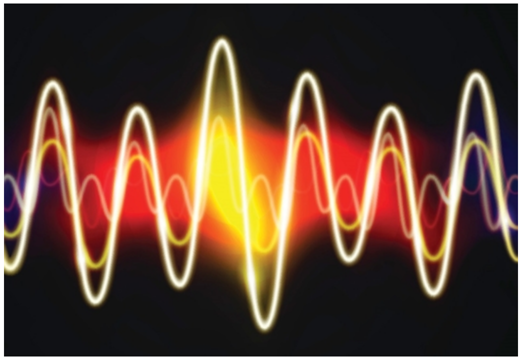
Optical cavities:
From phenomenology to field quantisation

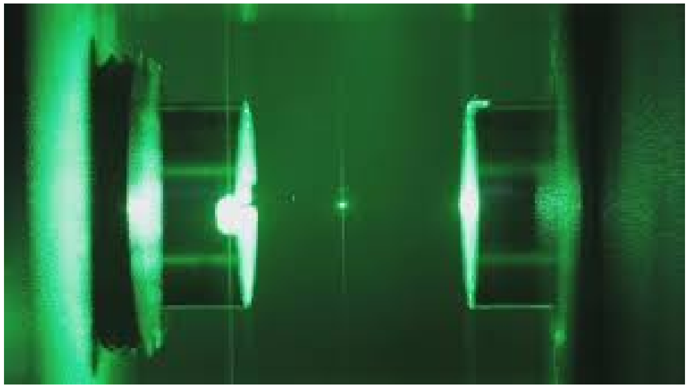


Almut Beige

University of Leeds, United Kingdom

Meeting at King's College London, March 2025

Outlook



1. Single mode cavities
2. The input-output formalism
3. Alternative models of FP cavities

—

4. Local photons
5. A local photon approach to optical cavities
6. Strange effects: Casimir, Doppler and Unruh

—

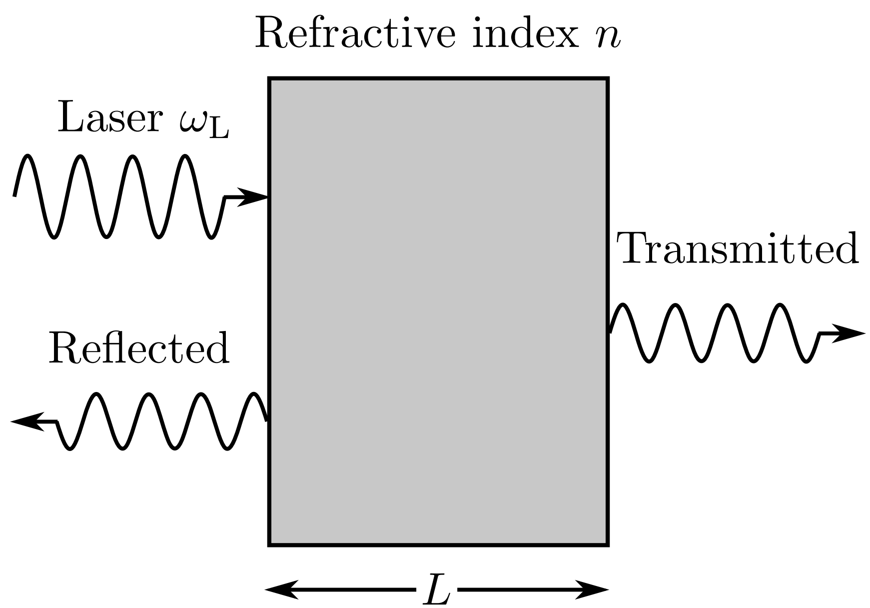
7. Final remarks

I

Single mode cavities

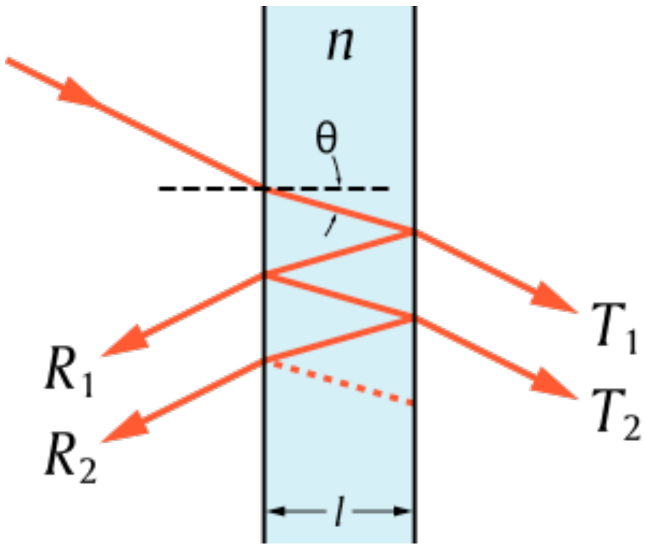
Experimental setup

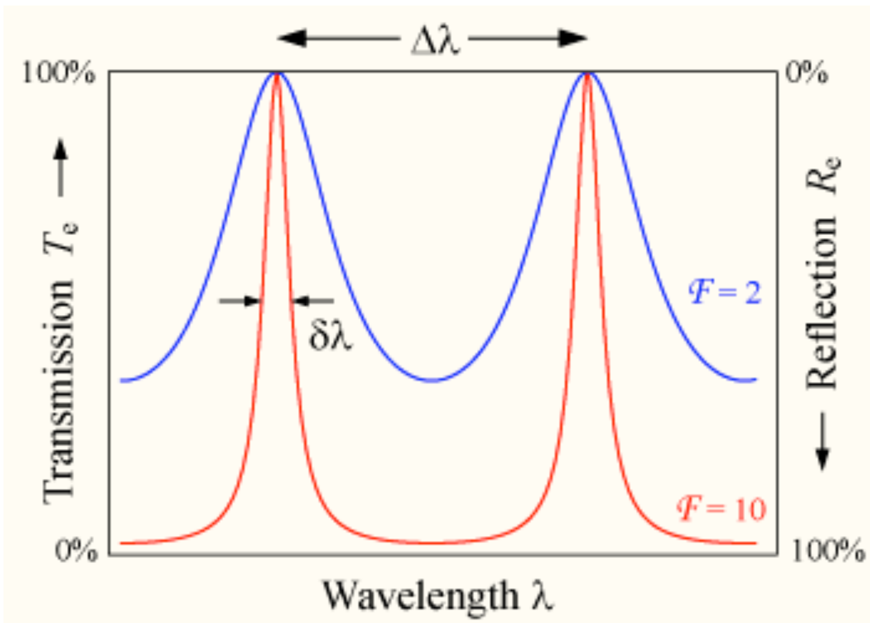
To start, we have a closer look at an experimental setup which can be modelled quantum and classically:



Predictions of Maxwell's equations

Fabry-Perot cavity:





Cavity transmission and reflection rates

These rates depend only on the cavity parameters n and L and on the frequency of the incoming light ω_{L}:

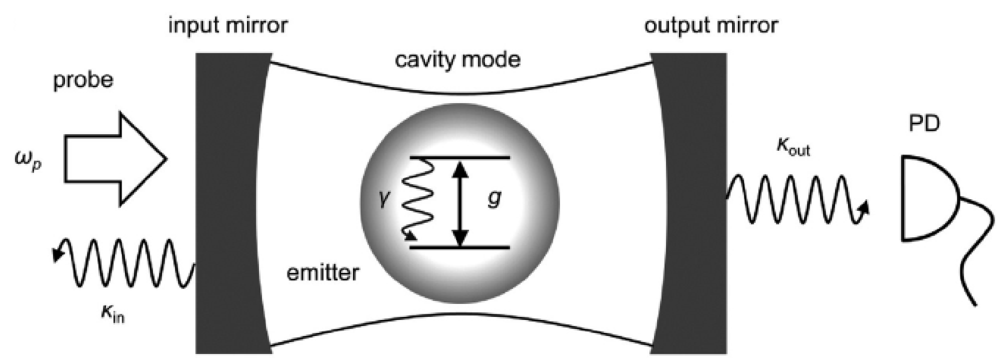
$$\begin{aligned} T_{\mathrm{cav}}(\omega_{\mathrm{L}}) &= \left| \frac{1 - r^2}{1 - r^2\, \mathrm{e}^{2\mathrm{i}nk_{\mathrm{L}}L}} \right|^2 , \\ R_{\mathrm{cav}}(\omega_{\mathrm{L}}) &= \left| \frac{1 - \mathrm{e}^{2inkL}}{1 - r^2\, \mathrm{e}^{2\mathrm{i}nk_{\mathrm{L}}L}}\, r \right|^2 , \end{aligned}$$

$$\text{where } r \equiv \frac{1-n}{1+n} : \qquad \text{Fresnel coefficient}$$

The quantum optics of atom-cavity systems

Questions:

- Is it enough to consider only a single cavity mode?
- How can we reproduce the Fabry-Perot cavity behaviour?
- How can we model reflection by mirror surfaces?
- How can we derive spontaneous cavity decay rates?



