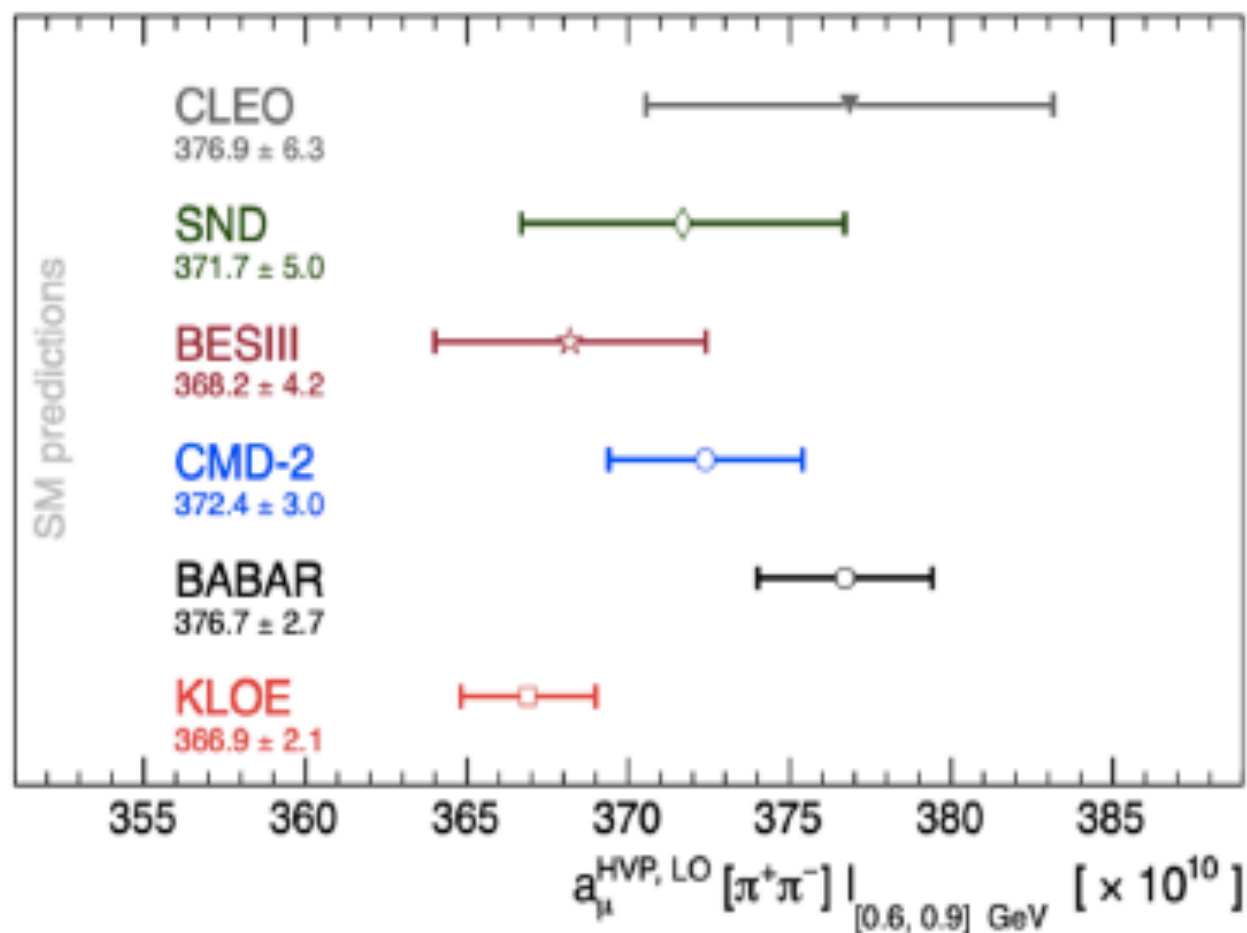


Figure 15: Comparison of results for  $a_{\mu}^{\text{HVP, LO}}[\pi\pi]$ , evaluated between 0.6 GeV and 0.9 GeV for the various experiments.

# NP in Bhabha scattering?

- What if the measurement of the KLOE luminosity is affected by NP ?

