

Waveform challenges and numerical relativity

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Panelists

- Marta Colleoni (UIB)
- Arnab Dhani (AEI)
- Keefe Mitman (Caltech)
- Lorenzo Pompili (AEI)
- Rodrigo Vicente (IFAE)

Chairs: Alessandra Buonanno (AEI) and Katy Clough (Queen Mary U)

Keefe Mitman
Advances and challenges in
numerical relativity

Challenges in Numerical Relativity (for vacuum GR)

What we need:

- simulate 9d parameter space
 - ⇒ mass ratio (1), spins (6), eccentricity (2)
- both efficiently and accurately,
 - ⇒ improve mismatches by a factor of ~ 10
 - ⇒ arXiv:1912.10055, 2006.04272
- extract physically-meaningful waveforms,
 - ⇒ ~~extrapolation~~
 - ⇒ evolution
 - Cauchy-characteristic evolution
 - Cauchy-characteristic matching
- and extend waveforms to be “infinite”
 - ⇒ by hybridizing with perturbative solutions
 - PN/PM/EOB and QNMs/tails

Code	Open Source	Catalog	Formulation
AMSS-NCKU [43–46]	Yes	No	BSSN/Z4c
BAM [47–49]	No	[18]	BSSN/Z4c
BAMPS [50, 51]	No	No	GHG
COFFEE [52, 53]	Yes	No	GCFE
Dendro-GR [54–56]	Yes	No	BSSN/CCZA
Einstein Toolkit [57, 58]	Yes	No	BSSN/Z4c
*Canada [59–62]	Yes	No	BSSN
*Illinois GRMHD [63]	Yes	No	BSSN
*LazEv [37, 64]	No	[65–68]	BSSN+CCZA
*Lean [69, 70]	Partially	No	BSSN
*MAYA [71]	No	[71]	BSSN
*NRPy+ [72]	Yes	No	BSSN
*SphericalNR [73, 74]	No	No	spherical BSSN
*THC [75–77]	Yes	[18]	BSSN/Z4c
ExaHyPE [78]	Yes	No	CCZA
FIL [79]	No	No	BSSN/Z4c/CCZA
FUKA [80, 81]	Yes	No	XCTS
GR-Athena++ [82]	Yes	No	Z4c
GRChombo [83–85]	Yes	No	BSSN+CCZA
HAD [86–88]	No	No	CCZA
Illinois GRMHD [89, 90]	No	Yes	BSSN
MANGA/NRPy+ [91]	Partially	No	BSSN
MHDuet [92, 93]	No	No	CCZA
SACRA-MPI [94]	No		BSSN+Z4c
SpEC [95, 96]	No	[96, 97]	GHG
SpECTRE [98, 99]	Yes	No	GHG
SPHINCS_BSSN [100]		No	BSSN

Challenges in Numerical Relativity (for vacuum GR)

What we have:

➤ simulate 9d parameter space

⇒ mass ratio (1), spins (6), eccentricity (2)

➤ both efficiently and accurately,

⇒ improve mismatches by a factor of ~ 10

⇒ arXiv:1912.10055, 2006.04272

➤ extract physically-meaningful waveforms,

⇒ ~~extrapolation~~

⇒ evolution

○ Cauchy-characteristic evolution

○ Cauchy-characteristic matching

➤ and extend waveforms to be “infinite”

⇒ by hybridizing with perturbative solutions

○ PN/PM/EOB and QNMs/tails

● ($q \lesssim 15$, $|\chi| \lesssim 0.8$, not much eccentricity + precession)

● (no significant changes since arXiv:1912.10055, but SpECTRE should hit this)

● (fails to capture memory)

● (working in SpECTRE!)

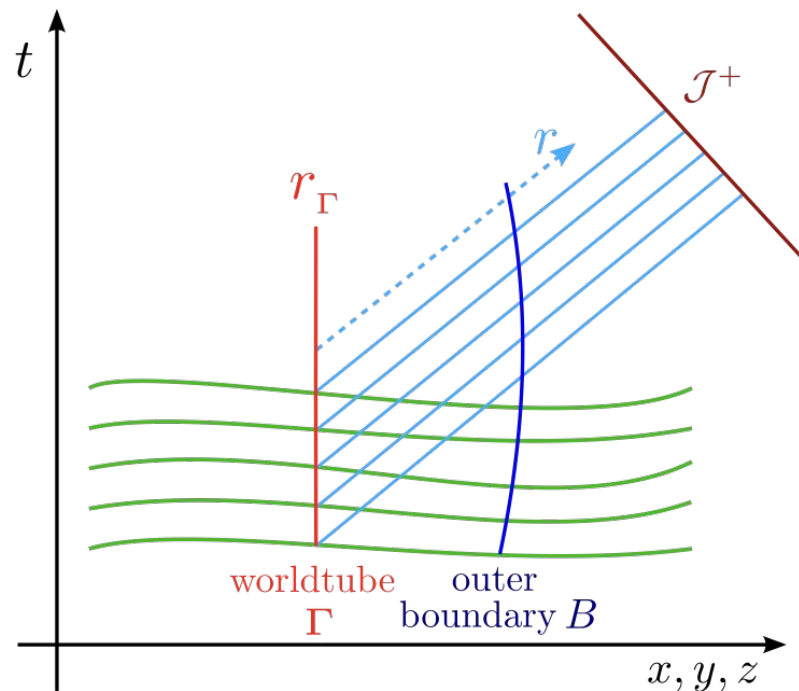
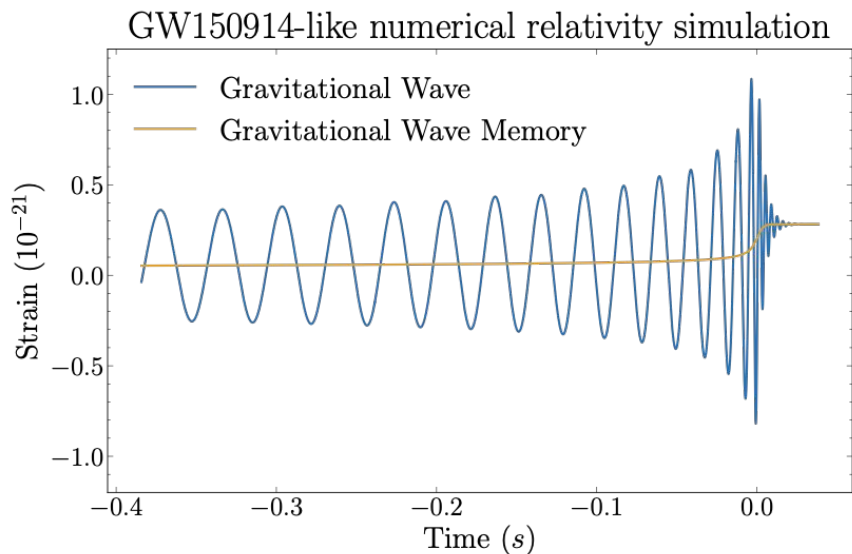
● (no BBHs, but in SpECTRE)