II

The input-output formalism

Experimental setup

uobabylon.edu.iq

Two-Sided Cavity

A two-sided cavity has two partially transparent mirrors with associated loss coefficients γ_1 and γ_2, as shown in Fig. 7.2. In this case there are two input ports and two output ports. The equation of motion for the internal field is then given by an obvious generalisation as

$$\frac{da(t)}{dt} = -i\omega_0 a(t) - \frac{1}{2}(\gamma_1 + \gamma_2)a(t) + \sqrt{\gamma_1}\, a_{IN}(t) + \sqrt{\gamma_2}\, b_{IN}(t). \tag{7.30}$$

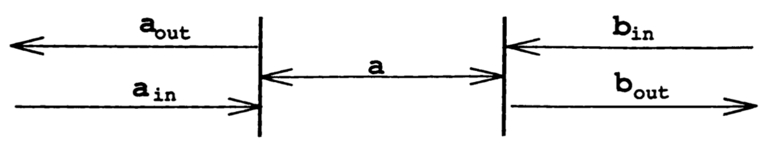


Fig. 7.2 A schematic representation of the cavity field and the input and output fields for a double-sided cavity

III

Alternative models of FP cavities

A quantum optical traveling-wave model

Traveling-wave cavity Hamiltonian:

$$H_{\rm laser} = \hbar\Omega\,\left(a_{\rm R}\,{\rm e}^{{\rm i}\omega_{\rm L}t} + {\rm H.c.}\right)$$

$$H_{\rm fiber} = \hbar\omega_{\rm L}\,\left(a_{\rm L}^\dagger a_{\rm L} + a_{\rm R}^\dagger a_{\rm R}\right) + \frac{1}{2}\hbar J\,\left(a_{\rm R} a_{\rm L}^\dagger + a_{\rm L} a_{\rm R}^\dagger\right)$$

$$\dot\rho = -\frac{\rm i}{\hbar}[H,\rho] + \sum_{\rm X=L,R}\kappa_{\rm X}\left(a_{\rm X}\rho a_{\rm X}^\dagger - \frac{1}{2}\rho a_{\rm X}^\dagger a_{\rm X} - \frac{1}{2}a_{\rm X}^\dagger a_{\rm X}\rho\right)$$

- The laser excites a traveling field mode inside the resonator.
- Photons in the $a_{\rm L,R}$ mode are converted into $a_{\rm R,L}$ photons at a rate J.
- We assign different decay channels to photons with different directions.

Consistency conditions

The predictions of this model are consistent with the predictions of Maxwell's equations, if we choose:

$$\kappa = -\frac{2c}{nL}\ln r = \frac{4}{(n+1)^2}\cdot\frac{c}{L}$$

$$J(\omega_{\mathrm{L}}) = \frac{2c}{nL}\cdot\frac{n-1}{n+1}\cdot\sin\left(nk_{\mathrm{L}}L\right)$$

- The spontaneous cavity decay rate κ depends only on n and L.
- The bouncing rate $J(\omega_{\mathrm{L}})$ contains an interference term.

Special cases

- **Very long cavity:** $J = 0$ and $\kappa = 0$ for $L \to \infty$
- **Resonant cavity:** $J = 0$ and $\kappa \neq 0$
- **Very short cavity:** J is relatively large.

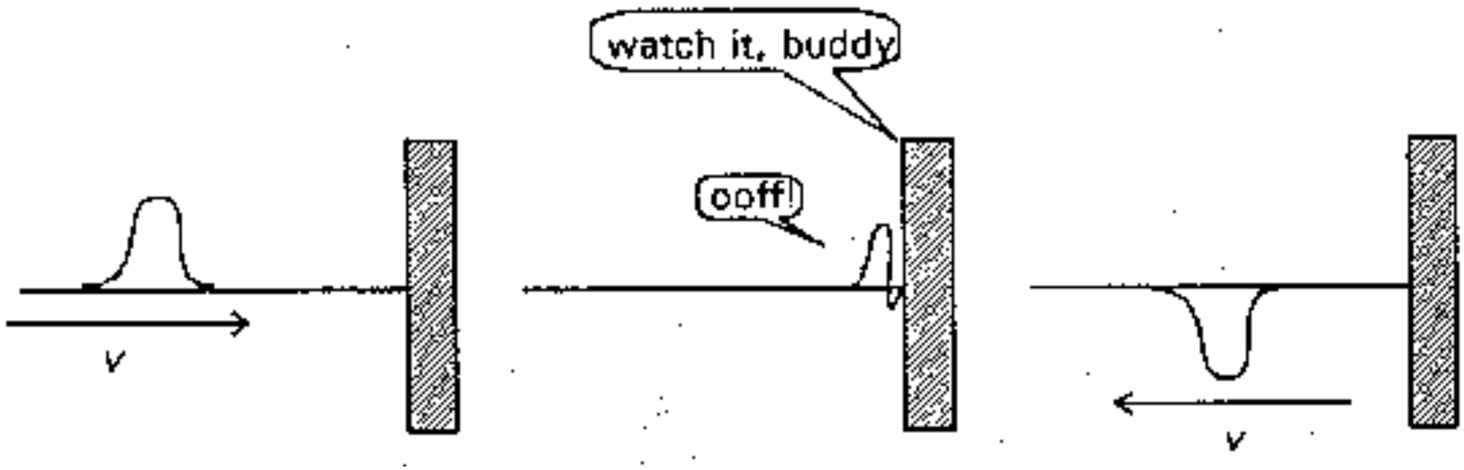
$$a_\pm \equiv \frac{1}{\sqrt{2}} (a_{\rm L} \pm a_{\rm R})$$

$$\Rightarrow \quad H_{\rm fiber} = \hbar(\omega_{\rm L} + J) a_+^\dagger a_+ + \hbar(\omega_{\rm L} - J) a_-^\dagger a_-$$

The standing wave modes are highly degenerate.
The cavity is effectively single-mode.

Our model reproduces known results!

Phenomenological approaches do not always work!!!!!