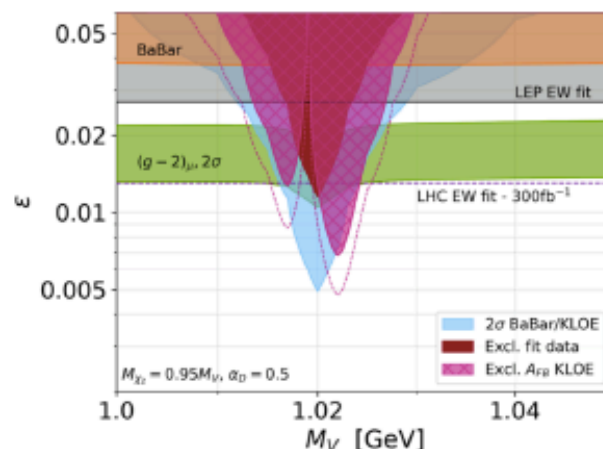
[Darmé, Grilli di Cortona, Nardi 21/2.09/39]

$$\mathcal{L}_{e^+e^-}^{\text{SM}} = \frac{N_{\text{Bha}}}{\sigma_{\text{eff}}^{\text{SM}}} \quad \longrightarrow \quad \mathcal{L}_{e^+e^-} = \mathcal{L}_{e^+e^-}^{\text{SM}} \frac{\sigma_{\text{eff}}^{\text{SM}}}{\sigma_{\text{eff}}}$$

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}^{\text{SM}} (1 + \delta_R)$$

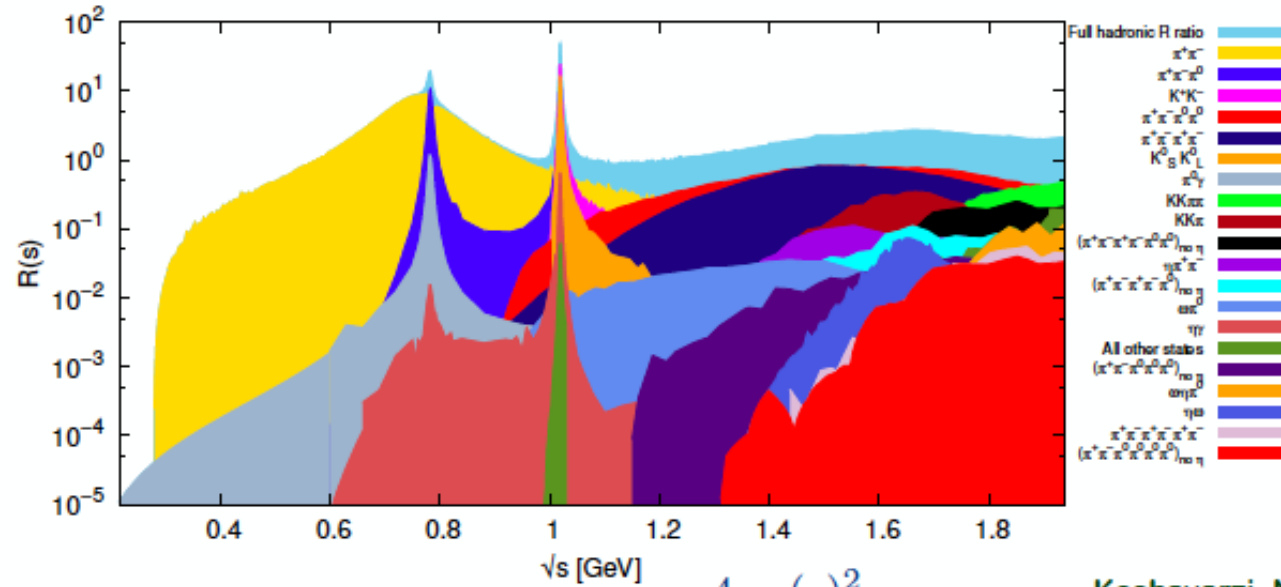
$$\sigma_{\text{had}} \propto N_{\text{had}} / \mathcal{L}_{e^+e^-} \quad \longrightarrow \quad \sigma_{\text{had}} \rightarrow \sigma_{\text{had}} (1 + \delta_R)$$