● (PN/PM/EOB w/ BMS!)

● (still figuring out tails)

Advances in Numerical Relativity: CCE and Memory

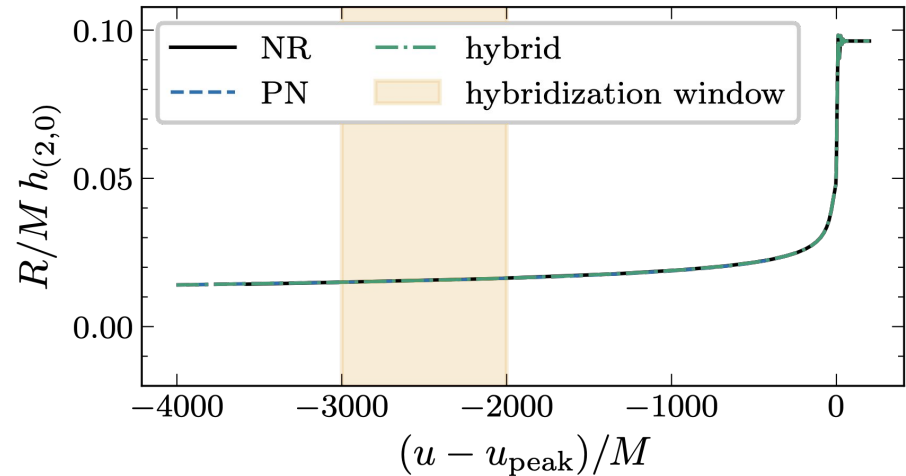
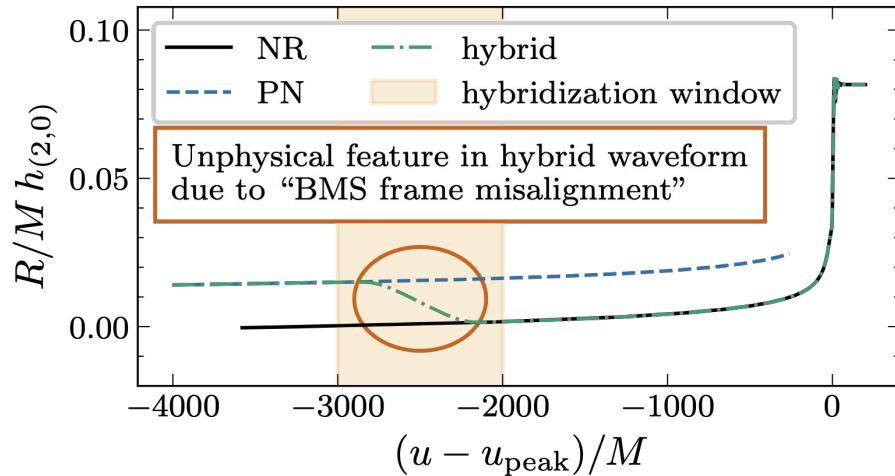
- Cauchy-characteristic Evolution (arXiv:2007.01339 and 2110.08635)
 - ⇒ After a Cauchy evolution, use worldtube data for a 2nd evolution on null slices
 - ⇒ Compactify radial coordinate to include null infinity on computational grid



Credit: N. Bishop, L. Rezzolla (2016)

Advances in Numerical Relativity: BMS Frame Fixing

- Coordinate freedom at future null infinity isn't just the Poincaré group, it's the BMS group = Poincaré group + supertranslations
 - ⇒ Needs to be accounted for when comparing waveforms, i.e., when performing convergence tests or hybridizing
 - ⇒ arXiv:1509.00862, 2208.04356, 2405.08868,



Upcoming Numerical Relativity Advances

- First BBH with SpECTRE Code
- SXS Catalog update with CCE waveforms and BMS frame fixing
- More efficient and accurate surrogate models for
 - ⇒ $q \leq 8$, precessing (extension of arXiv:1905.09300 and 2306.03148)
 - ⇒ eccentricity (extension of arXiv:2101.11798)
- First BBH with CCM (Cauchy-characteristic matching)
 - ⇒ CCM = CCE, but run at the same time as the Cauchy evolution, so more backscattering physics is captured
 - ⇒ arXiv:2308.10361
- And more?