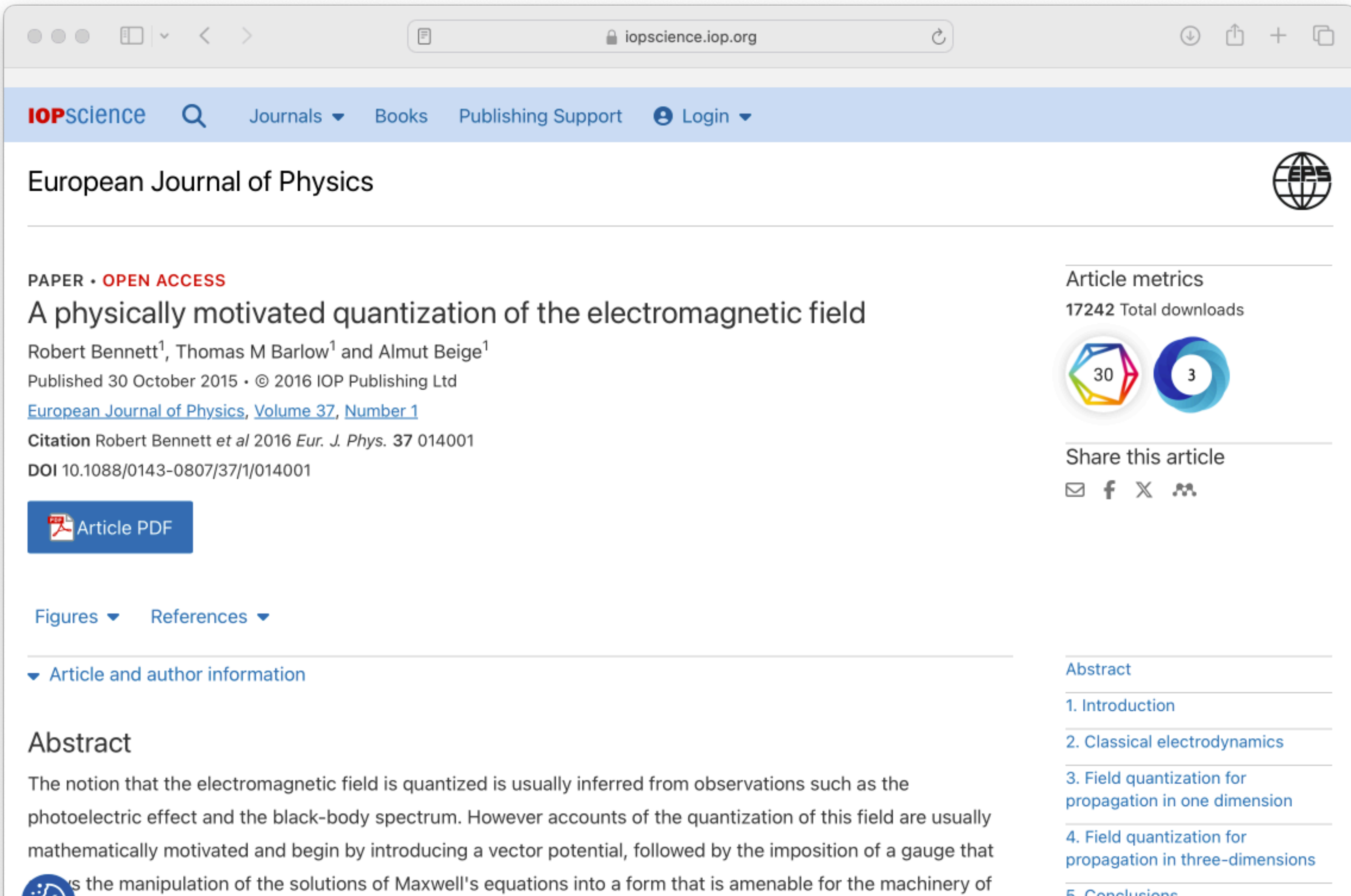


We do not know, how to model mirrors!

IV

Local photons

A gauge-independent quantisation of the EM field



iopscience.iop.org

IOPscience Journals Books Publishing Support Login

European Journal of Physics

PAPER • OPEN ACCESS

A physically motivated quantization of the electromagnetic field

Robert Bennett[1], Thomas M Barlow[1] and Almut Beige[1]

Published 30 October 2015 • © 2016 IOP Publishing Ltd

European Journal of Physics, Volume 37, Number 1

Citation Robert Bennett *et al* 2016 *Eur. J. Phys.* **37** 014001

DOI 10.1088/0143-0807/37/1/014001

Article PDF

Figures References

Article and author information

Abstract

The notion that the electromagnetic field is quantized is usually inferred from observations such as the photoelectric effect and the black-body spectrum. However accounts of the quantization of this field are usually mathematically motivated and begin by introducing a vector potential, followed by the imposition of a gauge that ...s the manipulation of the solutions of Maxwell's equations into a form that is amenable for the machinery of

Article metrics

17242 Total downloads

30

3

Share this article

Abstract

1. Introduction

2. Classical electrodynamics

3. Field quantization for propagation in one dimension

4. Field quantization for propagation in three-dimensions

5. Conclusions

A physically motivated quantisation of the EM field

Standard quantum physics approach:

- We identify canonical variables and impose canonical commutator relations.

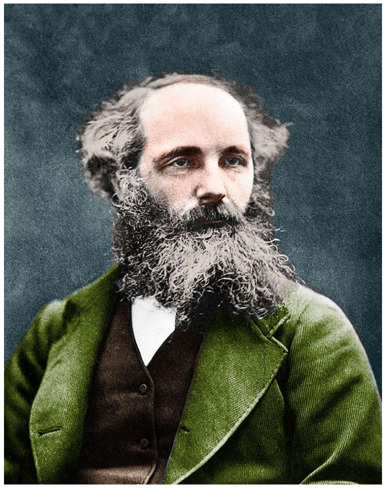
(Difficult task!)

A more "physically motivated" approach:

- We identify the distinguishable states of the classical system and map these onto pairwise orthogonal quantum states.
- Hamiltonian and observables must be such that the expectation values of the "most classical" quantum states evolve classically.

In this way, any classical experiment can also be described by quantum physics.

Light as waves



$$\begin{aligned}\vec{\nabla}\cdot\vec{E} &= 0\\ \vec{\nabla}\cdot\vec{B} &= 0\\ \vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t}\\ \vec{\nabla}\times\vec{B} &= \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}\end{aligned}$$

The basic solutions of MW's equations in free space are **waves**:

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{O}(x,t)=0 \quad \text{with } \boldsymbol{O}=\boldsymbol{E},\boldsymbol{B}$$

An alternative way of solving wave equations

$$\vec{\nabla} \cdot \vec{E} = 0$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \times \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

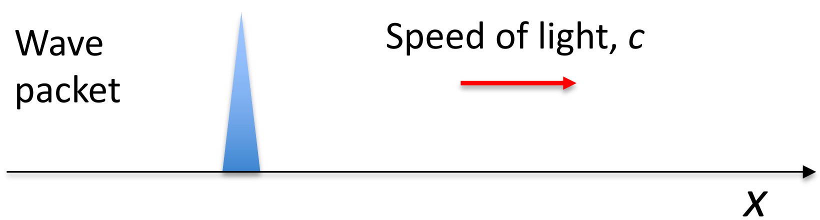
According to d'Alembert's principle, any **local wave packet** which moves at the speed of light solves MW's equations:

$$\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}\right)\mathbf{O}(x,t) = 0 \text{ with } \mathbf{O} = \mathbf{E}, \mathbf{B}$$

Light as "blips"

The basic solutions of MW's equations in free space are wave packets of any shape which travel at the speed of light either left or right:

$$\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}\right)\boldsymbol{O}(x,t) = 0 \quad \text{with } \boldsymbol{O} = \boldsymbol{E}, \boldsymbol{B}$$

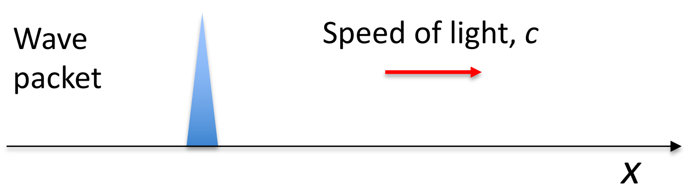


This includes highly-localised WPs which remain localised!

