Early Dark Energy and Phantom Crossing in axio-dilaton Systems

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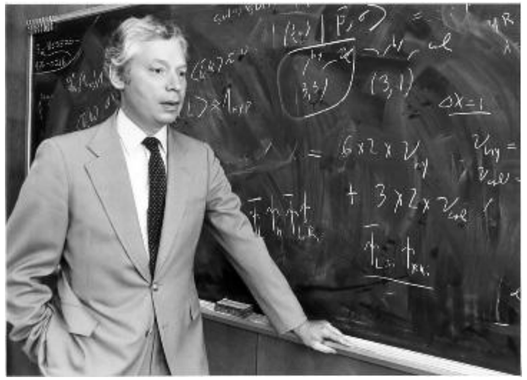
Collaboration with C. van de Bruck (Sheffield), C. Burgess (Perimeter), A. Davis (Cambridge), M. Mylova(Perimeter), F. Quevedo (Cambridge) and A. Smith (Sheffield).

The dark energy scale is in the ***meV range***: apparent fine-tuning compared to standard model scales.

$$\delta\rho_\Lambda = M^4, \quad M \sim 100\mathrm{GeV}$$

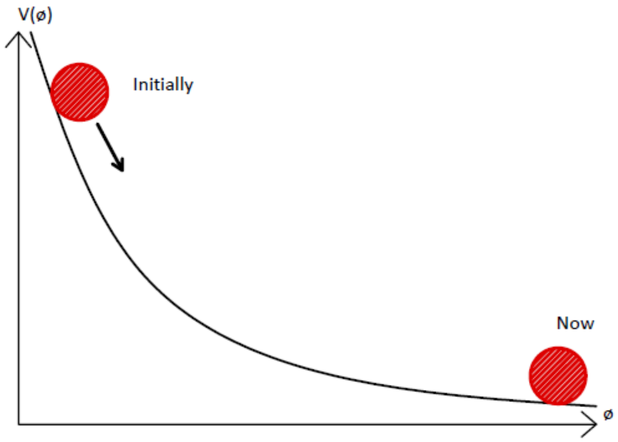
Weinberg's theorem states that there is no non fine-tuned nearly vanishing vacuum energy in a 4d quantum field theory respecting **Poincare invariance**.



Dynamical configurations

↓

Dark energy



Scalar field rolling down its potential

A good example: the dilaton of broken scale invariance can be such a candidate.

Important stringy ***conjectures*** for dark energy are:

In an ideal world, string theory or any other version of quantum gravity would be finite so the vacuum energy could be calculable. Not the case in ordinary Quantum Field Theory.

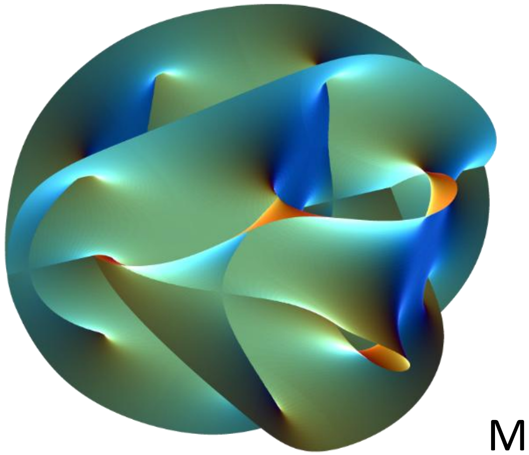
- ✓ ***The de Sitter conjecture***: a pure vacuum energy with no dynamics is not compatible with string theory (?)

- ✓ ***The vacuum conjecture:*** Empty space-time is described by the dynamics of at least one scalar field with a potential such that

$$|\frac{dV}{d\phi}| \geq c\frac{V}{m_{\rm Pl}}$$

$$c = \mathcal{O}(\sqrt{2})$$

This forbids very flat potentials. This favours runaway potentials where the field is a “modulus”.