Lorenzo Pompili
Advances and challenges in
waveform modelling

Waveform modelling challenges

Accuracy

Inaccuracies in waveform models can lead to biases in the inferred binary parameters or be misinterpreted as GR deviations.
For XG/LISA imperfect subtraction of high SNR signals can contaminate the analysis of other overlapping signals.

Parameter space coverage

Waveform models need to cover the full (9D) parameter space of expected binaries: mass ratio (1), spins (6), eccentricity (2) for BBHs in vacuum GR. Signals from unmodelled parts of parameter space can be missed or interpreted incorrectly.

Efficiency

Parameter estimation of a single event requires $10^6 - 10^8$ waveform evaluations with standard methods
=> models need to generate waveforms in milliseconds.

State-of-the art: quasi-circular orbits

State-of-the art models for BBHs in **quasi-circular orbits**, with **generic spins** and higher-modes.

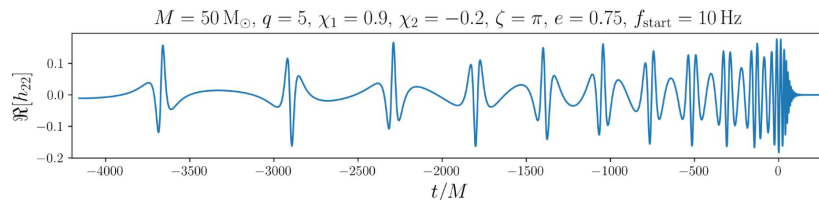
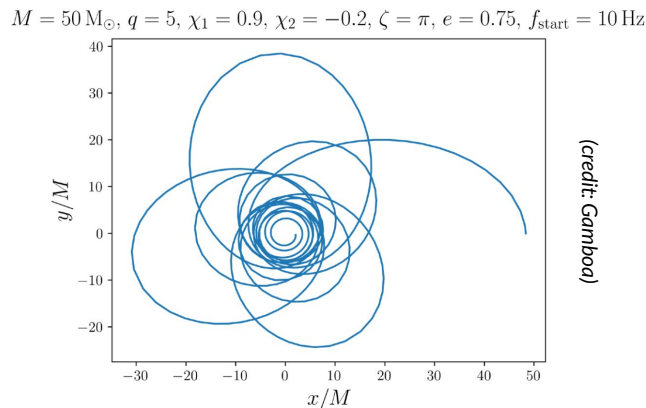
Different families, based on synergy between **analytical** and **numerical relativity** (NR), addressing trade-offs in different ways.

- **Effective-one-body (EOB)**: semi-analytical models calibrated to NR. Model binary dynamics and waveform => “easy” to include extra physics. Integrate ODEs numerically => speed is challenging.
 - **SEOBNRv5PHM** [Ramos-Buades+23, Pompili+23, Khalil+23, van de Meent+23].
 - **TEOBResumS-Giotto** [Akcaay+21, Gamba+22, Nagar+23].
- **IMRPhenom**: piecewise closed-form expressions for the waveform calibrated to PN/EOB - NR hybrids => fastest models available, slightly less accurate than EOB.
 - **IMRPhenomXPHM** [Pratten+20, García-Quiros+20]: frequency-domain. Recent updates include calibration to spin-precessing NR (**IMRPhenomXO4a**) [Thompson+23, Ghosh+23, Hamilton+21] and improved spin-precession prescriptions [Colleoni+23, Yu+23].
 - **IMRPhenomTPHM** [Estellés+21]: time-domain, improved modeling of spin-precession.
- **NR surrogate**: directly interpolate NR waveforms => most accurate models, but limited to NR length and parameter-space coverage.
 - **NRSur7dq4** [Varma+19]: $q \leq 4$ and spin magnitudes $\chi_1, \chi_2 \leq 0.8$, covering around 20 orbits before merger.

Parameter space challenge: generic orbits

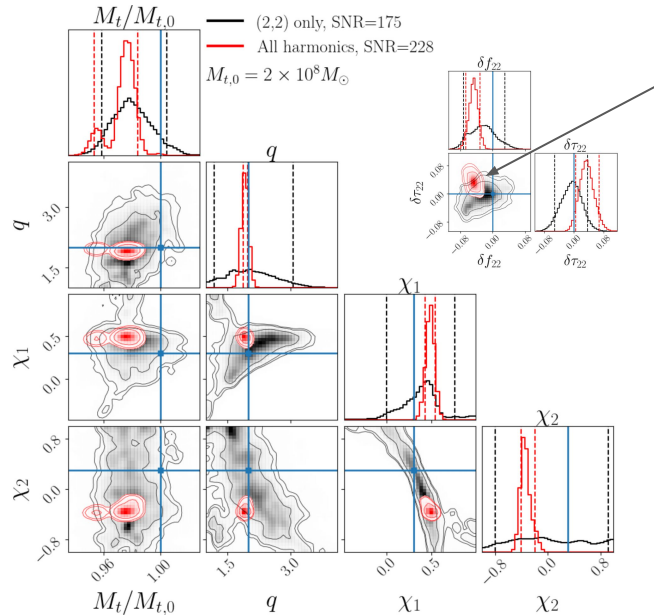
GWs efficiently circularize binary systems, so the measurement of **orbital eccentricity** can provide robust evidence for dynamical binary formation channels [Zevin+21].