Local carriers of light

Suppose the EM field consists of **blips** ($=$ bosons localised in position) with annihilation operators $a_{s\lambda}(x)$ which travel at the speed of light.

$$
\begin{aligned}
s = \pm 1 &\quad : \quad \text{direction of propagation} \\
\lambda = \mathsf{H}, \mathsf{V} &\quad : \quad \text{polarisation} \\
x \in (-\infty, \infty) &\quad : \quad \text{position}
\end{aligned}
$$



Commutator relations

The state of a single local excitation:
$|1_{s\lambda}(x)\rangle = a^{\dagger}_{s\lambda}(x)|0\rangle$

These states are pairwise orthogonal for **bosonic** blips:

$$\begin{aligned}
\langle 1_{s\lambda}(x)|1_{s'\lambda'}(x')\rangle &= \langle 0|a_{s\lambda}(x)a^{\dagger}_{s'\lambda'}(x')|0\rangle \\
&= \left[a_{s\lambda}(x), a^{\dagger}_{s'\lambda'}(x')\right] \\
&= \delta_{s,s'}\,\delta_{\lambda,\lambda'}\,\delta(x-x')
\end{aligned}$$

Transformation into momentum space

x-space solutions:

$x \in (-\infty, \infty)$
$\lambda = \mathsf{H}, \mathsf{V}$
$s = \pm 1$

k-space solutions:

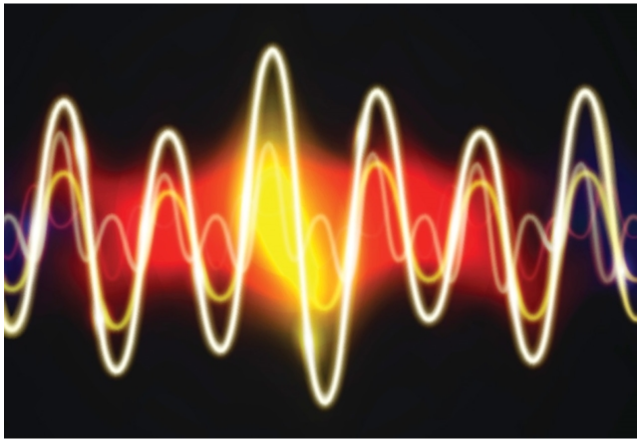
$k \in (-\infty, \infty)$
$\lambda = \mathsf{H}, \mathsf{V}$
$s = \pm 1$

When transferring the $a_{s\lambda}(x)$ via a Fourier transform into momentum space, we obtain bosonic $\tilde{a}_{s\lambda}(k)$ for monochromatic photons:

$$\tilde{a}_{s\lambda}(k) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \mathsf{d}x \, \mathrm{e}^{-\mathrm{i}skx} \, a_{s\lambda}(x)$$

$$a_{s\lambda}(x) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \mathsf{d}k \, \mathrm{e}^{\mathrm{i}skx} \, \tilde{a}_{s\lambda}(k)$$

The basic building blocks of light



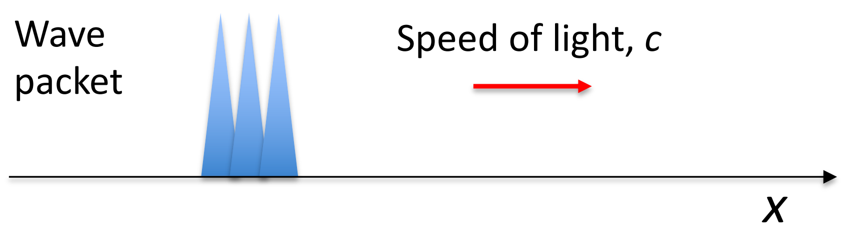
There are different carriers of light, local and non-local:

1. **Monochromatic waves:**
   These correspond either to standing or to travelling waves.

2. **Localised wave packets:**
   blips = bosons localised in position

Single-photon wave packets of any shape can be obtained by superposing either waves or blips.

The Schrödinger equation



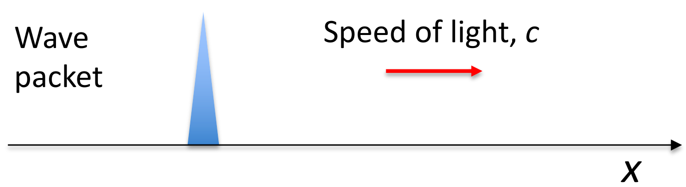
$s = -1$: left-moving WPs
$s = +1$: right-moving WPs

The dynamical Hamiltonian H_{dyn}
is the generator of dynamics:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\frac{\mathrm{i}}{\hbar} H_{\text{dyn}}|\psi(t)\rangle$$

The Hamiltonian H_{dyn} has positive and negative eigenvalues.

The dynamical Hamiltonian

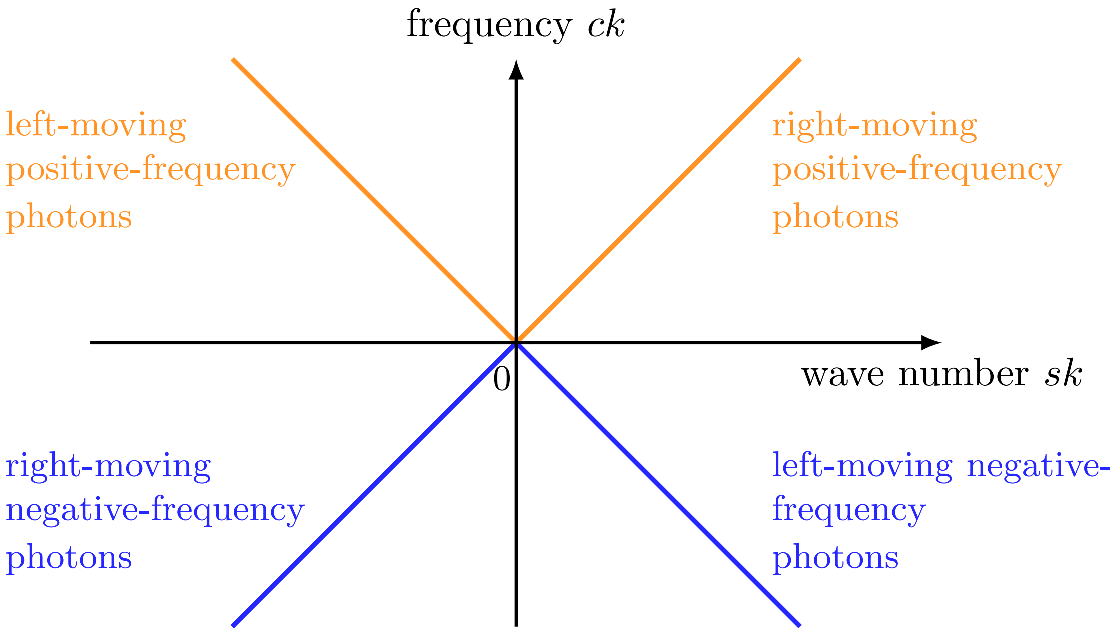


In the Heisenberg picture: $a_{s\lambda}(x,t) = a_{s\lambda}(x - sct, 0)$

$$\implies \quad H_{\rm dyn} = -i\hbar \sum_{s,\lambda} \int_{-\infty}^{\infty} {\rm d}x \, sc \, a^\dagger_{s\lambda}(x') \, \frac{\partial}{\partial x} \, a_{s\lambda}(x)$$

$$= \sum_{s,\lambda} \int_{-\infty}^{\infty} {\rm d}k \, \hbar ck \, \tilde{a}^\dagger_{s\lambda}(k) \tilde{a}_{s\lambda}(k)$$

Frequencies and wave numbers



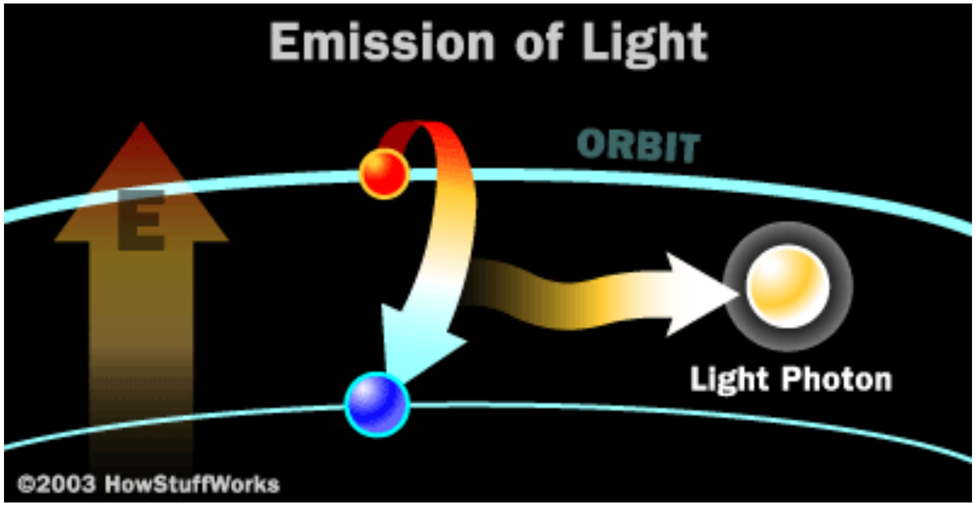
Electric and magnetic field observables

Consistency with MW's equations applies when

$$\begin{aligned}\boldsymbol{\mathcal{E}}(x) &= \sum_{s=\pm 1} \mathcal{R}\Big(a_{s\mathsf{H}}(x)\,\hat{\boldsymbol{y}} + a_{s\mathsf{V}}(x)\,\hat{\boldsymbol{z}}\Big) \\ \boldsymbol{\mathcal{B}}(x) &= -\sum_{s=\pm 1} \frac{s}{c}\,\mathcal{R}\Big(a_{s\mathsf{V}}(x)\,\hat{\boldsymbol{y}} - a_{s\mathsf{H}}(x)\,\hat{\boldsymbol{z}}\Big)\end{aligned}$$

