

# An Analytic Approach to Light Dark Matter Propagation

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March 24, 2023

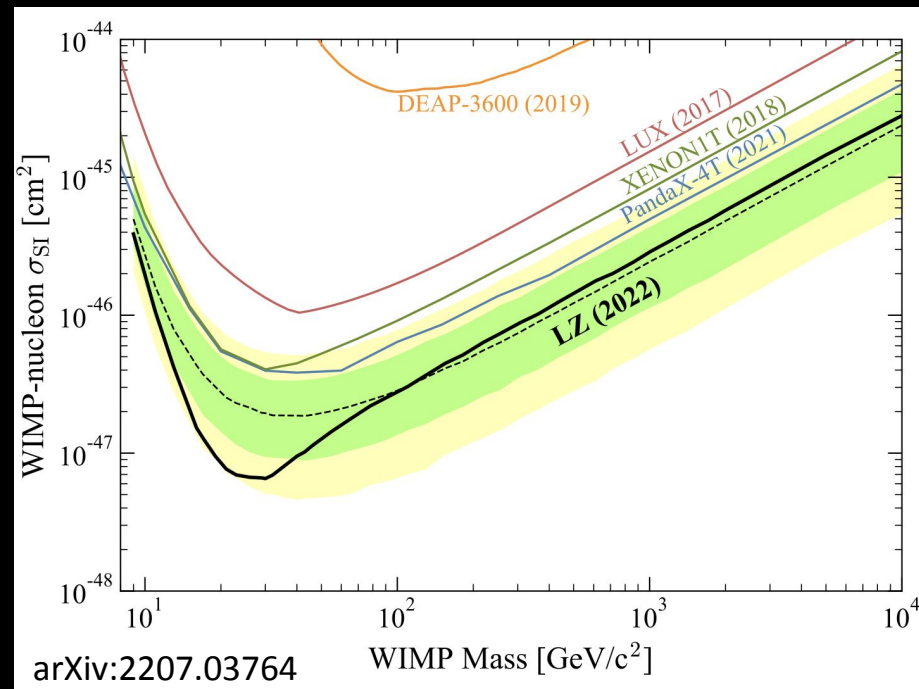


Arthur B. McDonald  
**Canadian Astroparticle Physics Research Institute**

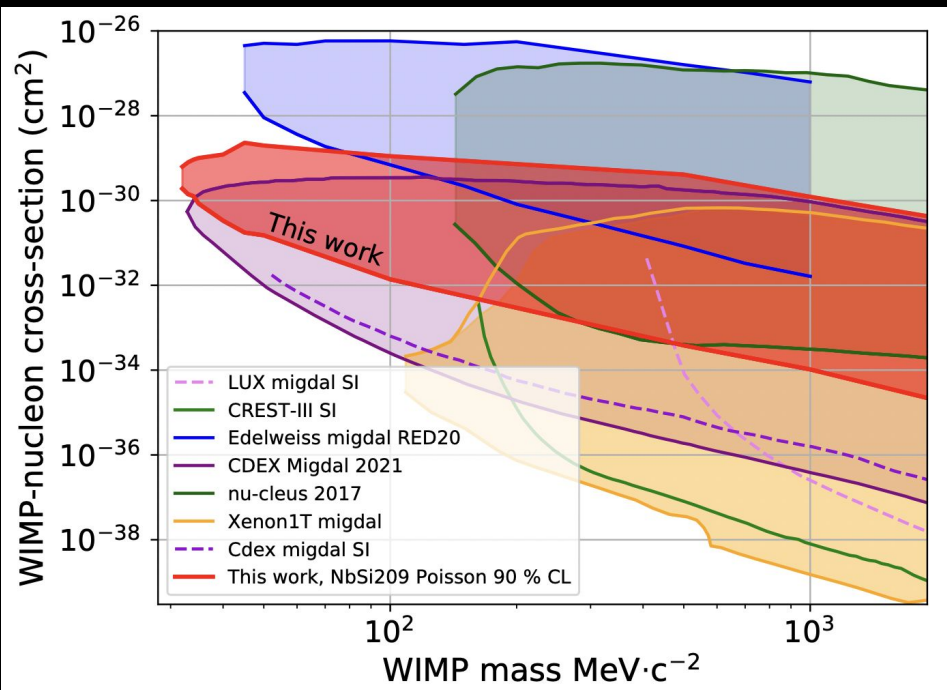


# An Intro to Dark Matter Attenuation

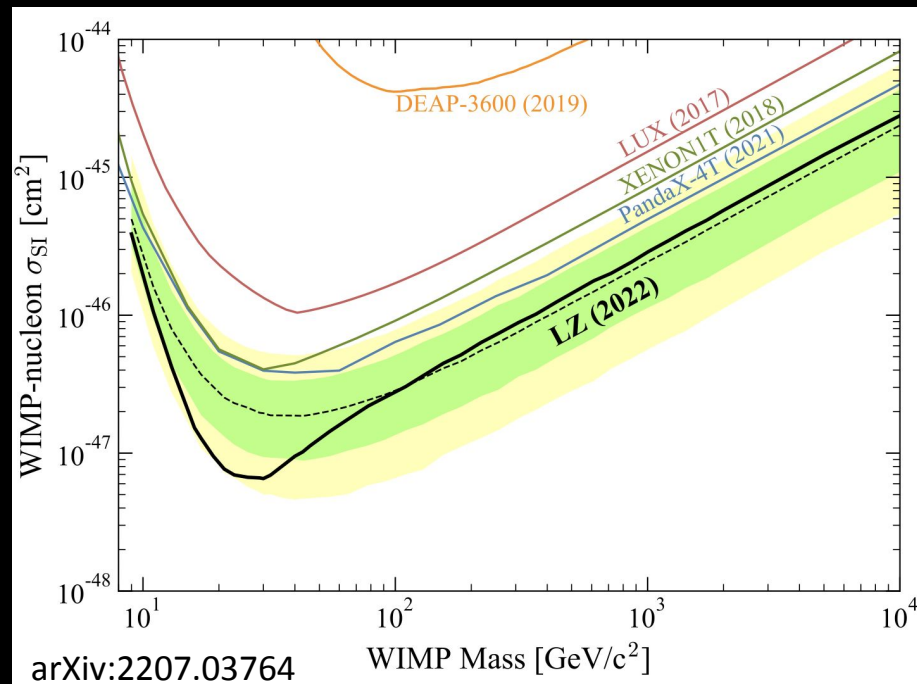
# Direct Detection Limits



# Direct Detection Limits



arXiv:2203.03993 (Edelweiss)



arXiv:2207.03764

# Attenuation: Straight Line Approximation

PHYSICAL REVIEW D

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## Opening the window on strongly interacting dark matter

Glenn D. Starkman and Andrew Gould

*Institute for Advanced Study, Princeton, New Jersey 08540*

Rahim Esmailzadeh

*Center for Particle Astrophysics, University of California, Berkeley, California 94720*

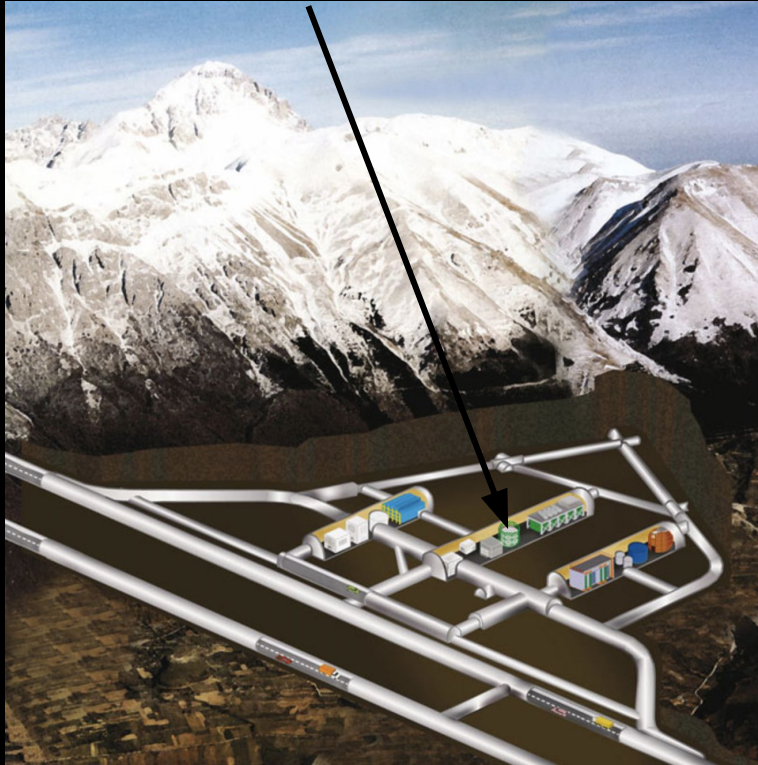
Savas Dimopoulos\*

*CERN TH-Division, 1211 Geneva 23, Switzerland*

(Received 2 February 1990)

We discuss the possibility that the dark matter consists of strongly interacting massive particles (SIMP's) which have cross sections with ordinary matter which are larger than characteristic weak-interaction cross sections. We show that, while results from  $\beta\beta$  decay, cosmic-ray detectors, galactic-halo stability, the cooling of molecular clouds, proton-decay detectors, and the existence of old neutron stars and the Earth constrain the interactions of the missing matter with ordinary matter over a broad range of parameter space, there still exist several windows for SIMP's. It is noteworthy that there are two regions of less than geometric cross sections: one with masses of  $10^5 - 10^7$  GeV and another with masses above  $10^{10}$  GeV.