- Significant progress in modeling binaries in eccentric/scattering orbits: waveform models for BBHs in **generic orbits** with **aligned-spins** and higher-modes have reached a mature stage:
 - SEOBNRv5EHM [Gamboa+] - SEOBNRv4EHM [Ramos-Buades+21].
 - TEOBResumS-Dali [Nagar+24, Nagar+21, Chiamarello+20].
 - Ongoing efforts to include eccentricity in the IMRPhenom family [Ramos-Buades+, Planas+].
- Models lack calibration to eccentric NR, and employ a quasi-circular merger-ringdown waveform (e.g. assume the system has circularized by the time the binary merges) => only accurate up to moderate eccentricities.
 - First works studying the merger-ringdown morphology of eccentric systems with NR [Albanesi+23, Carullo+23, Carullo 24, Nee+].
- First progress towards modeling BBHs in **generic orbits** with **generic spins** [Gamba+24, Liu+23].
 - Important to model both effects to confidently distinguish eccentricity from spin precession [Romero-Shaw+22].



Accuracy challenge

A **significant improvement** in the **accuracy** of current waveform models (by \sim two orders of magnitude [Pürrer+19]) and of NR simulations is needed to **avoid systematic biases** in parameter estimation with high SNR signals in XG/LISA [Dhani+24, Kapil+24, Toubiana+23].

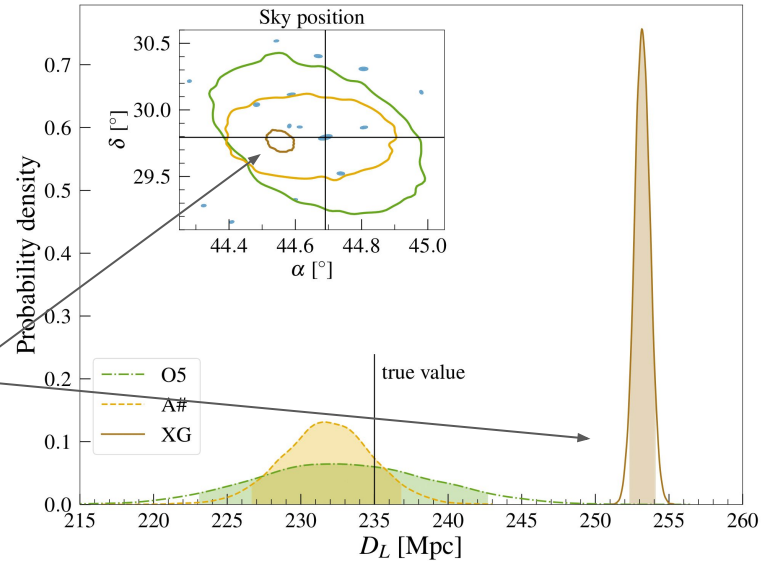


False GR deviation in the quasi-normal-mode frequency and decay time of the ringdown due to systematics.

Biased measurement of Hubble-Lemaître parameter H_0 due to systematics.

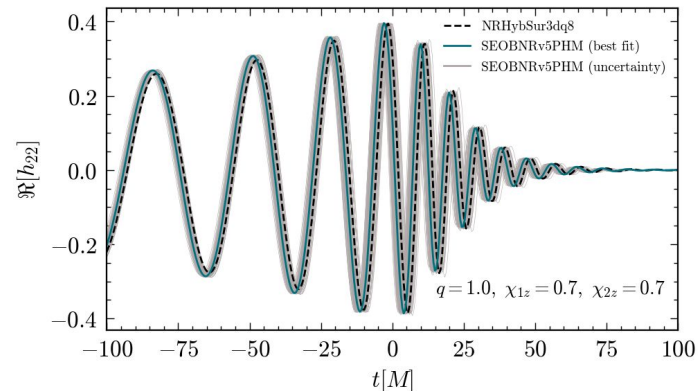
$$m_1 = 23.2 M_\odot, m_2 = 2.6 M_\odot \quad \chi_1 = 0.7, \chi_2 = 0.4$$

$$\text{SNR}_{\text{O5}} = 75, \text{SNR}_{\text{A\#}} = 137, \text{SNR}_{\text{XG}} = 1040$$



How to address the accuracy challenge?

- **More NR simulations:** challenge in covering 9D parameter space with the required length and increased numerical resolution [Ferguson+21, Jan+23].
- **More analytical information:**
 - Current models primarily rely on the post-Newtonian (PN) approximation (weak-field, small-velocity expansion).
 - Develop innovative ways to incorporate information from other perturbative methods, such as the **gravitational self-force** (small mass-ratio expansion) [Akca+12, Antonelli+20, van de Meent+23] and the **post-Minkowskian** (weak-field, arbitrary velocity expansion) [Damour 16, Antonelli+19, Khalil+22, Damour+22, Retegno+23, Buonanno+24] approximations.
- **Integrate uncertainty estimates into waveform models:**
 - Several proposals: **Gaussian process regression** to interpolate waveform residuals or directly NR waveforms [Moore+14, Doctor+17, Williams+20, Andrade+23, Khan 24], **frequency-dependent amplitude and phase corrections**, as in the case of detector calibration uncertainty [Read 23], introducing **higher-order parameters** to capture currently unknown PN terms [Owen+23].
 - Account for modeling uncertainties by marginalizing over these additional degrees of freedom => parameter estimates with reduced precision (e.g. wider posteriors), but robust even in the presence of systematics.