$\mathcal{R}$: regularisation operator;
independent of x and of t

The energy of monochromatic photons



- Suppose **one** atom with energy $\hbar\omega_0$ emits exactly **one** photon.
- Due to resonance, the photon resembles a monochromatic ω_0-wave.
- Energy conservation implies that the photon has the energy $\hbar\omega_0$.

Field observables in position space

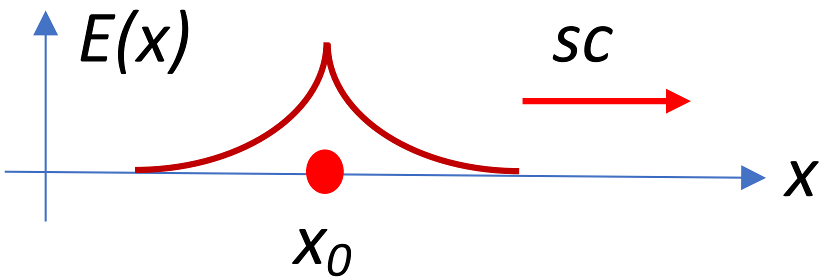
$$\Rightarrow \quad \mathcal{E}_{s\lambda}(x) = \int_{-\infty}^{\infty} \mathrm{d}x'\, g(x,x')\, a_{s\lambda}(x')$$

$$\mathcal{B}_{s\lambda}(x) = \frac{s}{c}\int_{-\infty}^{\infty} \mathrm{d}x'\, g(x,x')\, a_{s\lambda}(x')$$

$$\text{with} \quad g(x,x') = \left(\frac{\hbar c}{2\pi^2\varepsilon_0 A}\right)^{1/2} \int_{-\infty}^{\infty} \mathrm{d}k\, \sqrt{|k|}\, \mathrm{e}^{\mathrm{i}k(x-x')}$$

$$= -\left(\frac{\hbar c}{4\pi\varepsilon_0 A}\right)^{1/2} \frac{1}{|x-x'|^{3/2}}$$

A physical picture of blips



Comments:

- Local blips at x_0 can be felt everywhere.
- Localised fields can only be created by a non-local source.
- We now have positive and negative frequency photons.

Similarities with gravitational fields

Local photons are similar to massive particles: they are **local carriers** of fields. In analogous classical situations, it is often easier to model the dynamics of the carriers than of the fields.

The energy of light

- By definition, H_{dyn} also represents the energy of the quantised EM field.
- The eigenvalues of the energy observable H_{eng} should be only positive.

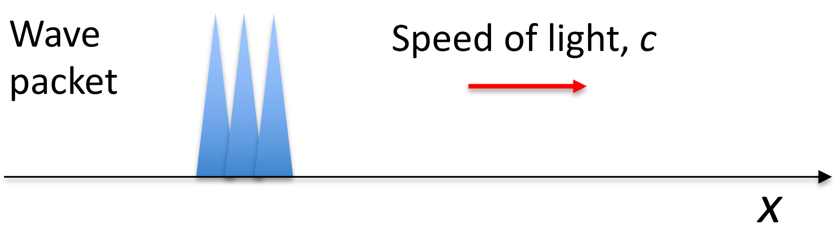
Hence:

$$\begin{aligned} H_{\mathsf{eng}} &= \begin{cases} -H_{\mathsf{dyn}} & \text{for } k < 0 \\ +H_{\mathsf{dyn}} & \text{for } k \geq 0 \end{cases} \\ &= \sum_{s,\lambda} \int_{-\infty}^{\infty} \mathrm{d}k \, \hbar c |k| \, a_{s\lambda}^{\dagger}(k) a_{s\lambda}(k) \end{aligned}$$

With respect to the complex field vectors $\boldsymbol{\mathcal{E}}^{\dagger}(x)$ and $\boldsymbol{\mathcal{B}}^{\dagger}(x)$:

$$H_{\mathsf{eng}} = \frac{A}{4} \int_{-\infty}^{\infty} \mathrm{d}x \left[\varepsilon \, \boldsymbol{\mathcal{E}}^{\dagger}(x) \cdot \boldsymbol{\mathcal{E}}(x) + \frac{1}{\mu} \boldsymbol{\mathcal{B}}^{\dagger}(x) \cdot \boldsymbol{\mathcal{B}}(x) \right]$$

The dynamical momentum



$s = -1$: left-moving WPs
$s = +1$: right-moving WPs

The momentum p_{dyn} is the generator
for the spatial translation of quantum states:

$$\frac{\partial}{\partial x}|\psi(t)\rangle = \frac{\mathrm{i}}{\hbar} p_{\mathsf{dyn}} |\psi(t)\rangle$$

$$\Longrightarrow \quad p_{\mathsf{dyn}} = \sum_{s,\lambda} \int_{-\infty}^{\infty} \mathrm{d}k \, \hbar s k \, a_{s\lambda}^{\dagger}(k) a_{s\lambda}(k)$$

Relation to field observables

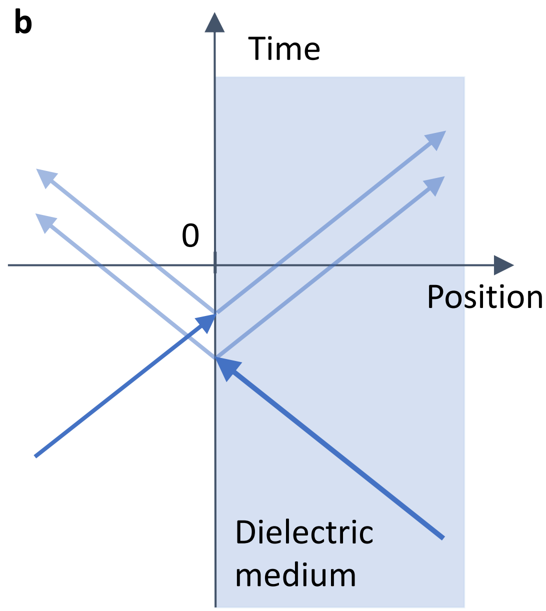
The dynamical momentum p_{dyn} of light can be written in the form

$$p_{\mathsf{dyn}} = \begin{cases} -p & \text{for } k < 0\,, \\ +p & \text{for } k \geq 0 \end{cases}$$

with p defined such that

$$p\,\hat{x} = \frac{\varepsilon A}{4} \int_{-\infty}^{\infty} \mathrm{d}x \left[\boldsymbol{\mathcal{E}}^{\dagger}(x) \times \boldsymbol{\mathcal{B}}(x) - \boldsymbol{\mathcal{B}}^{\dagger}(x) \times \boldsymbol{\mathcal{E}}(x) \right]$$

The Abraham-Minkowski controversy



In classical optics, it is not known whether the momentum of light increases **(Minkowski, 1910)** or decreases **(Abraham, 1909)** when light enters a dielectric medium.

A locally-acting mirror Hamiltonian

Minkowski
was right!