# Attenuation: Straight Line Approximation



Assume that dark matter follows a straight (ballistic) trajectory, every particle losing the average possible amount of energy:

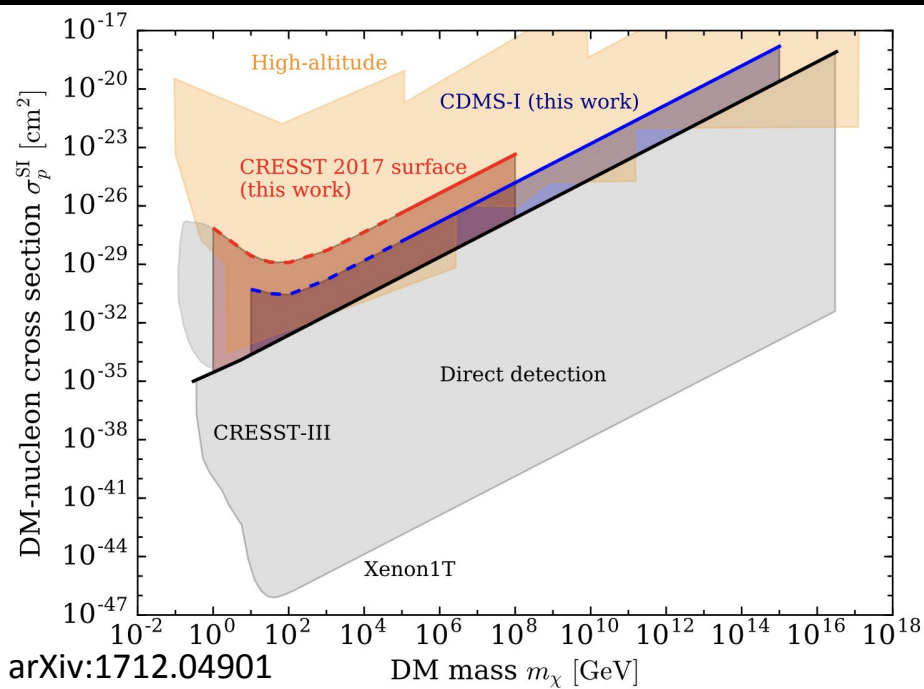
$$\frac{d\langle E_{\chi} \rangle}{dt} = - \sum_i n_i(\mathbf{r}) \langle E_R \rangle_i \sigma_i(v) v ,$$

- $n_i$  : number density of nuclei in overburden
- $\langle E_R \rangle_i$  : average recoil energy
- $\sigma_i$  : DM-nucleus cross section

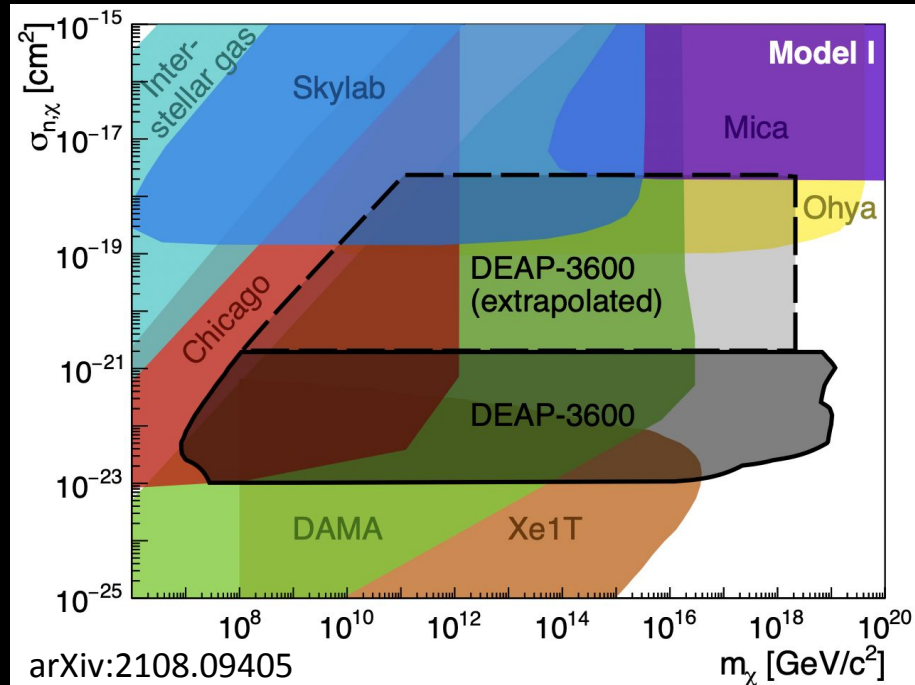
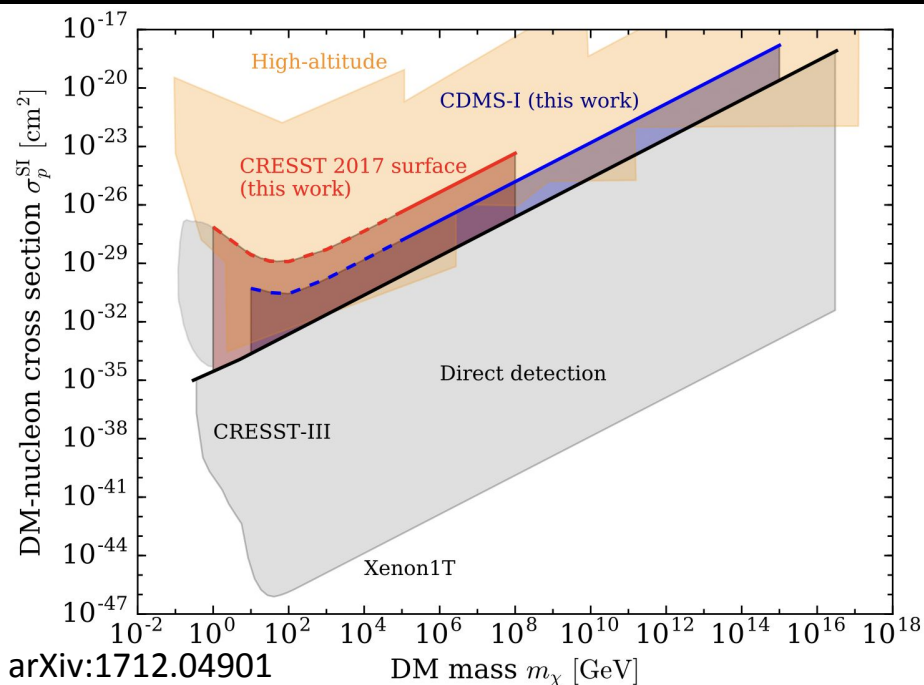
Left: Schematic of LNGS