Marta Colleoni

My highlights and challenges

Waveform models with matter effects

Model(s)	Higher Modes	Precession	Domain	Refs
NRHybSur3dq8Tidal	Yes	No	TD	Barkett+ 19
SEOBNRv4T	No	No	TD	Hinderer+ 16, Steinhoff+ 16, 21
SEOBNRv4T_surrogate	No	No	FD	Lackey+ 18
TEOBResumS-GIOTTO	Yes	Yes	TD/FD	Bernuzzi+ 15, Akcay+ 18, Nagar+ 18, Gamba+ 20,23
PhenomX+NRTidalv*	No	Yes	FD	Dietrich+ 18, Abac+ 23, Colleoni+ 23
SEOBNRv5_ROM+NRTidalv*	No	No	FD	Dietrich+ 18, Pompili+ 23, Abac+ 23
PhenomGSF	No	No	FD	Williams+ 24

} Damour+ 08, Damour&Nagar 09

- Post-Newtonian (PN) phasing with adiabatic tidal effects computed up to 7.5 PN order ([Henry+ 20](#))
- Several models include some treatment of dynamical f-mode tides ([Hinderer +16](#), [Steinhoff+ 16](#), [Schmidt&Hinderer 19](#), [Steinhoff+ 21](#))
- Neutron-star-black-hole binaries: development spurred by observations!
 - TEOBResumS ([Gonzalez+ 22](#))
 - SEOB/Phenom models ([Matas+ 20](#), [Thompson+ 20](#)) with NRTidal phasing + NR-tuned amplitude model ([Pannarale+ 15](#)).
Baselines and physics content being updated!

Challenges/open issues

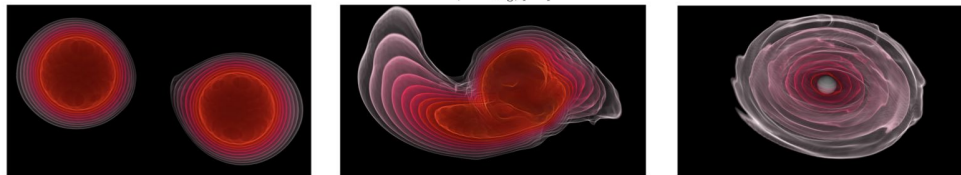
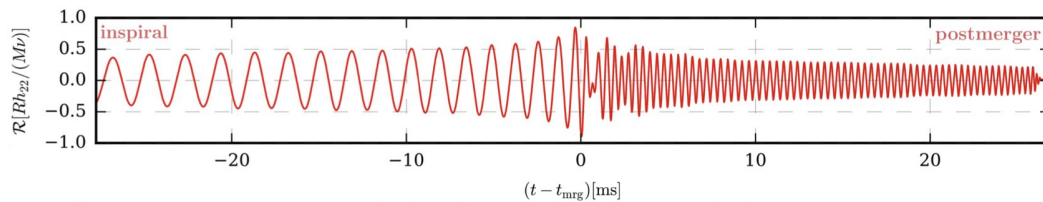
- Non-uniform degree of development of different waveform families (**we are catching up!**)
- Several open NR databases (SACRA, CoRe, SXS) but par space less populated than BBH
- More exotic scenarios explored: dark-matter admixed neutron stars ([Bezares+ 19](#), [Emma+ 22](#)), neutron stars with sub-solar mass companion ([Markin+ 23](#))

Going towards 3G detectors

kHz gravitational waves

Peaks in post-merger spectrum correlated with EOS properties ([Bauswein&Janka 12](#), [Bauswein+ 12](#))

- Salient features captured by post-merger models ([Breschi+ 19](#), [Breschi+ 22](#), [Soultanis+ 22](#), [Puecher+ 23](#), [Tringali+ 23](#))
 - if combined with inspiral-merger model, enable pre/post-merger consistency tests of Quasi Universal Relations ([Breschi+ 23](#))
- Ringdown: from prompt collapse ([Dhani+ 23](#)), “long ringdown” of hypermassive neutron stars ([Ecker+ 24](#))
- Significant challenges on the NR side (thermal effects, neutrino transport, phase transitions, magnetic fields...) ([Palenzuela+ 22](#), [Fields+ 23](#))



[Dietrich+ 20](#)

Increased sensitivity in the inspiral range

- r-modes ([Ma+ 21](#)), p-g mode instabilities, g-modes ([Ho&Andersson 23](#))...
- Improving understanding/treatment of dynamical tides in existing waveform models ([Steinhoff+ 21](#), [Gupta+ 23](#), [Pnigouras+ 23](#), [Yu+ 24](#)), eccentricity? ([Chirenti+ 16](#), [Gamba&Bernuzzi 22](#), [Dutta Roy&Saini 24](#), [Yu+ 24](#))
- Impact of BBH baseline

Arnab Dhani

My highlights and challenges

☁ What do you think is the biggest data analysis challenge for future detectors?