In the presence of a mirror interface at $x = 0$, the Hamiltonian of the quantised EM field becomes $H_{\rm mir} = H_{\rm dyn} + H_{\rm int}$ with

$$H_{\rm int} = \sum_{\lambda = \text{H,V}} \hbar\Omega \, a^{\dagger}_{-1\lambda}(0) \, a_{1\lambda}(0) + \text{H.c.}$$

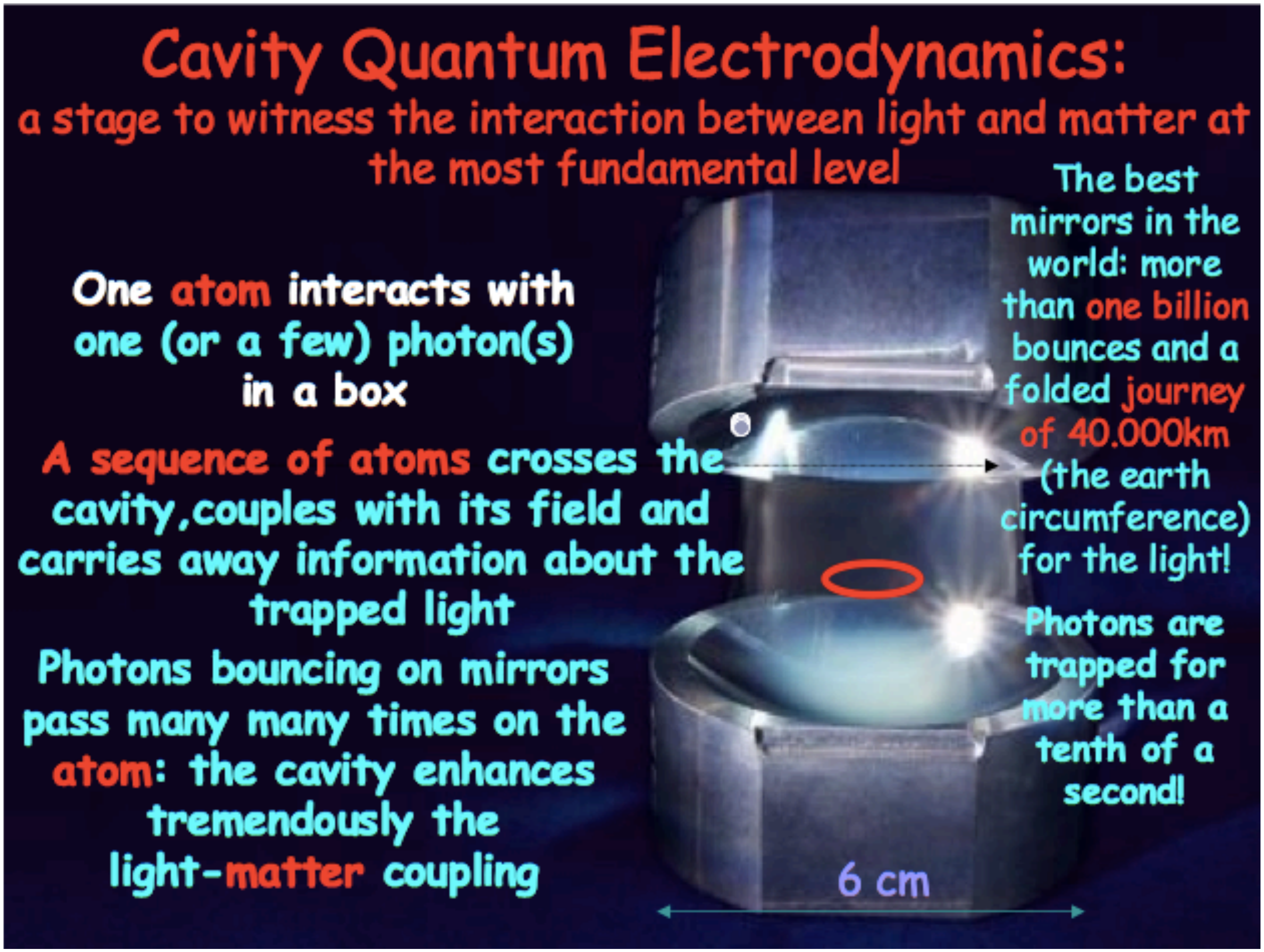
HERMITIAN!

This Hamiltonian can be analysed using a Dyson series expansion to predict the dynamics of the momentum of any incoming light.

V

A local photon approach to optical cavities


Photo: U. Montan
Serge Haroche



A locally-acting cavity Hamiltonian

In the presence of mirror interfaces at $x = 0$ and at $x = L$, the Hamiltonian of the quantised EM field becomes

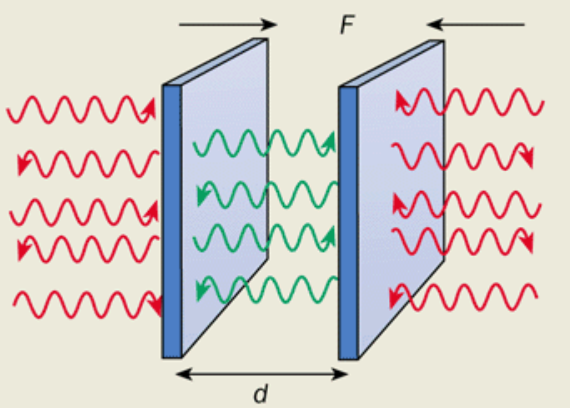
$$\begin{aligned} H_{\rm int} &= H_{\rm dyn} + \sum_{\lambda=\mathsf{H,V}} \hbar\Omega \, a^\dagger_{-1\lambda}(0) \, a_{1\lambda}(0) + {\rm H.c.} \\ &\quad + \sum_{\lambda=\mathsf{H,V}} \hbar\Omega \, a^\dagger_{-1\lambda}(L) \, a_{1\lambda}(L) + {\rm H.c.} \end{aligned}$$

This Hamiltonian can be analysed using a Dyson series expansion or master equations.

VI

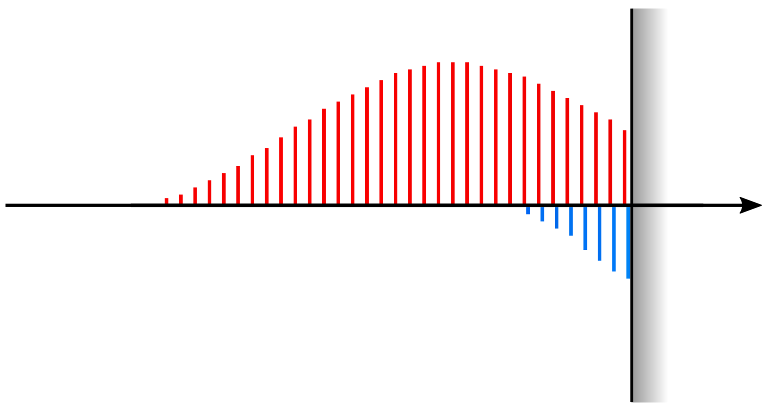
Strange effects:
Casimir, Doppler and Unruh

Casimir's effect



- Already in 1948, Casimir predicted an attractive force between two metallic mirrors for small distances d.
- This effect can be understood as a consequence of boundary conditions imposed by the mirrors and is attributed to vacuum fluctuations.
- Obtaining a finite force requires regularisation procedures.

The mirror image method



A mirror changes amplitude and direction of the incoming blip excitations.

Southall, Hodgson, Purdy and Beige, *Locally acting mirror Hamiltonians*, J. Mod. Opt. **68**, 647 (2021).