Moduli could be “sizes” of extra-dimensions

Some expected features of dark energy:

- Dark energy is determined by the position of the field now:

$$3\Omega_\Lambda H_0^2 m_{\rm Pl}^2 = V(\phi_{\rm now})$$

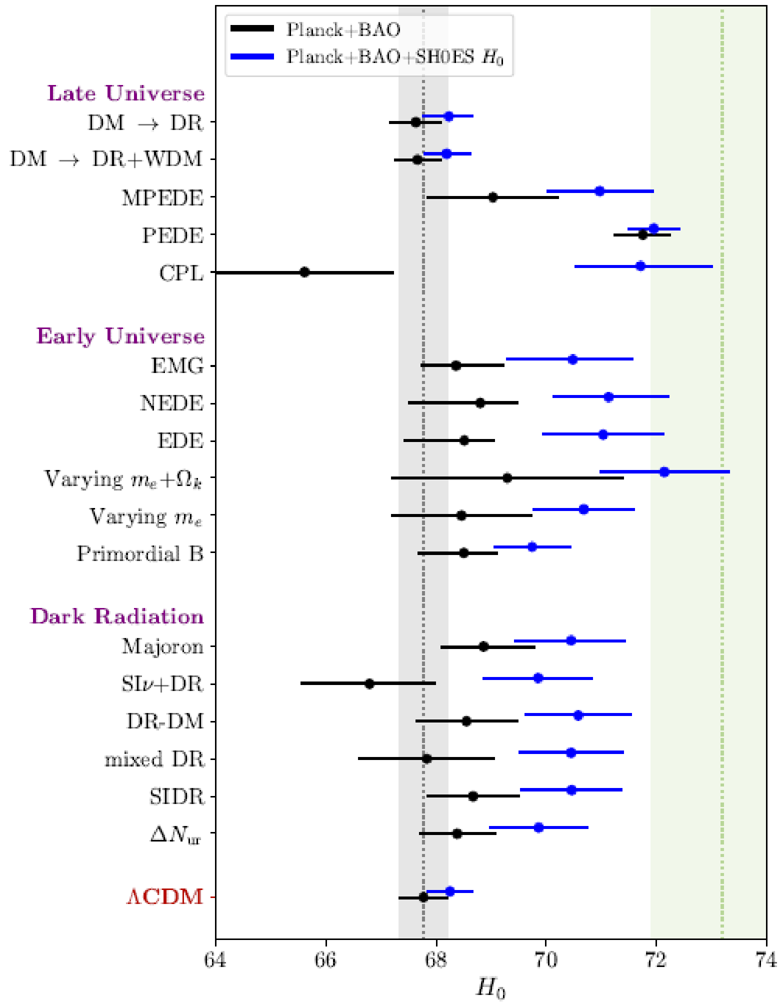
- The field is ***extremely light***:

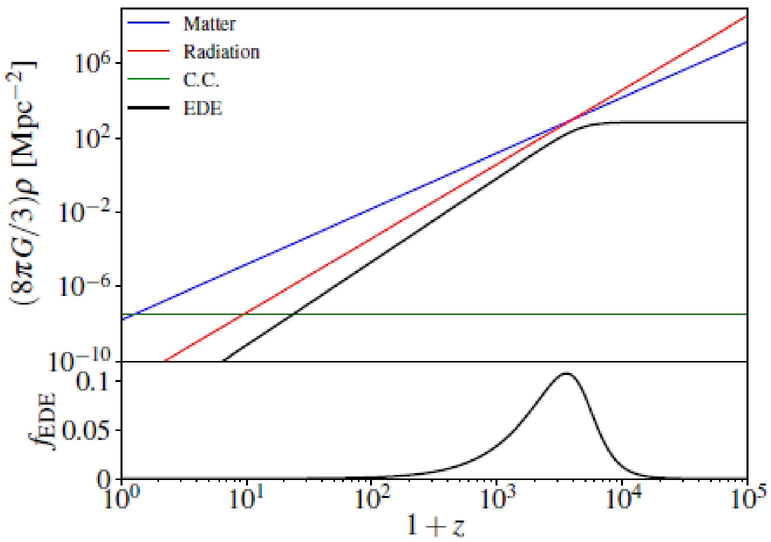
$$m_\phi^2 = \frac{d^2V}{d\phi^2}|_{\rm now} \sim \frac{V_{\rm now}}{m_{\rm Pl}^2} = 3\Omega_\Lambda H_0^2$$