$$a_{\mu}^{\text{LO,HVP}} \rightarrow a_{\mu}^{\text{LO,HVP}} (1 + \delta_R)$$



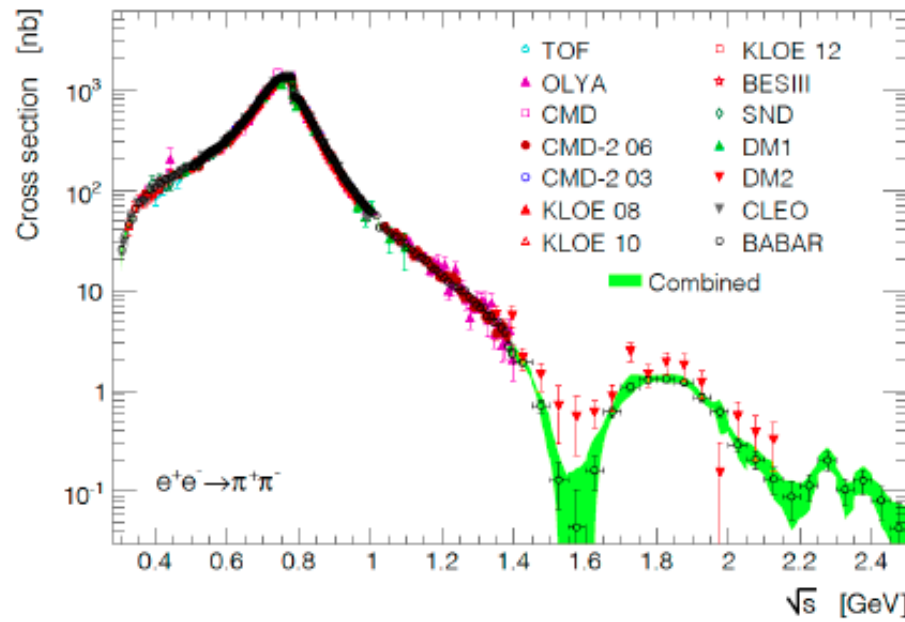
**Figure 3.** Parameter range compatible at  $2\sigma$  with the experimental measurement of  $\Delta a_{\mu}$  (green region) resulting from a redetermination of the KLOE luminosity, for  $\alpha_D = 0.5$ ,  $m_{\chi_2} = 0.95m_V$  and  $m_{\chi_1} = 25$  MeV. In the blue region the KLOE and BaBar results for  $\sigma_{\text{had}}$  are brought into agreement at  $2\sigma$ . The red region corresponds to a shift of the KLOE measurement in tension with BaBar (and with the other experiments) at more than  $2\sigma$ .

# $e^+e^- \rightarrow \pi^+\pi^-$ dominance of the low-energy hadronic cross-section



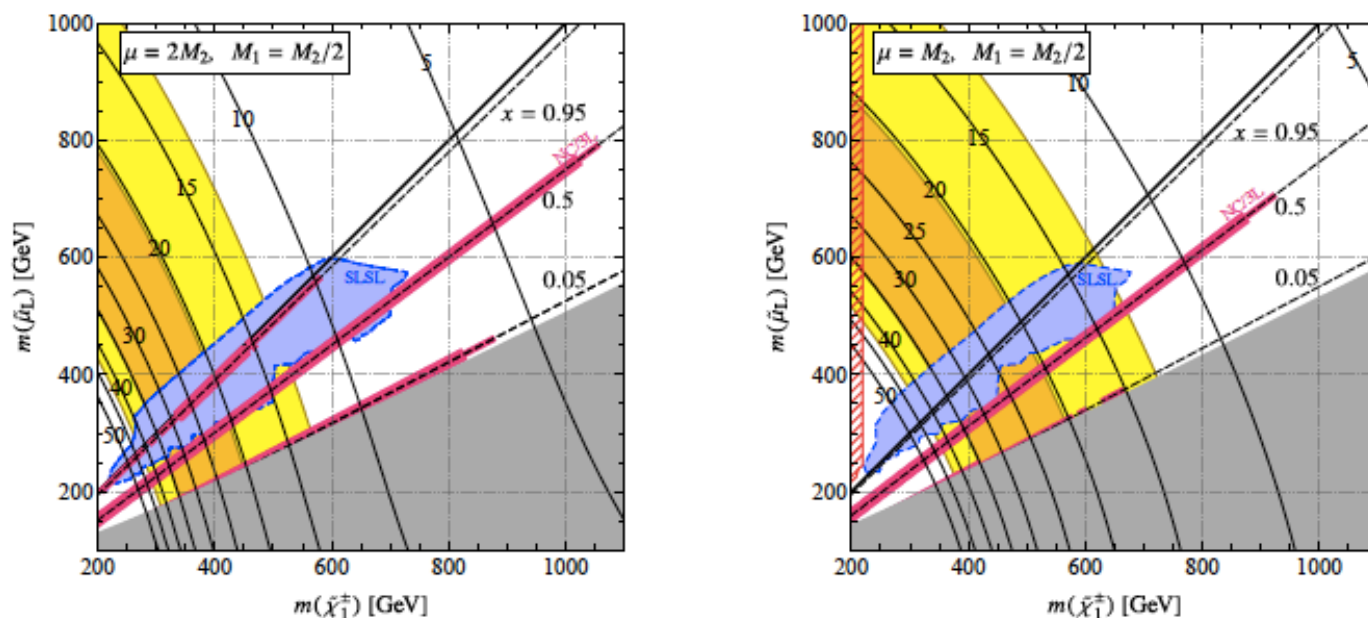
$$R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