The origin of the Casimir effect

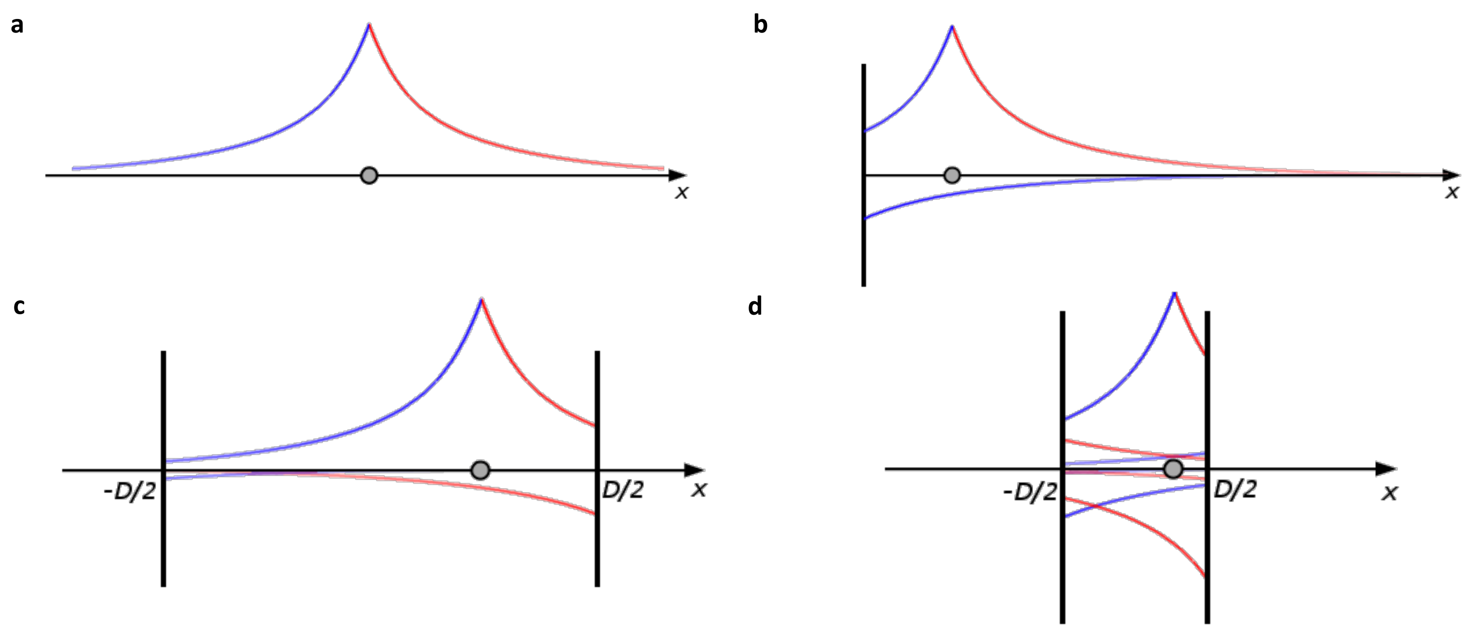


Figure 2: **a.** Because of the regularisation operator $\mathcal{R}$ in Eq. (6), local blip excitations contribute to local electric and magnetic field expectation values everywhere along the x axis (cf. Eq. (8)). **b.** Since a blip on one side of a highly reflecting mirror cannot contribute to the field expectation value on the other side, its field contribution must be folded back on itself. This effect alters the electric and magnetic field observables in the presence of a mirror. **c.** In the presence of two highly reflecting mirrors, blips outside the cavity cannot contribute to field expectation values on the inside. Moreover, the field contributions of blips on the inside need to be folded as in the case of one mirror. Now, however, the field contributions must be folded infinitely many times (cf. Eq. (18) in Methods). **d.** Comparing two cavities of different sizes, we see that the behaviour of the field contribution is now dependent on the cavity width.

The electric field inside the cavity

Suppose $\mathcal{X}$ restricts the Hilbert space to blip excitations at positions $x \in (-D/2, D/2)$.

$$\boldsymbol{E}^{(\mathrm{in})}_{s\lambda}(x,t) = \sum_{n=-\infty}^{\infty} \mathcal{X} \Big(\boldsymbol{E}^{(\mathrm{free})}_{s\lambda}(x + 2nD, t) - \boldsymbol{E}^{(\mathrm{free})}_{-s\lambda}(-x + (2n-1)D, t) \Big)$$

blips inside the cavity cannot create fields outside and vice versa.

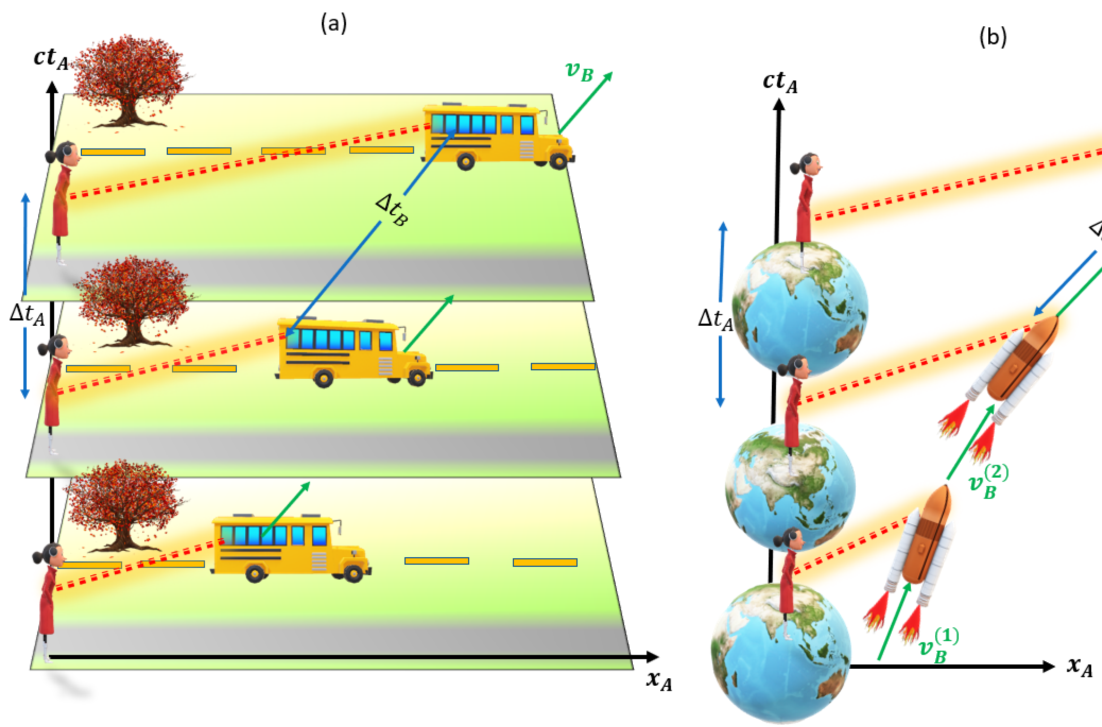
The zero point energy of the EM field

$$
\begin{aligned}
H_{\mathsf{ZPE}}^{(\mathsf{in})} &= \frac{\hbar c}{4\pi} \sum_{n,m=-\infty}^{\infty} \int_{-D/2}^{D/2} \mathsf{d}x \int_{-D/2}^{D/2} \mathsf{d}x' \\
&\qquad \Big[\left| (x + x' + (2n-1)D)(x + x' + (2m-1)D) \right|^{-3/2} \\
&\qquad + \left| (x - x' + 2nD)(x - x' + 2mD) \right|^{-3/2} \Big] \\
&= -\frac{\hbar c}{2\pi D} \sum_{m=-\infty}^{\infty} \frac{1}{m^2} \quad \Rightarrow \quad F_{\mathrm{Casimir}} = -\frac{\mathrm{d}H_{\mathrm{ZPE}}}{\mathrm{d}D} = -\frac{\pi \hbar c}{6D^2}
\end{aligned}
$$