-Credit: <https://www.appec.org>

# Attenuation: Straight Line Approximation

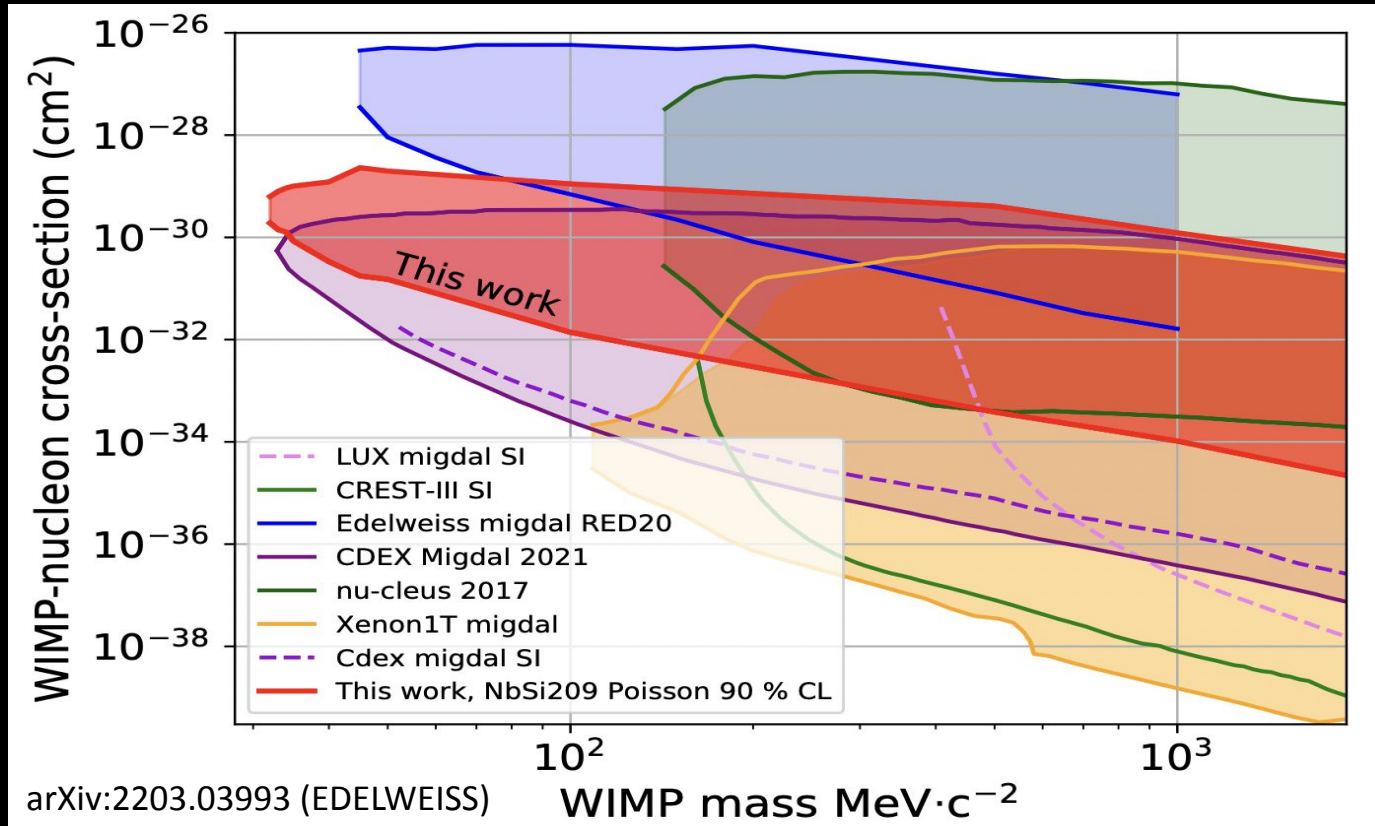


# Attenuation: Straight Line Approximation

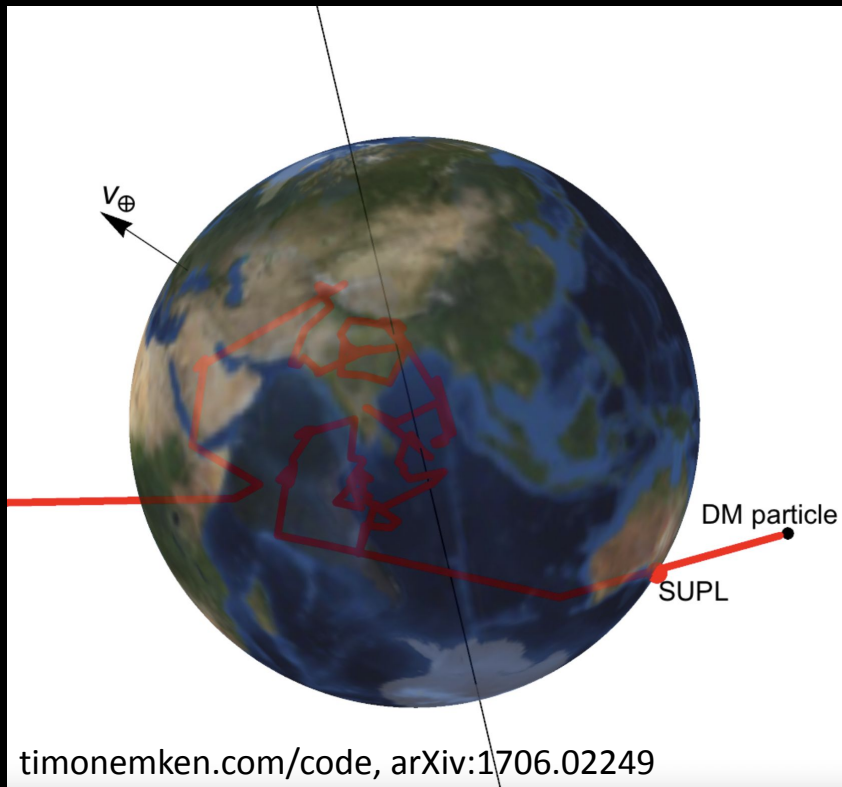




# Attenuation: Straight Line Approximation



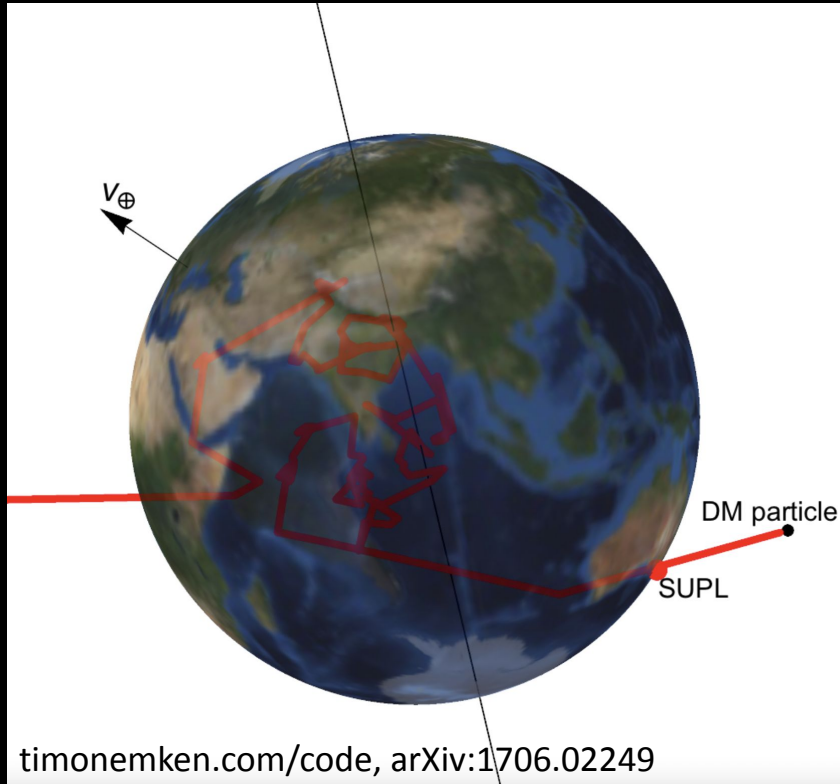
# Attenuation: Monte Carlo Simulations



Alternative approach: numerically simulate millions of dark matter particles scattering in Earth

- Initialize a particle above the Earth with initial velocity  $\mathbf{v}$
- Choose distance particle travels without scattering from exponential path length distribution
- Choose target + scattering angle, update velocity
- Stop if particle is detected, loses too much energy, or leaves Earth

# Attenuation: Monte Carlo Simulations



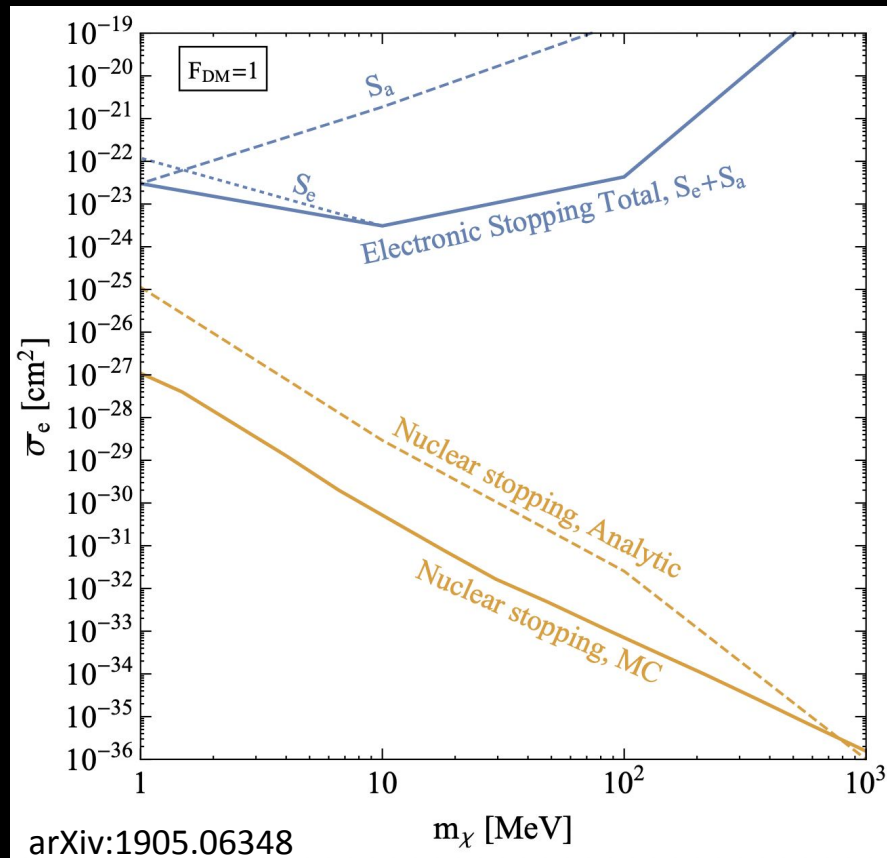
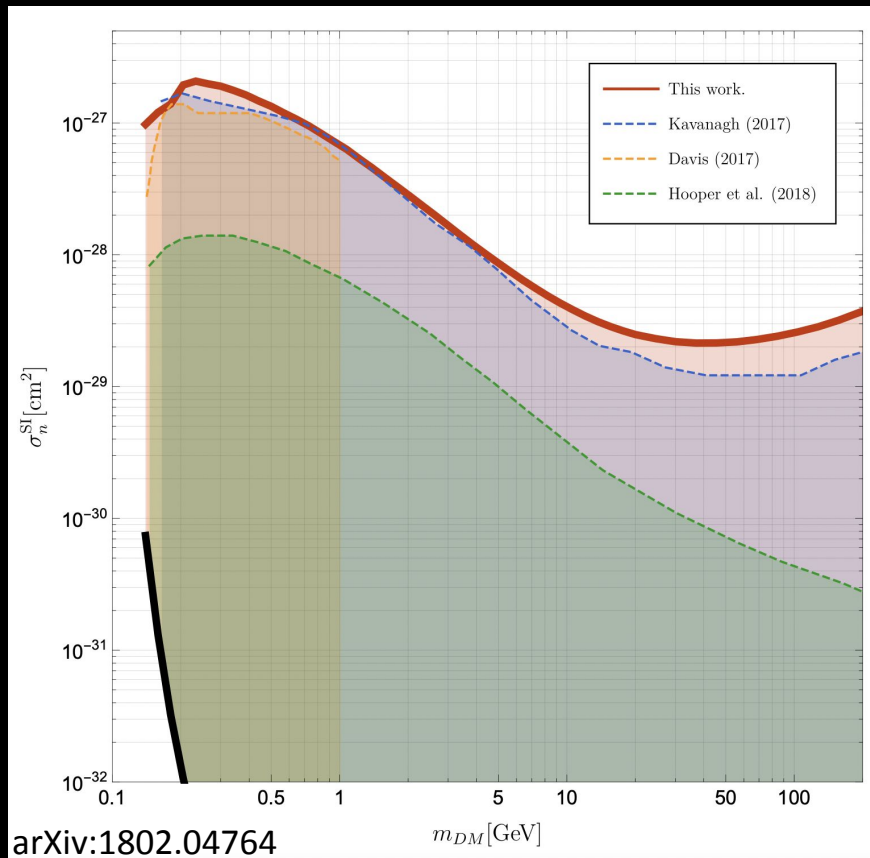
## Examples:

- DaMaSCUS: Dark Matter Simulation Code for Underground Scatterings, Timon Emken
- DMATIS: Dark Matter Attenuation Importance Sampling, M. Shafi Mahdawi
- DarkProp (for cosmic ray boosted dark matter), Chen Xia

## See also:

**CVC** and Beacom: PRD 100, 103011 (2019)  
PROSPECT Collaboration, **CVC**: PRD 104, 012009 (2021)

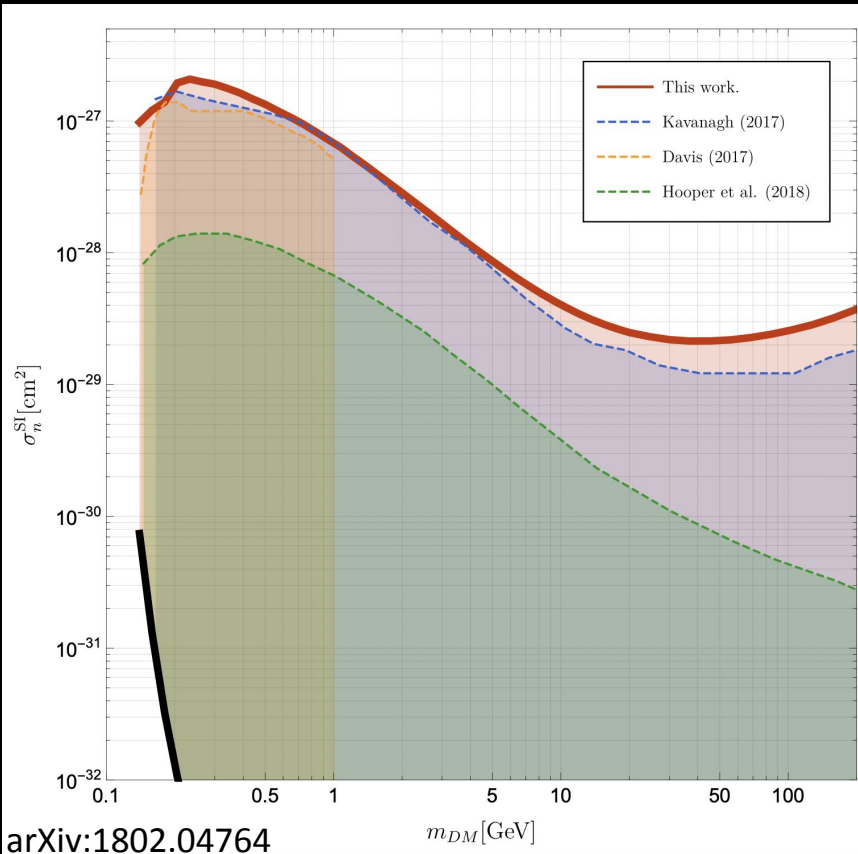
# Straight Line vs. Monte Carlo



# A New Analytic Approximation

Based on arXiv:2301.07728

# Probability of Not Scattering



We want an alternative to both Monte Carlo and straight-line approaches

Hooper & McDermott (Green):

- Compute fraction of dark matter reaching CRESST surface detector without scattering

$$P_{\text{initial}}(x) = \frac{1}{l} e^{-x/l}$$

$$P(z, \theta) = \frac{1}{l \cos(\theta)} e^{-z/(l \cos(\theta))}$$

$$P(z) = \int_0^1 \frac{d \cos(\theta)}{l \cos(\theta)} e^{-z/(l \cos(\theta))} = \frac{1}{l} \Gamma(0, z/l)$$

# What about scattering?

## Signatures of Earth-scattering in the direct detection of Dark Matter

Bradley J. Kavanagh,<sup>a</sup> Riccardo Catena<sup>b</sup> and Chris Kouvaris<sup>c</sup>

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Published January 9, 2017

**Abstract.** Direct detection experiments search for the interactions of Dark Matter (DM) particles with nuclei in terrestrial detectors. But if these interactions are sufficiently strong, DM particles may scatter in the Earth, affecting their distribution in the lab. We present a new analytic calculation of this ‘Earth-scattering’ effect in the regime where DM particles scatter at most once before reaching the detector. We perform the calculation self-consistently, taking into account not only those particles which are scattered away from the detector, but also those particles which are deflected towards the detector. Taking into account a realistic model of the Earth and allowing for a range of DM-nucleon interactions, we present the EARTHSHADOW code, which we make publicly available, for calculating the DM velocity distribution after Earth-scattering. Focusing on low-mass DM, we find that Earth-scattering reduces the direct detection rate at certain detector locations while increasing the rate in others. The Earth’s rotation induces a daily modulation in the rate, which we find to be highly sensitive to the detector latitude and to the form of the DM-nucleon interaction. These distinctive signatures would allow us to unambiguously detect DM and perhaps even identify its interactions in regions of the parameter space within the reach of current and future experiments.

JCAP01(2017)012

# What about scattering?

$$P_0(z) = \frac{1}{l} \Gamma(0, z/l)$$

$$P_1(z) = \int P_0(z') P(z - z') dz'$$



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Assume isotropic scattering

# What about scattering?

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$$P(z - z') = \frac{1}{2l} \Gamma(0, |(z - z')/l|)$$

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$$P_n(z) = \int_0^\infty P_{n-1}(z') \frac{1}{2l} \Gamma(0, |z - z'|/l) dz'$$

This process is iterative!  
Can compute probabilities through  
n scatterings

# Energy Loss

$$P(\Delta E) = \frac{\sum_A \frac{n_A \sigma_{\chi A}}{E_{max,A}} \theta(E_{max,A} - \Delta E)}{\sum_A n_A \sigma_{\chi A}}$$

$$\frac{dN}{dE}_n(E) = \int \left( \frac{E}{E - \Delta E} \right) \frac{dN}{dE}_{n-1} \left( \frac{E^2}{E - \Delta E} \right) P(\Delta E) d\Delta E$$

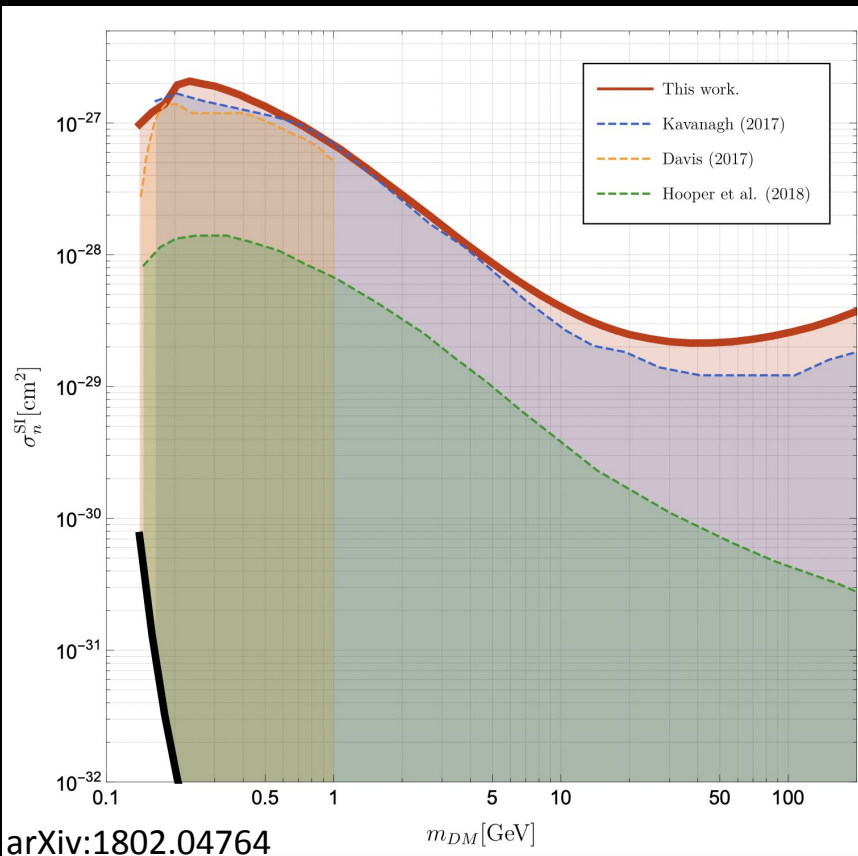
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$$\frac{dN}{dE}_{total}(z, E) = \sum_{n=0}^{\infty} \int_z^{\infty} P_n(z') dz' \frac{dN}{dE}_n(E)$$