Keshavarzi, Nomura Teubner  
PRD 2018



Davier, Hoecker, Malaescu, Zhang  
EPJC 2020

# $\Lambda \approx \nu$ : SUSY and the muon ( $g - 2$ )



**Figure:** LHC Run 2 bounds on SUSY scenario for the muon  $g - 2$  anomaly for  $\tan \beta = 40$ . Orange (yellow) regions satisfy the muon  $g - 2$  anomaly at the  $1\sigma$  ( $2\sigma$ ) level [Endo et al., '20].

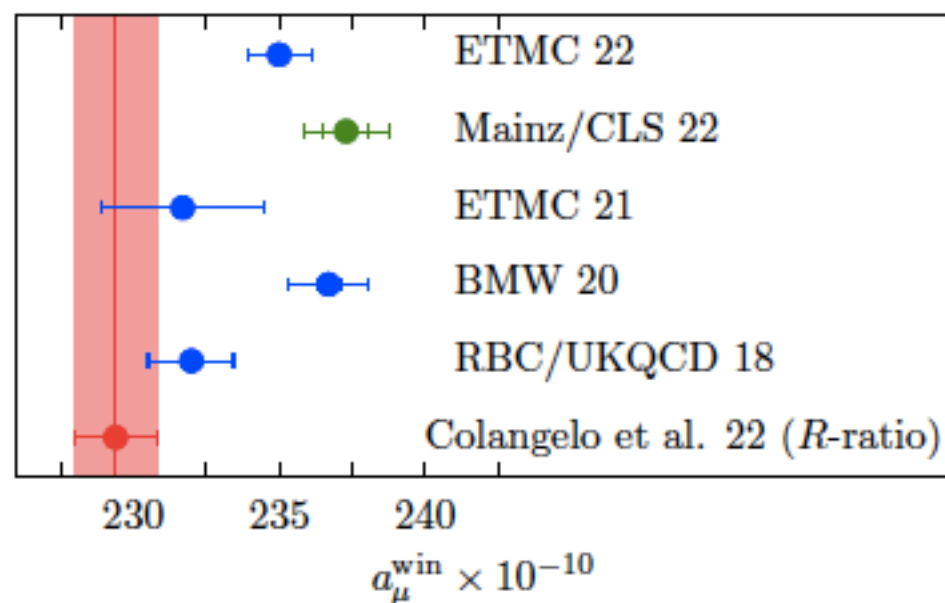
$$(a_\mu^{\text{SM}})_{\text{weak}} \approx \frac{g^2 m_\mu^2}{32\pi^2 M_W^2} \approx 2 \times 10^{-9}$$

$$a_\mu^{\text{SUSY}} \approx \frac{g^2 m_\mu^2 \tan \beta}{32\pi^2 \tilde{m}^2} \underbrace{\approx 2 \times 10^{-9}}_{\tilde{m} = 500\text{GeV} \ \& \ \tan \beta = 40}$$