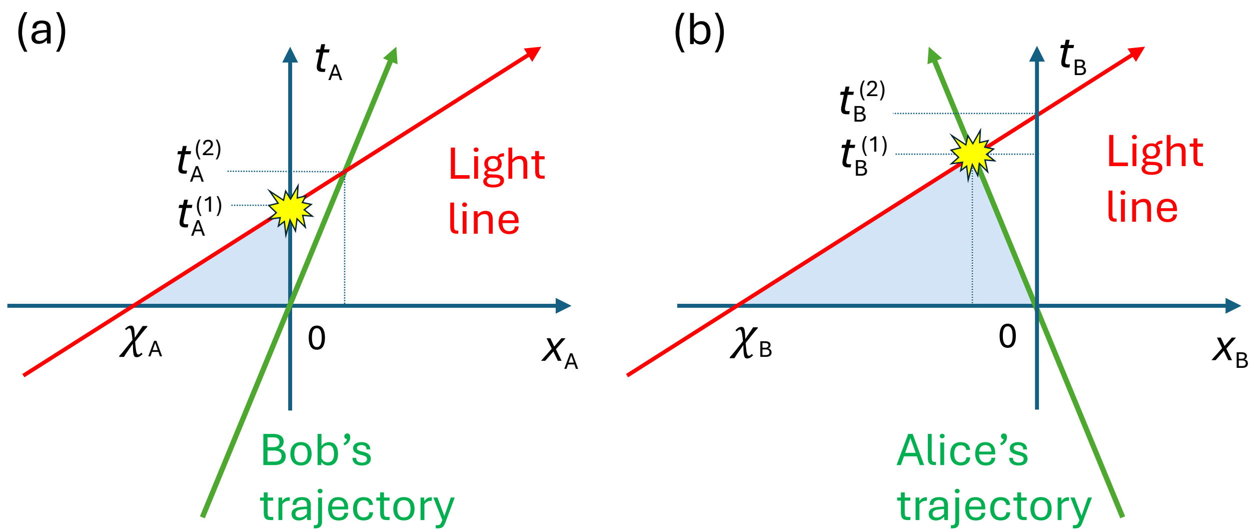
The Casimir effect is due to interference effects of evanescent fields belonging to opposite sides of the cavity.

The relativistic Doppler effect

The relativistic Doppler effect predicts frequency, wavelength and amplitude changes between two moving observers.



Alice and Bob experience space and time differently



We describe each light-like world-line by natural coordinates $\chi_{\rm A} = x_{\rm A} - sct_{\rm A}$ and $\chi_{\rm B} = x_{\rm B} - sct_{\rm B}$ and blip annihilation operators $a(\chi_{\rm A})$ and $b(\chi_{\rm B})$.

Operator transformations

Alice and Bob experience the same blips when referring to the same point in the spacetime diagram.

$$\chi_{\rm A} = -ct_{\rm A}^{(1)} = -(c - v_{\rm B})t_{\rm A}^{(2)}$$

$$\chi_{\rm B} = -(c + v_{\rm B})t_{\rm B}^{(1)} = -ct_{\rm B}^{(2)}$$

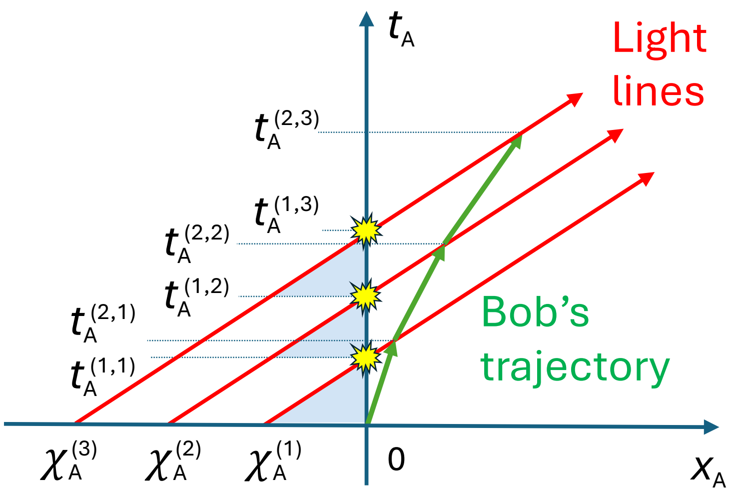
$$\chi_{\rm B}/\chi_{\rm A} = \kappa\,, \quad t_{\rm B}^{(1)}/t_{\rm A}^{(1)} = t_{\rm A}^{(2)}/t_{\rm B}^{(2)} = \gamma$$

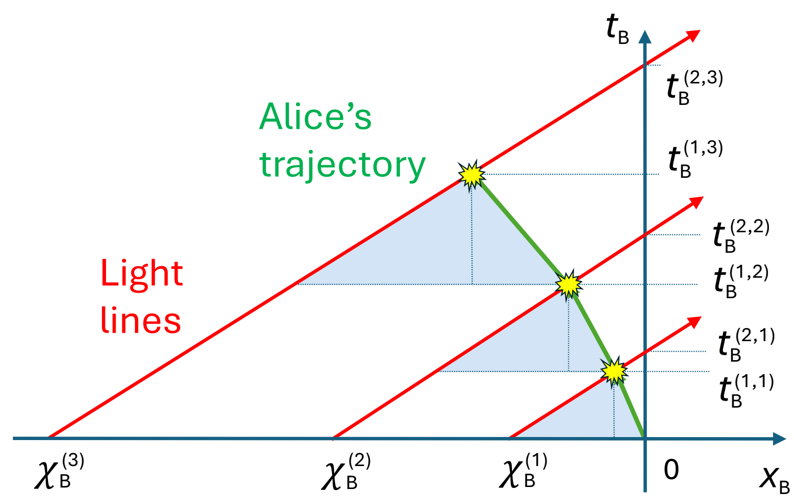
Hence:

$$\begin{aligned} \chi_{\rm A} &= \gamma(1 - s\beta)\,\chi_{\rm B} \;\; \text{with} \;\; \gamma = [\sqrt{1-\beta^2}]^{-1/2}, \quad \beta = v_{\rm B}/c \\ \Rightarrow \quad b_{s\lambda}(\chi_{\rm B}) &= [\gamma(1 - s\beta)]^{1/2}\, a_{s\lambda}(\chi_{\rm B}) \end{aligned}$$

The Unruh effect

The Unruh effect predicts the presence of thermal photons in the reference frame of an accelerating observer. However, it can also be modelled consistently while all observers share a **common vacuum**:





Acceleration without photon pair creation

Suppose, Alice and Bob are placed at the origins of their diagrams and meet at an initial time when $t_{\rm A} = t_{\rm B} = 0$. Bob's velocity $v_{\rm B}$ in Alice's coordinate system is known at any time $t_{\rm A}$. She sends signals at regular time intervals.

$$\chi_{\rm B} = \int_0^{\chi_{\rm A}} {\rm d}\chi'_{\rm A}\, \gamma(\chi'_{\rm A})\, [1 + s\beta(\chi'_{\rm A})]$$

$$\Rightarrow \quad b_{s\lambda}(\chi_{\rm B}) = \sqrt{\gamma(\chi_{\rm A})\big(1 - s\beta(\chi_{\rm A})\big)}\, a_{s\lambda}(\chi_{\rm A})$$

VII

Final remarks

Comments

We quantised the EM field in 1D **in position space.** No-go theorems have been overcome by doubling its usual Hilbert space and by allowing for negative frequency photons.

This approach allows us to introduce a locally-acting mirror **Hamiltonian** and to calculate **zero point energies**.

See also related work by Dirac, Hawton, Cook, Mostafazadeh and Pendry and others.

References:

1. **A physically-motivated quantisation of the electromagnetic field**, Bennett, Barlow and Beige, Eur. J. Phys. **37**, 014001 (2016).

2. **Locally-acting mirror Hamiltonians**, Southall, Hodgson, Purdy and Beige, J. Mod. Opt. **68**, 647 (2021).

3. **Local photons**, Hodgson, Southall, Purdy and Beige, Front. Photon. **3**, 978855 (2022).

4. **Local photon model of the momentum of light**, Hodgson, Lang, Alapatt and Beige, Phys. Rev. A **111**, 023703 (2025).

5. **An intuitive picture of the Casimir effect**, Hodgson, Burgess, Altaie, Beige and Purdy, arXiv:2203.14385 (2025).

6. **A simple quantum picture of the relativistic Doppler effect**, Hodgson, Kanzi and Beige, Symmetry **16**, 279 (2024).

7. **Acceleration without photon pair creation**, Kanzi, Hodgson and Beige, arXiv:2503.09863 (2025).