# Velocity Distribution



We want an alternative to both Monte Carlo and straight-line approaches

Hooper & Mcdermott (Green):

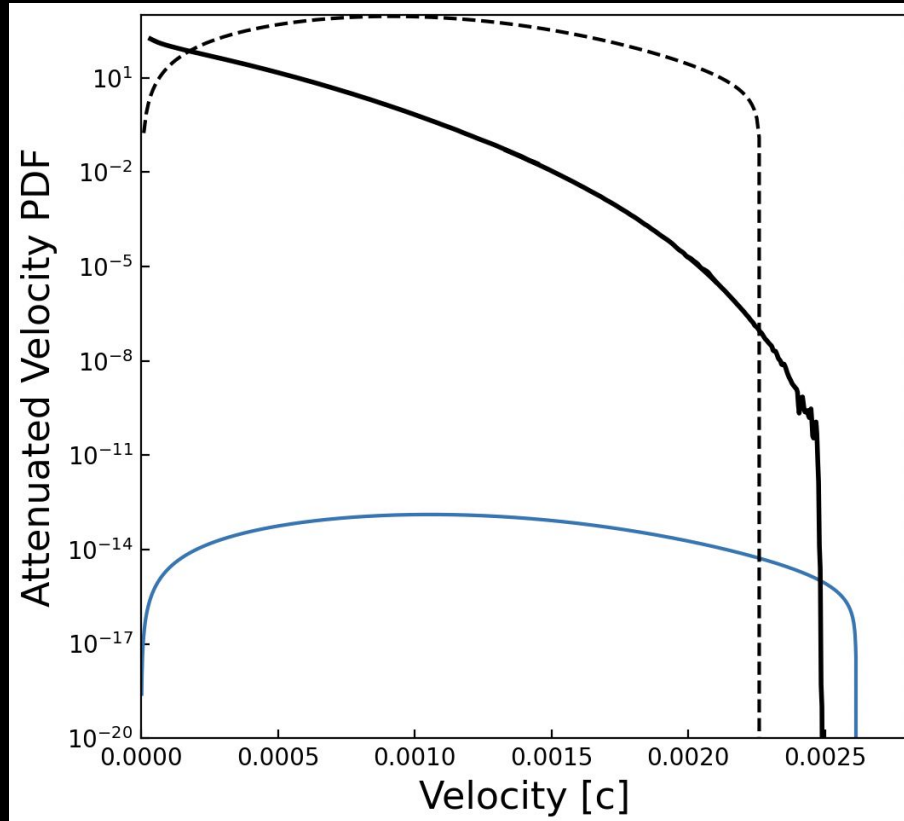
- Compute fraction of dark matter reaching CRESST surface detector without scattering

$$P_{\text{initial}}(x) = \frac{1}{l} e^{-x/l}$$

$$P(z, \theta) = \frac{1}{l \cos(\theta)} e^{-z/(l \cos(\theta))}$$

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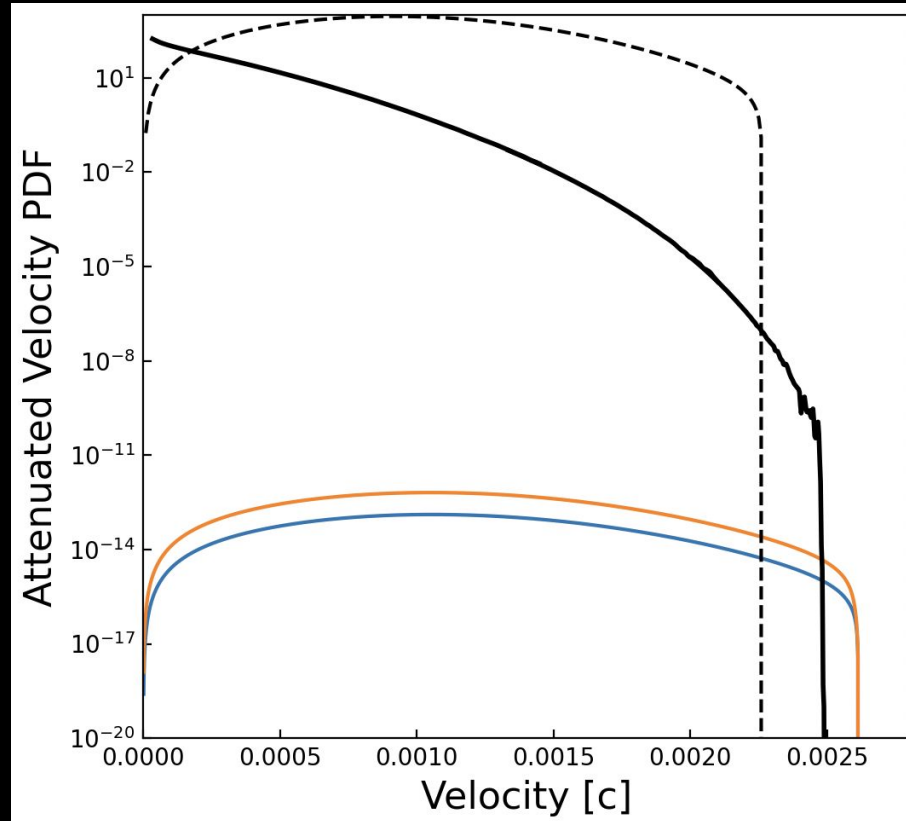
# Velocity Distribution



$M_X = 100 \text{ MeV}$   
 $\sigma_{XN} = 5 \cdot 10^{-30} \text{ cm}^2$   
Depth = 1400 m

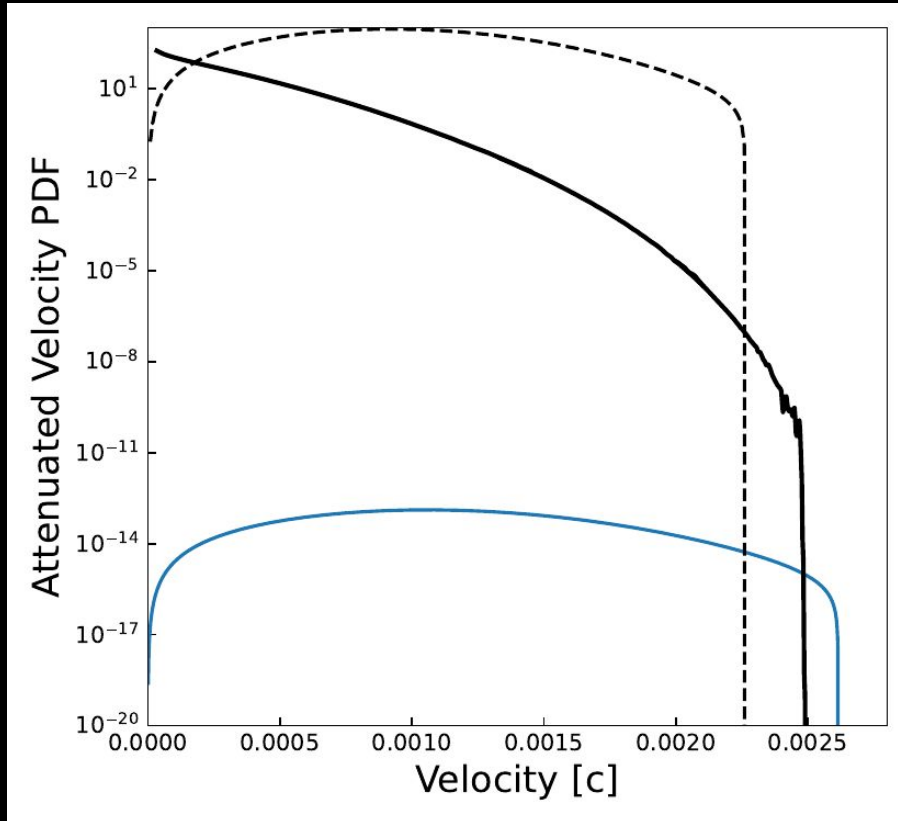


# Velocity Distribution



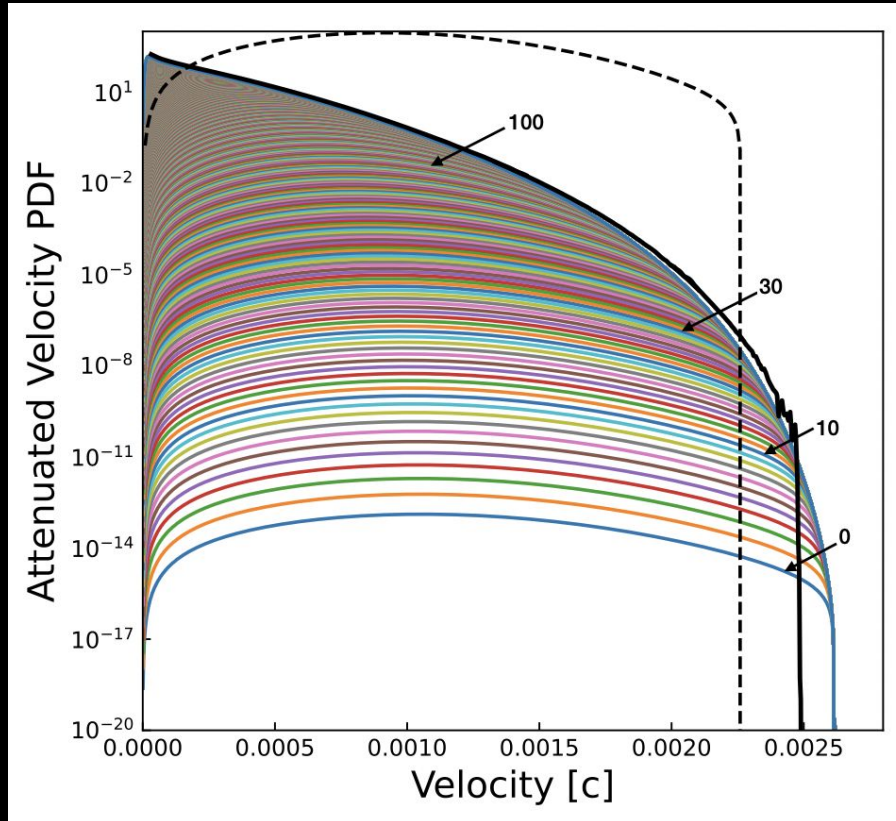
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# Velocity Distribution