## COMPARISON WITH RESULTS FOR $a_\mu^{\text{win}}$

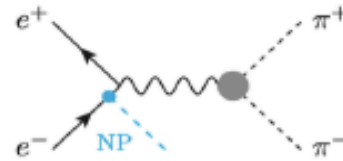
- Isospin-breaking correction  $+(0.70 \pm 0.47) \times 10^{-10}$  included:

$$a_\mu^{\text{win}} = (237.30 \pm 0.79_{\text{stat}} \pm 1.13_{\text{syst}} \pm 0.05_{\text{Q}} \pm 0.47_{\text{IB}}) \times 10^{-10}$$



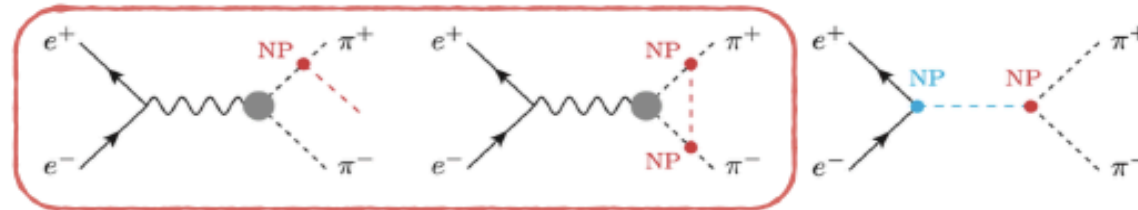
- $3.9\sigma$  tension with data-driven estimate in [2205.12963, Colangelo et al.].
- Genuine difference between lattice and data-driven results?

- Light new physics inducing a sub-GeV modification of  $\sigma_{\text{had}}$  is the only possibility



1. NP coupled only to **electrons**  $\rightarrow$  severe bounds

[See however  
Darmé, Grilli di Cortona, Nardi 21/12/09/139  
NP in Bhabha scattering?  $\rightarrow$  backup slides]