Complexity of data

Unknown number of signals

Burst-like searches

cats

Non stationarity

number of signals

Quantity of signals

Sensitivity estimates

Overlapping signals

Glitches

Unmodelled background

Nonstationarity

Waveform systematics

Weak modeled events

Understanding detector no

Overlap

Testing GR

Overlaps

Population analysis

Unknown unknowns

Find golden binaries

population modelling

Noise modelling

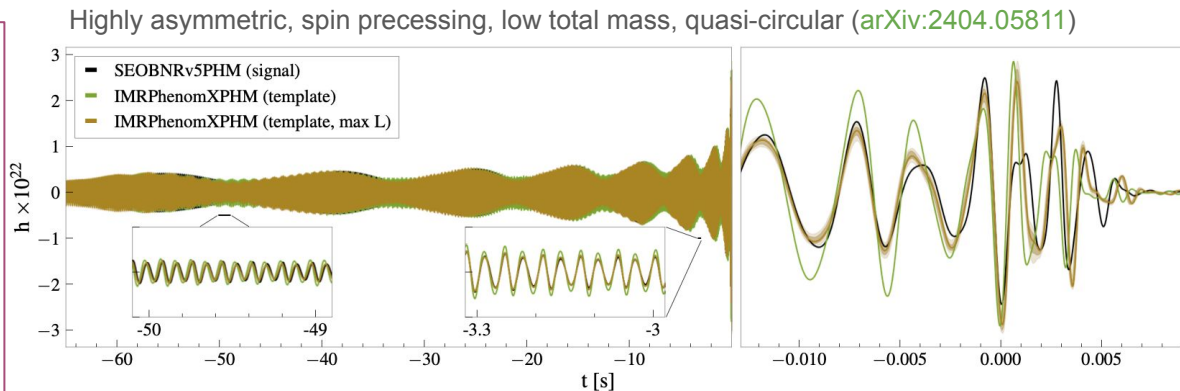
Add response

Systematic bias and waveform accuracy

Inaccurate waveforms \Rightarrow inaccurate parameter estimation \Rightarrow inaccurate inference of GW physics

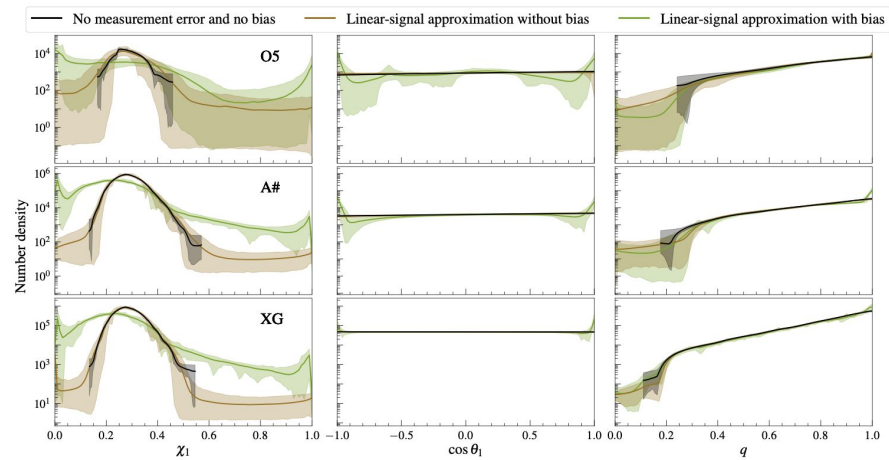
What does it look like?

- SNR ~ 75
- Mismatch $\sim 4\%$
- Systematic bias:
 - $M \sim 3\%$, $q \sim 6\%$, $\chi_p \sim 13\%$



How does it impact population inference?

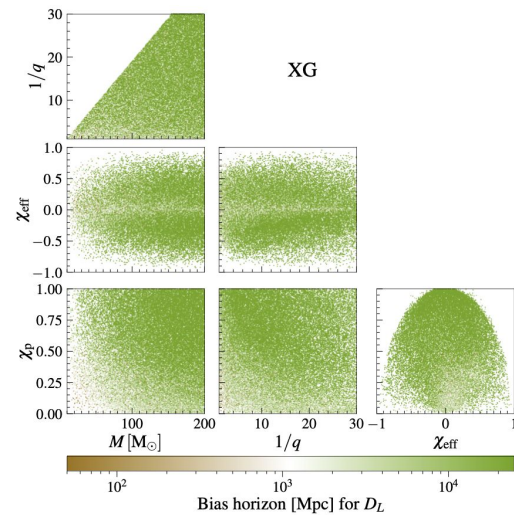
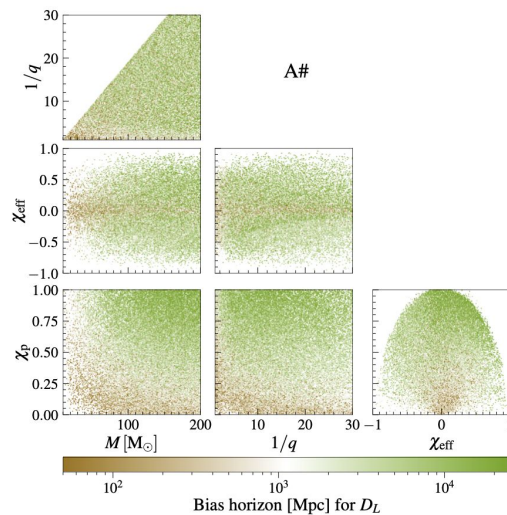
- Not only individual event science cases are affected (as in Lorenzo's slides), but also population inference
- Primary spin magnitude distribution is biased even in A#



Systematic bias and waveform accuracy

What parameter space is affected?