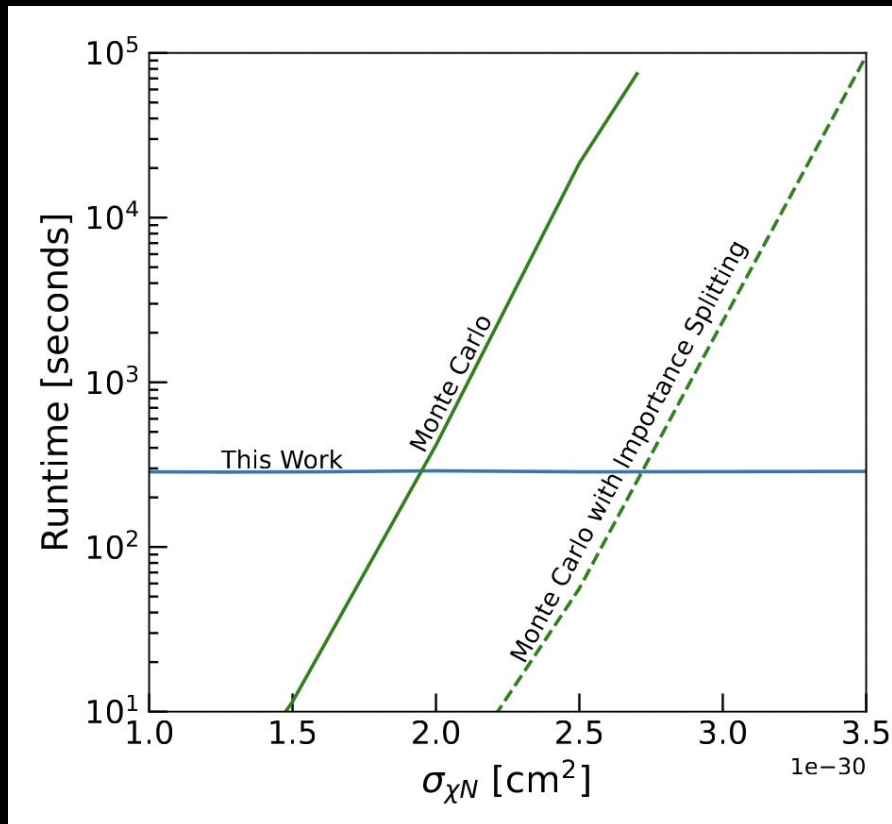
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# Velocity Distribution



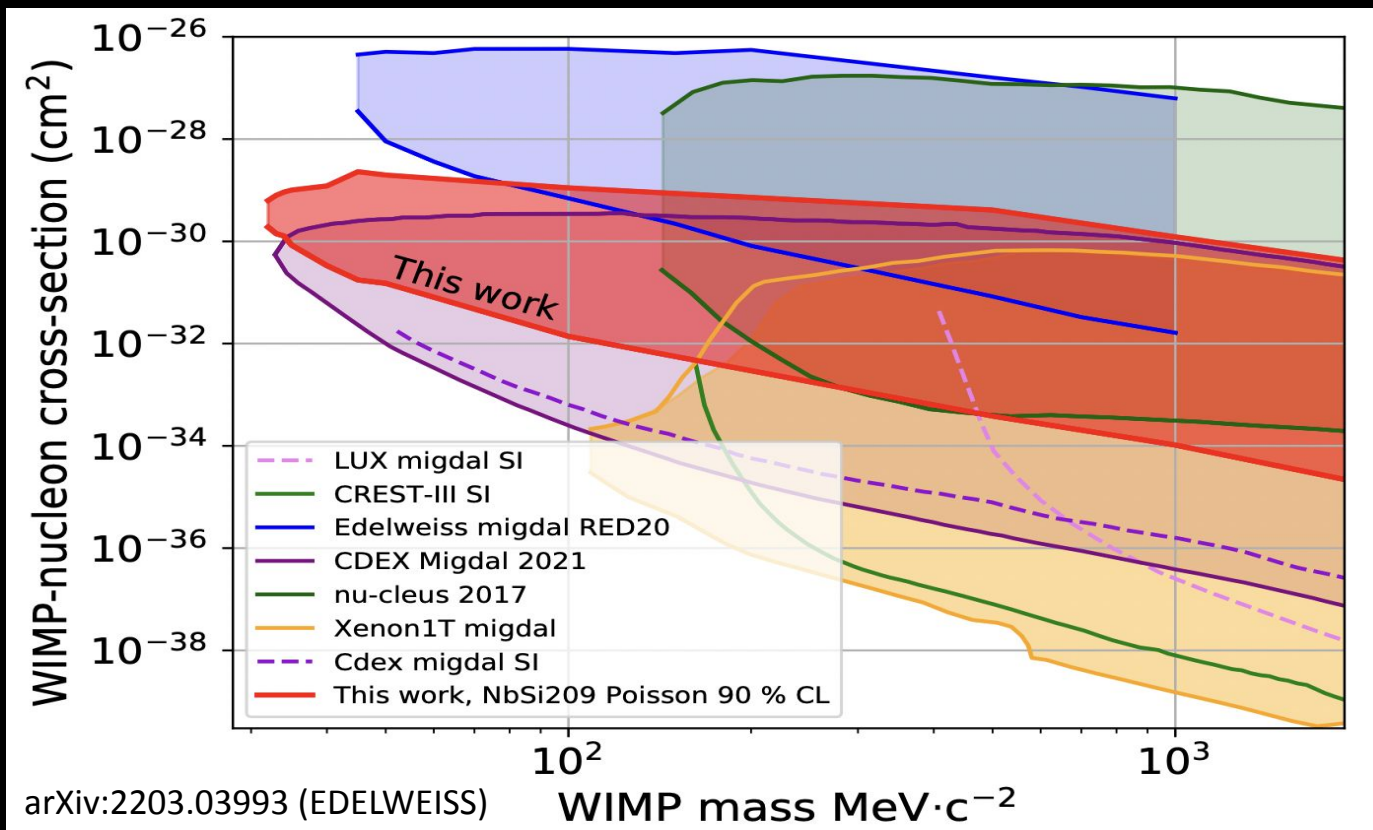
$M_X = 100 \text{ MeV}$   
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Depth = 1400 m