Mass of the order of the Hubble rate

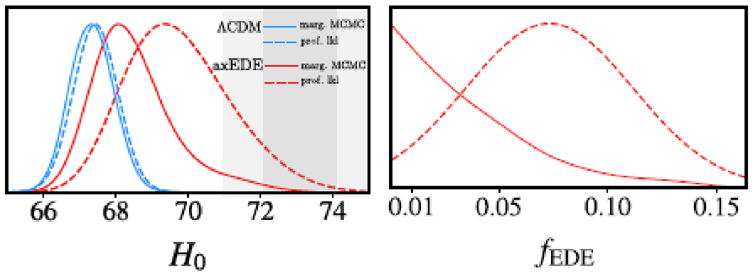
$H_0 \sim 10^{-42}$ GeV

Early Dark Energy

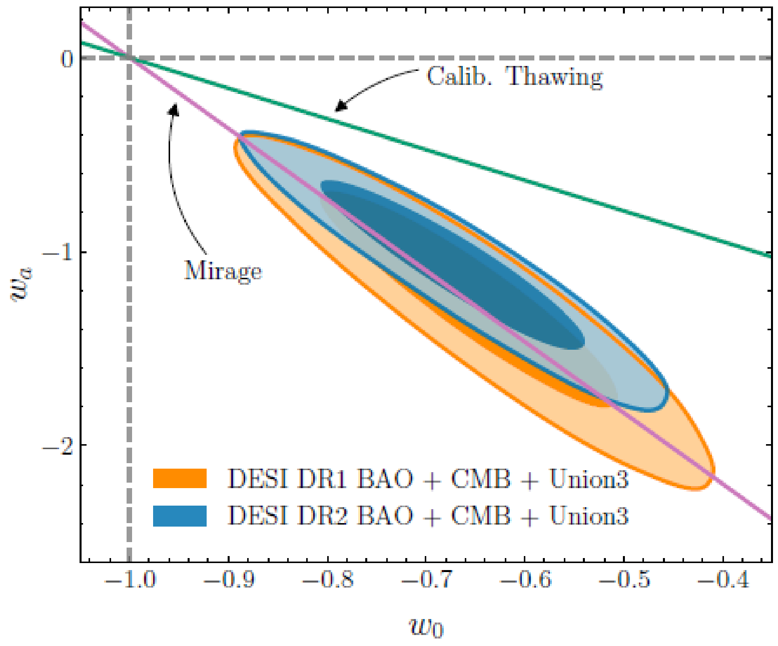




Early dark energy fraction



Late Dark Energy



$$\omega = \frac{p}{\rho}$$

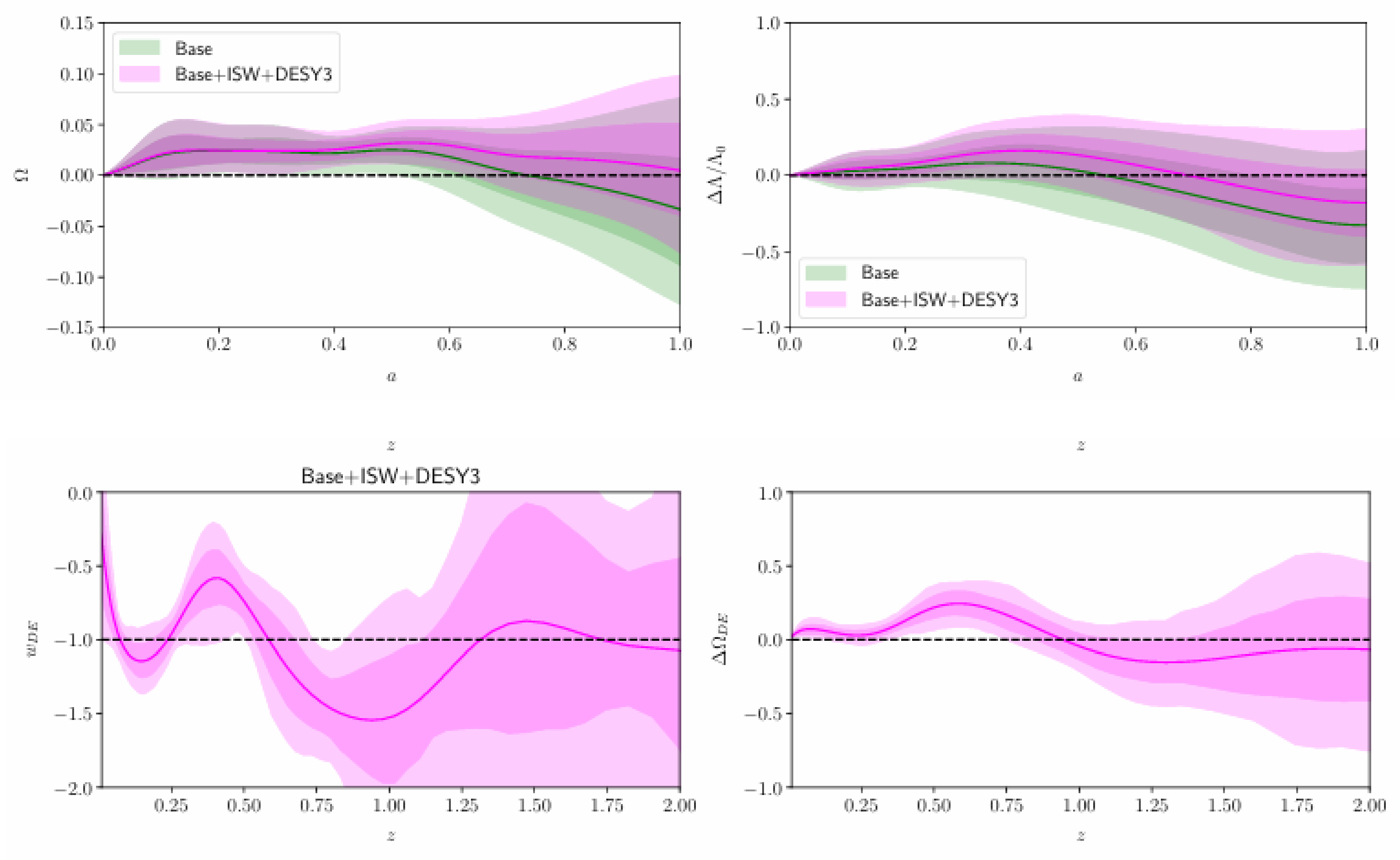
$$w_0 + w_a < -1$$