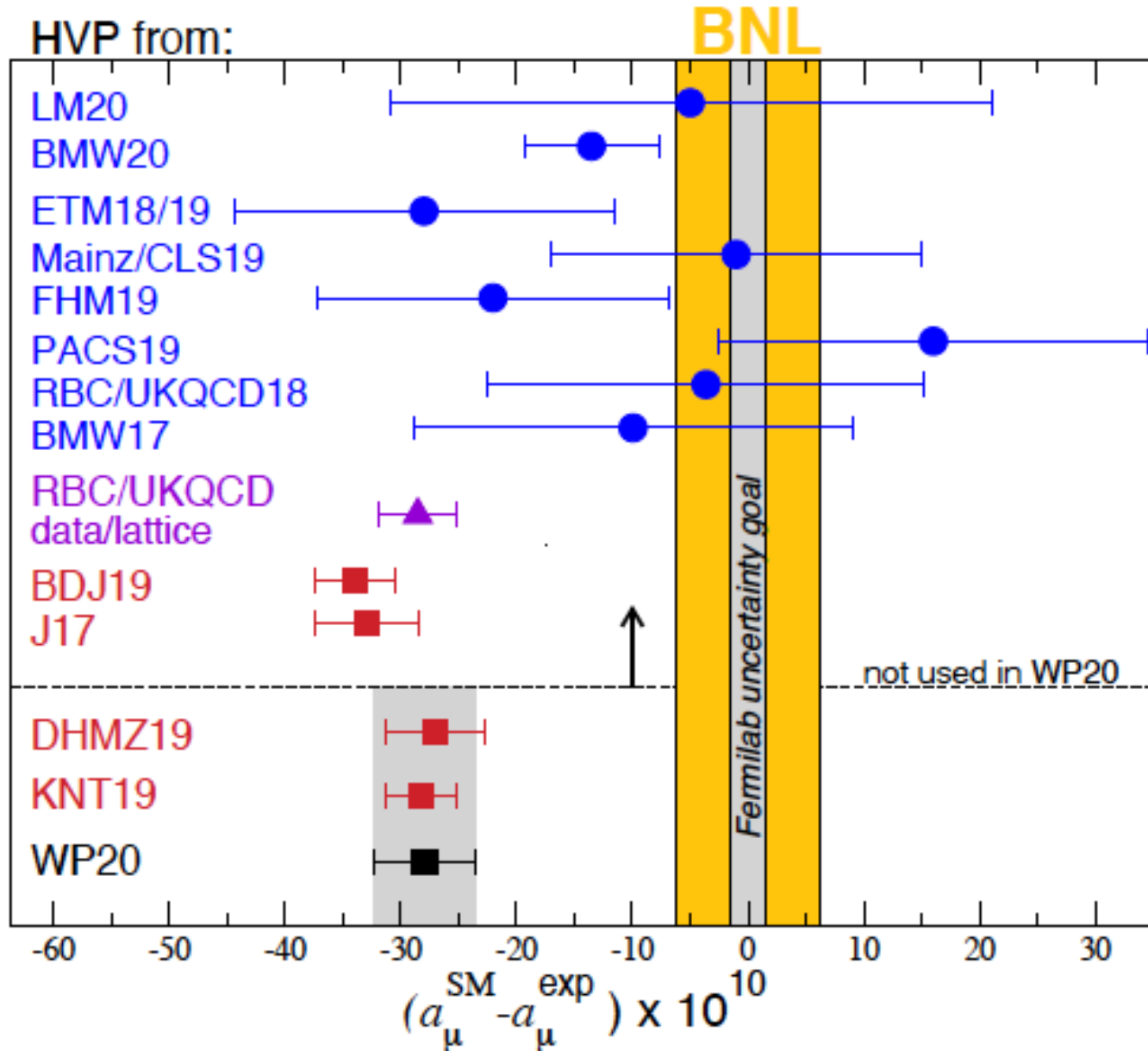
2. NP coupled only to **hadrons**

*FSR effects due to NP should be included into  $\sigma_{\text{had}}(s)$ , not easy to be accounted for...  
(depend on exp. cuts and mass of NP)*

$\rightarrow$  however, we know that in the QED case

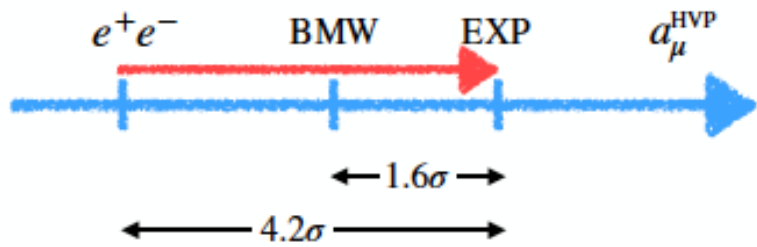
$$(a_{\mu}^{\text{HVP}})^{\text{FSR}}_{e^+e^-} \approx 50 \times 10^{-11} \quad \longleftrightarrow \quad |(a_{\mu}^{\text{HVP}})_{\text{BMW}} - (a_{\mu}^{\text{HVP}})^{\text{WP20}}_{e^+e^-}| \approx 150 \times 10^{-11}$$

# HADRONIC VACUUM POLARIZATION CONTRIBUTION



Ab-initio lattice calculations

Dispersive relations,  
 $e^+e^- \rightarrow \text{hadrons}$  exps.



*some conclusive thoughts:*

- attempt to solve the "new" muon g-2 puzzle introducing NP which modifies  $\sigma(e^+e^- \rightarrow \text{hadrons})$ , but without affecting  $a_\mu^{\text{HVP}}$ :
  - a) NP  $\rightarrow$  light ( $<1$  GeV) vector **Z'** coupling only to electrons and hadrons;
  - b) the **experimental constraints** on the size of such couplings **prevent** the Z' exchange to provide the needed enhancement of the hadronic  $\sigma$  to suitably address the new g-2 puzzle
- **Two** directions to be vigorously pursued:
  - i) perform **new** independent **lattice QCD** computations of the HVP contribution to  $a_\mu$  to assess the validity of the **BMWc result** ;
  - ii) identifies **new** experimental ways to probe  $a_\mu^{\text{HVP}}$  (the **MUonE** exp. can (hopefully reasonably) soon provide an **independent determination** of the leading hadronic contributions to  $a_\mu$  alternative to both the dispersive and lattice methods)