# Runtime Comparison

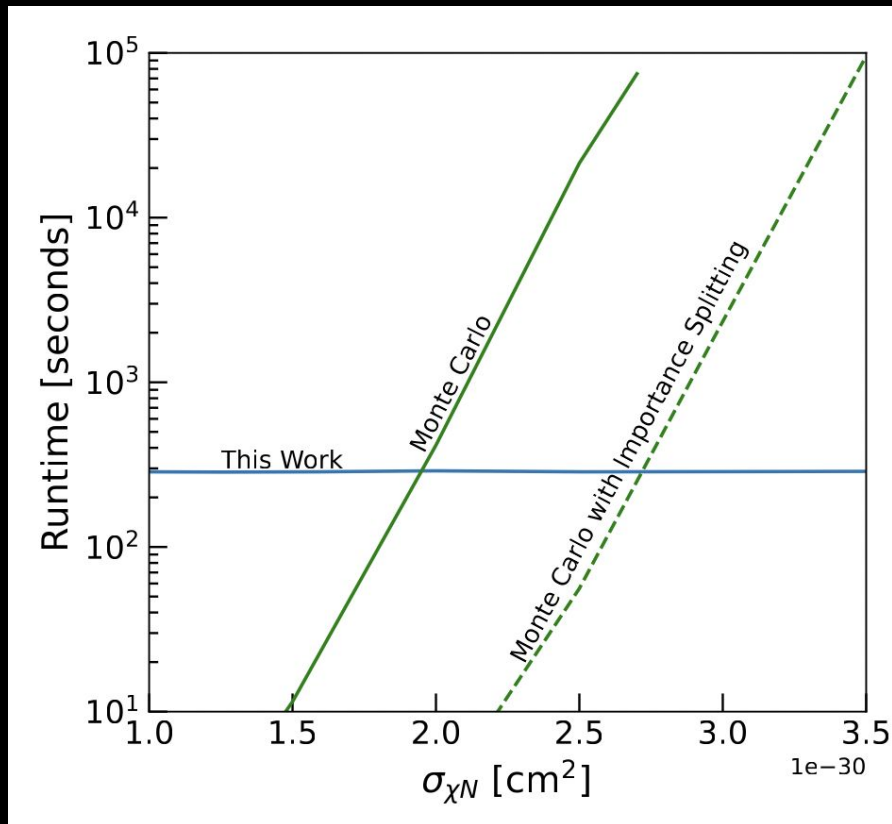


$M_\chi = 200 \text{ MeV}$   
Depth = 1400 m

# Attenuation: Straight Line Approximation



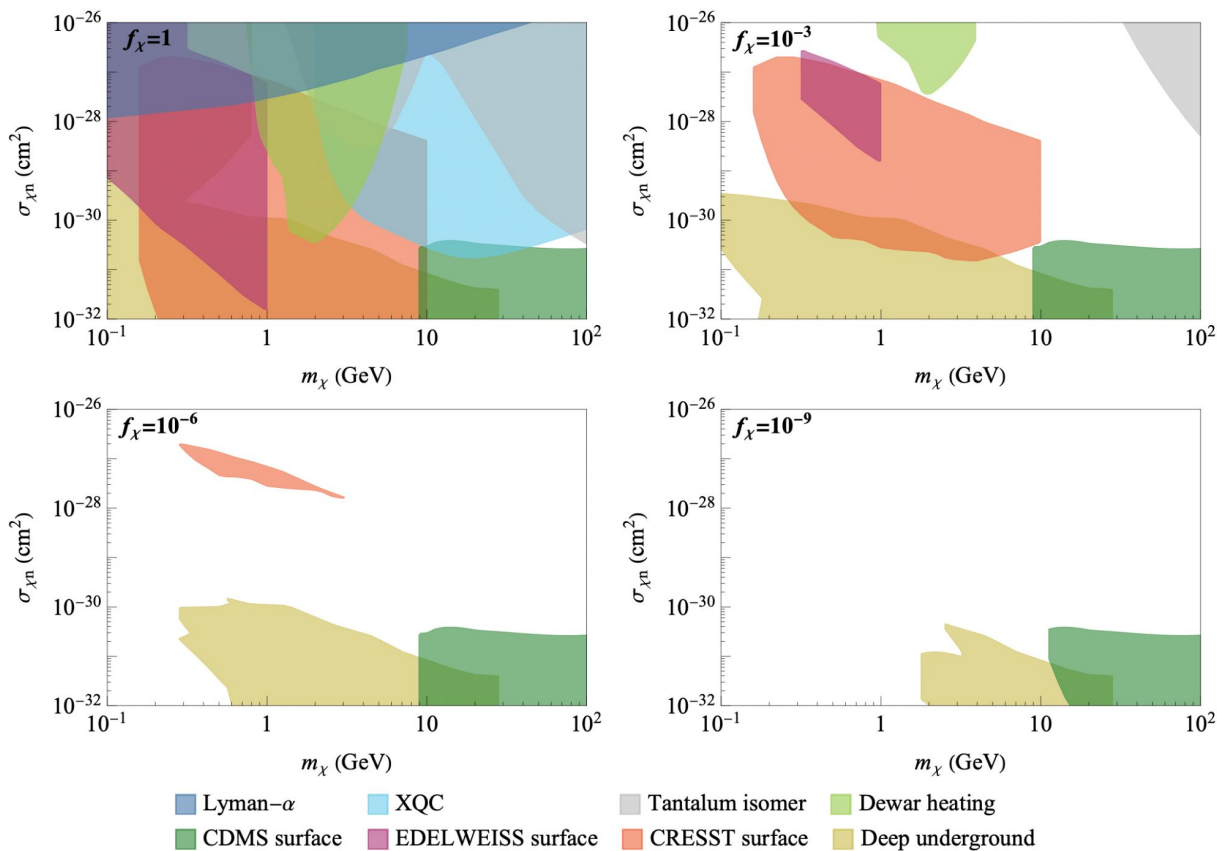
# Runtime Comparison



$M_\chi = 200 \text{ MeV}$   
Depth = 1400 m

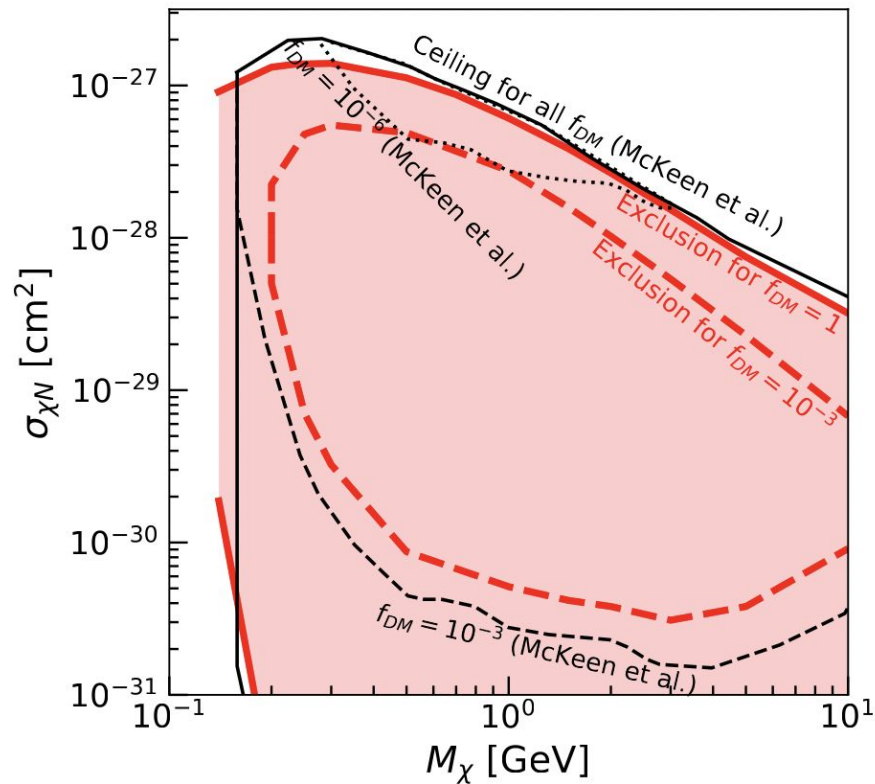
# Recomputing Constraints on Sub-Dominant Dark Matter