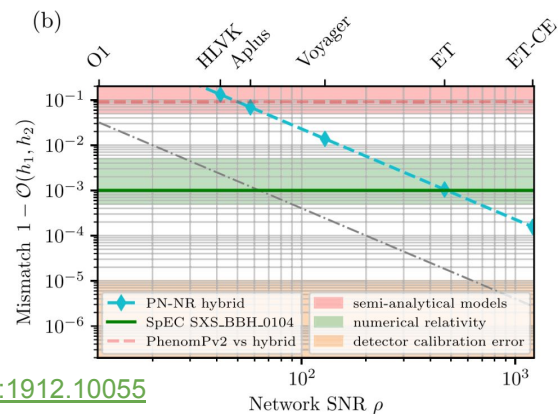
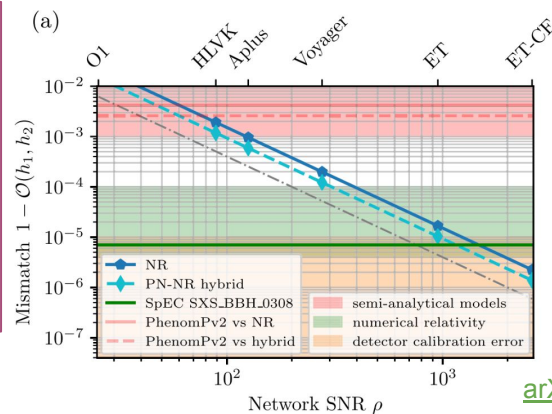
- Plots show the distance *upto* which a binary's DL parameter will be biased.
- A# has a BBH detection efficiency of 50% at $z = 1$ (DL ~ 7000 Mpc)
- XG has a BBH detection efficiency of 98% at $z = 3$ (DL ~ 25 Gpc)



Ready for what lies ahead?

[arXiv:1912.10055](https://arxiv.org/abs/1912.10055)

- 100k BBHs per year, 10k with SNR > 100
- 25k cumulative SNR per year



[arXiv:1912.10055](https://arxiv.org/abs/1912.10055)

Rodrigo Vicente
Environmental Effects

How to model environmental effects (dynamical friction, accretion, etc) on GW waveforms?

Most effects are effective in early inspiral (Newtonian modelling is enough)

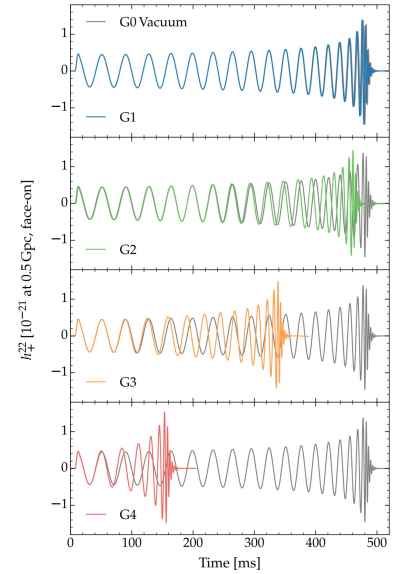
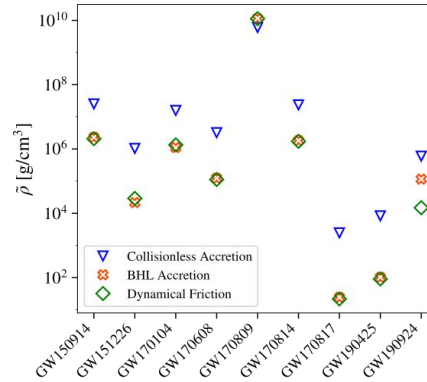
1. find how orbital dynamics is modified [(semi)analytic model vs numerical simulations]
2. use SPA to get waveform in Fourier domain

$$A \approx \frac{h[t(f)]}{2\sqrt{\dot{f}}}$$

$$\Phi \approx 2\pi f t_c - \varphi_c - \frac{\pi}{4} - 2\pi \int_f^\infty df' \left(\frac{f - f'}{\dot{f}} \right)$$

How empty is the vacuum of LVK binaries?

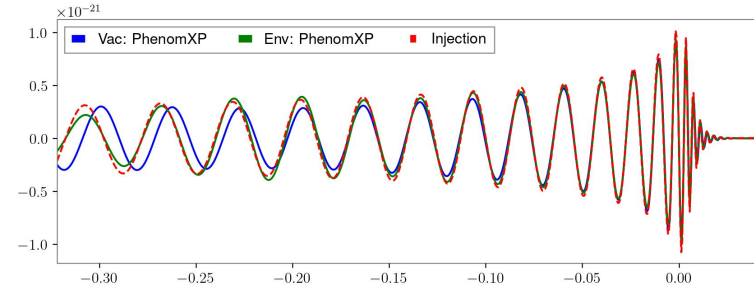
- No evidence for environments (but $< 10^4 - 10^7 \text{ g/cm}^3$)
- Disfavour formation via dynamic fragmentation and other exotica (superradiant boson clouds)



Fedrow et al. [1704.07383]

How accurate is this modelling?