# Sub-Dominant Dark Matter

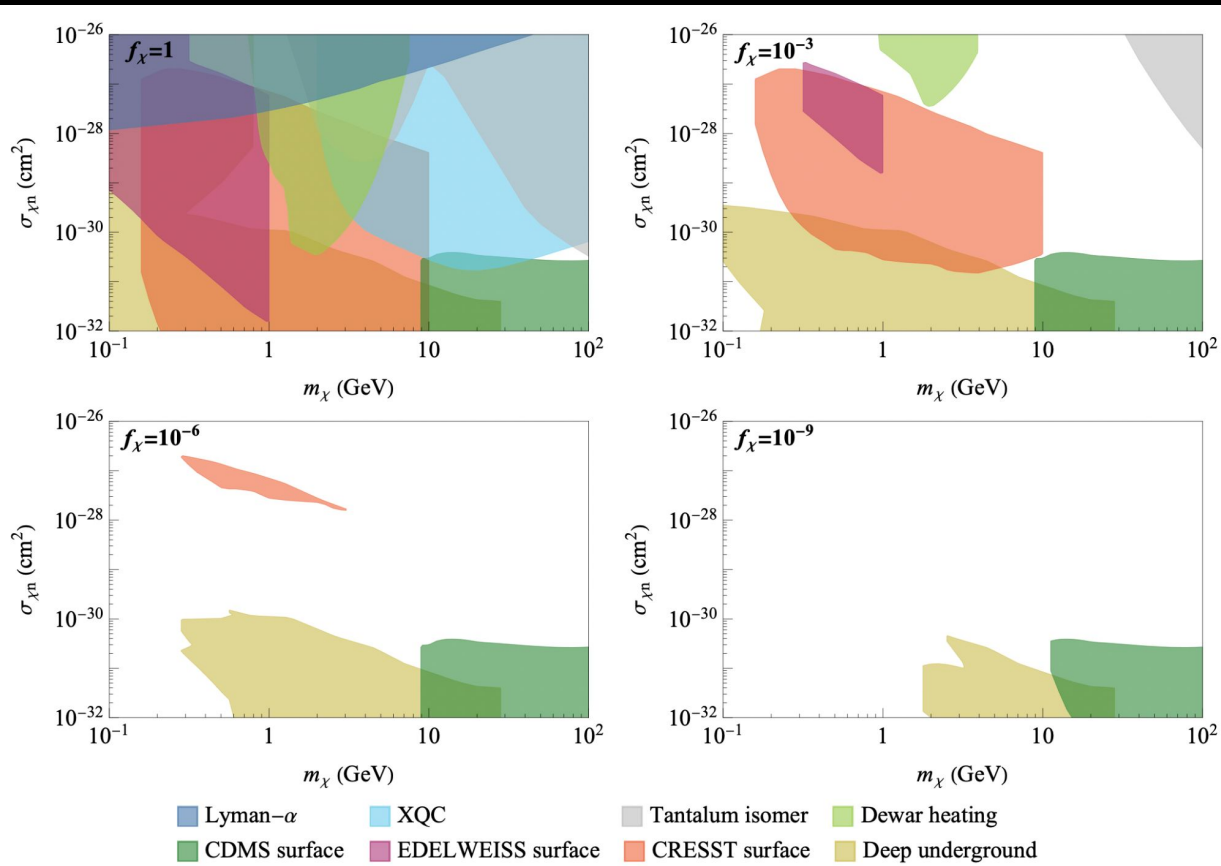




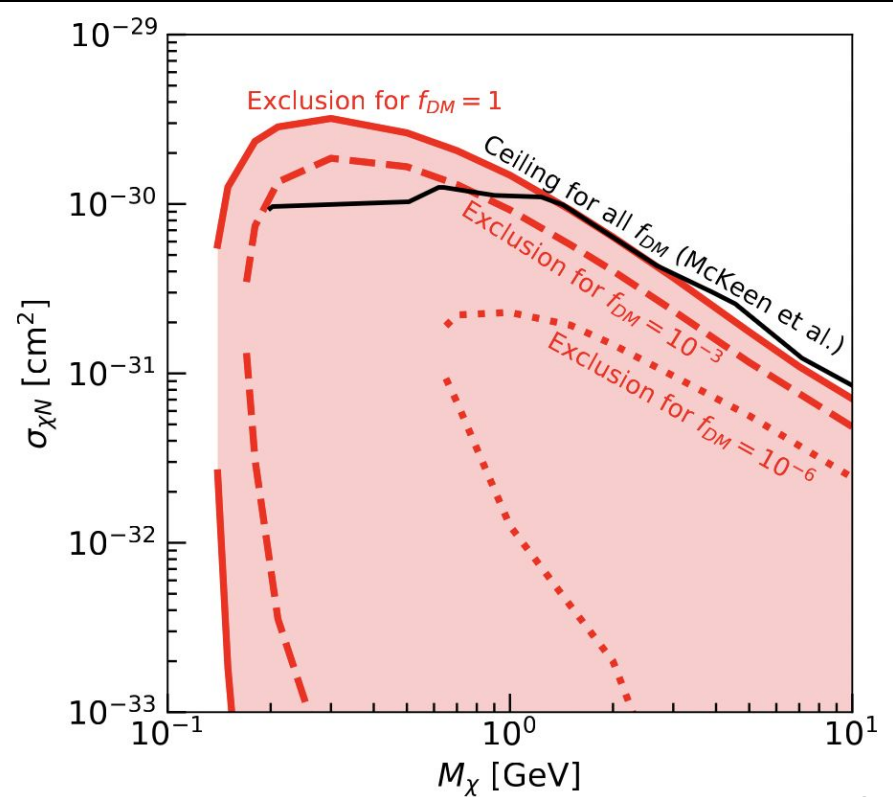
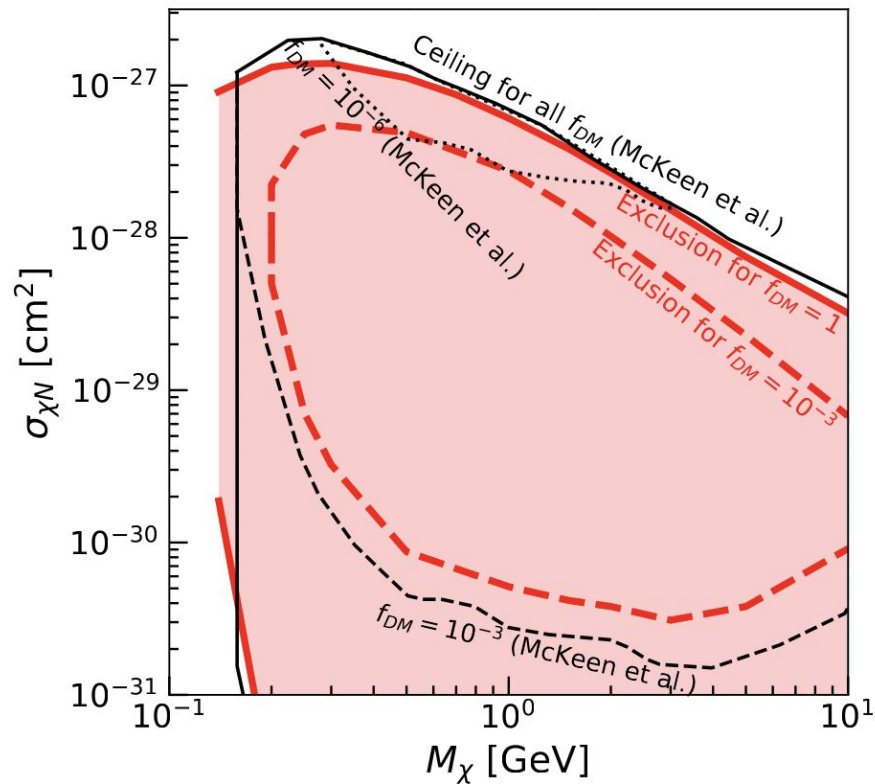
# Sub-Dominant Dark Matter



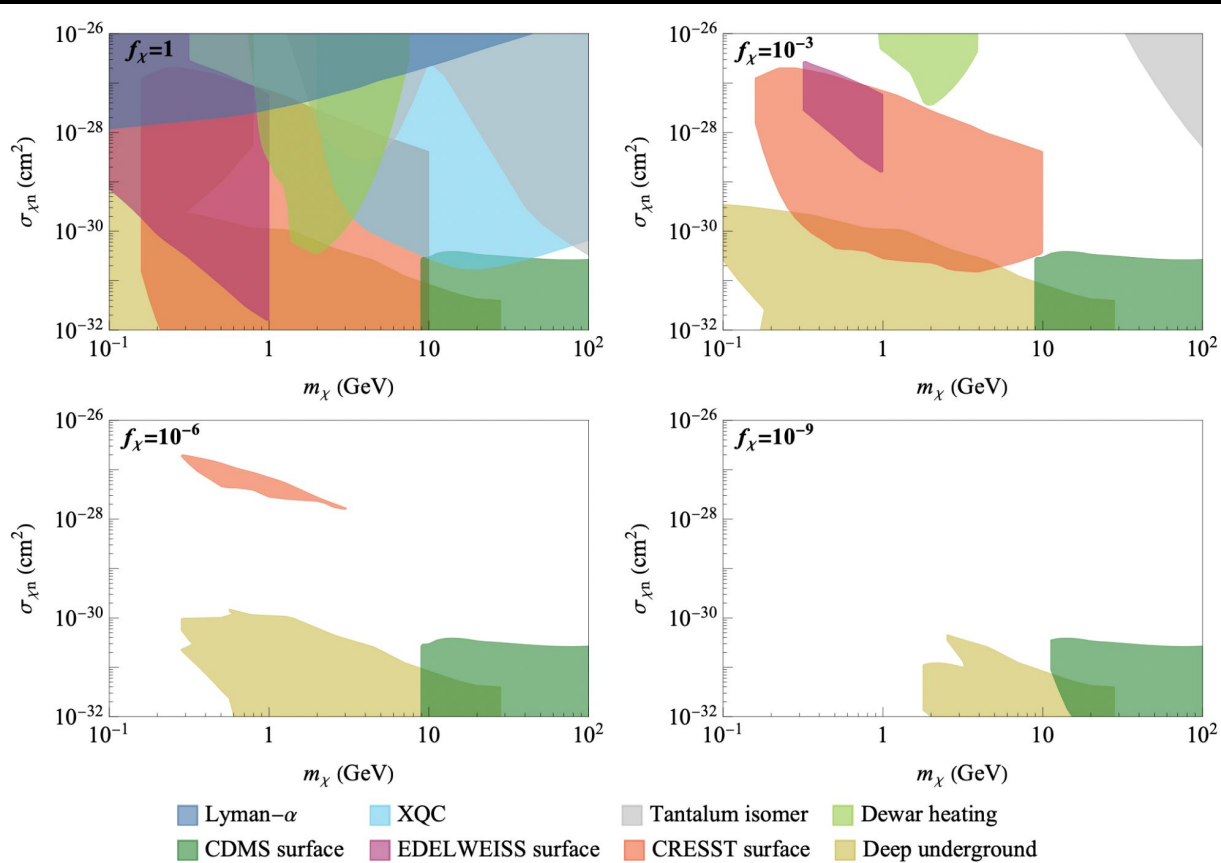
# Sub-Dominant Dark Matter



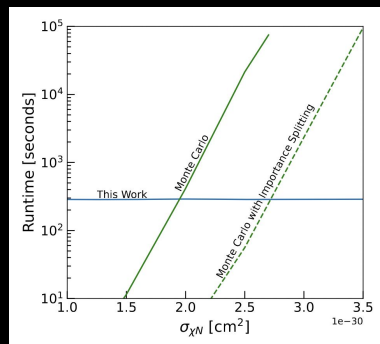
# Sub-Dominant Dark Matter



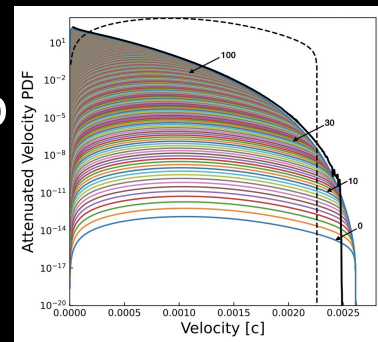
# Sub-Dominant Dark Matter



For light DM, this approach approximates Monte Carlo results much better than the straight line approximation

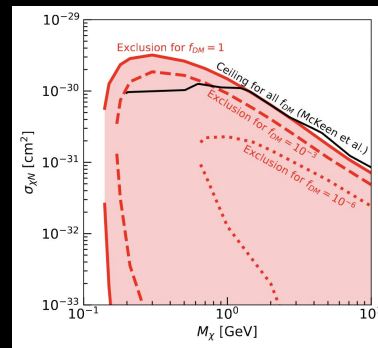


For large cross sections, this approach is much faster than Monte Carlo simulations



Can be used to efficiently recompute/recast exclusion regions from light dark matter searches

- Code: <https://github.com/ccapp413/DMpropPublic>



# Thank you!

Many thanks to Timon Emken for details on the DaMaSCUS Monte Carlo code

Thank you to Ivan Esteban for suggestions on speeding up integration

Backup: 500 MeV,  $10^{-30} \text{ cm}^2$