- Quite good! [overlap of 0.99, $\log B > 2$]
- But it may be worse for longer signals (near-future detectors), or more complex environments
- Much harder modelling if environmental effects drive the inspiralling (as may happen in EMRIs)

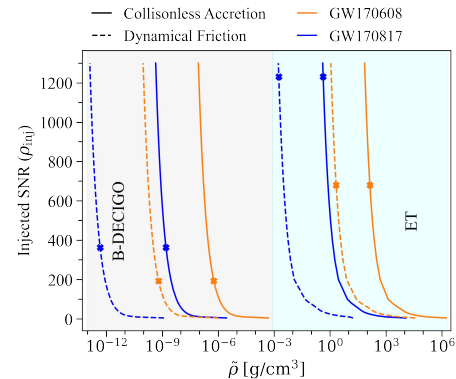


Why should we care?

- Near-future GW observations will be sensitive to the densities of astrophysical environments

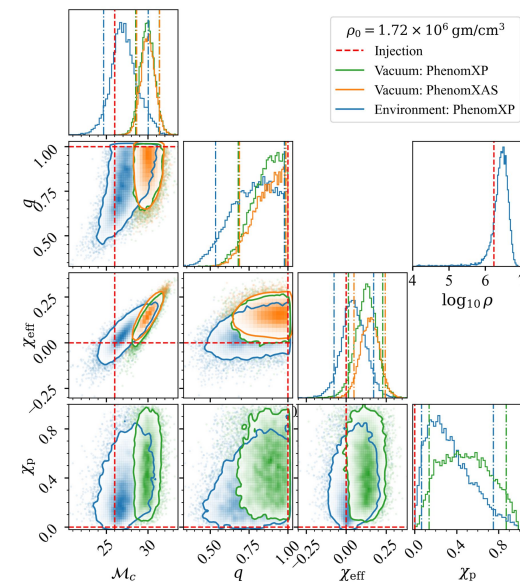
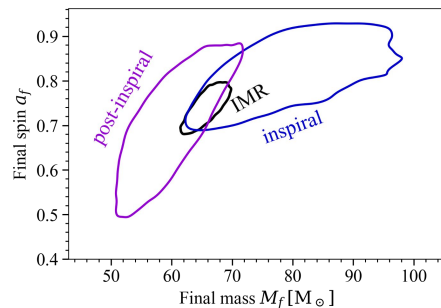
$$\rho_{\text{thin}} [\text{g}/\text{cm}^3] \lesssim 10^{-1} (10^5 M_{\odot} / m_3)^{7/10}$$

$$\rho_{\text{thick}} [\text{g}/\text{cm}^3] \lesssim 10^{-8} (10^5 M_{\odot} / m_3)$$
- Unmodelled environmental effects will lead to systematics [even when the signal passes (vacuum) consistency tests]



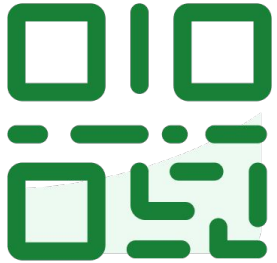
Challenges:

- Environmental effects are effective in early inspiral & accumulate over many cycles [where (NR) simulations are expensive; synergy with analytical methods and data analysis is crucial]
- Many different environments [distinguishability?]
- (Partial) degeneracy with vacuum parameters
- Environmental effects vs higher (vacuum) PNs
- Environmental effects driving the inspiral



Discussion session

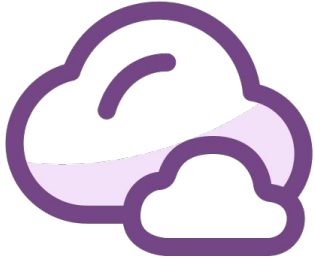
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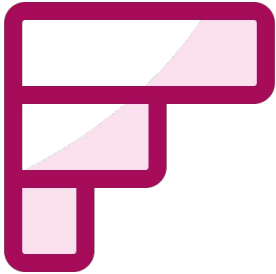
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What's the biggest challenge for numerical relativity and waveform modelling for next generation detectors?

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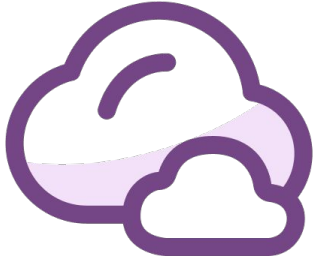
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What do you consider most important missing physics in NR and waveform modelling?

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What is the most important feature of a waveform model for you? (e.g. speed, accuracy, usability...)

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Participant slides
(these points have been
incorporated into the main
talks)

Gravitational Wave Extraction and Interpretation

- Numerical Relativity is the main tool for analyzing the strong-field regime.
- While perturbation theory is used when the configuration is close to a known solution (e.g. Schwarzschild). It is also used to validate the Numerical analyses in some appropriate limit.
- If the initial state of the Compact Object(s) are stationary, then the subsequently emitted Gravitational Waves can be used to infer the main properties of the emitting body.
- Typical methods - Post-Newtonian, Regge-Wheeler, Zerilli, Teukolsky.

Eccentric binary black hole systems

- Detection (or confirmation) of an eccentric GW signal is very close.
- Ongoing effort to improve the control over eccentric NR simulations -> enhancement of eccentric waveform models.
- We now have more mature eccentric, *aligned-spin* waveform models. But models for eccentric, *precessing-spin* systems are still under early developments.
- Waveform models with eccentricity and precession will be fundamental to better understand the possible biases in parameter estimation.