$$\omega = \omega_0 + \omega_a(1 - a)$$

Phantom crossing

$$\omega(z) < -1, \quad z \geq 0.5$$

E pur si muove ! G.G.(1633)



Parameterisation-independent reconstruction of the cosmological background in the effective dark energy action.

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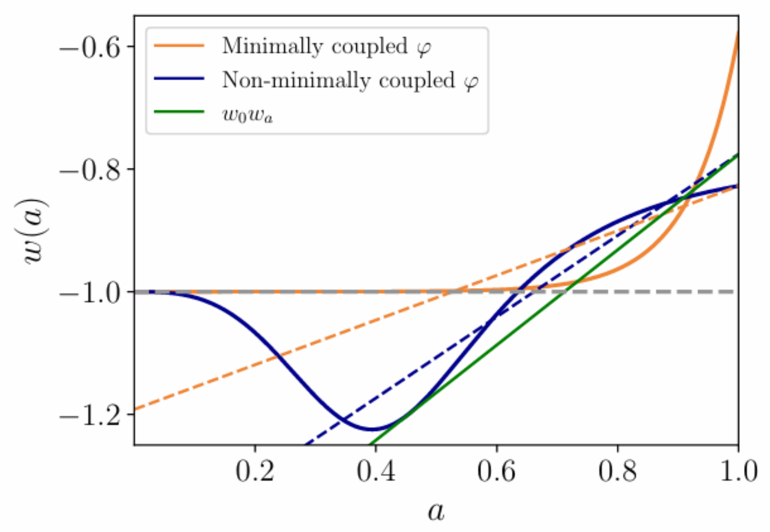
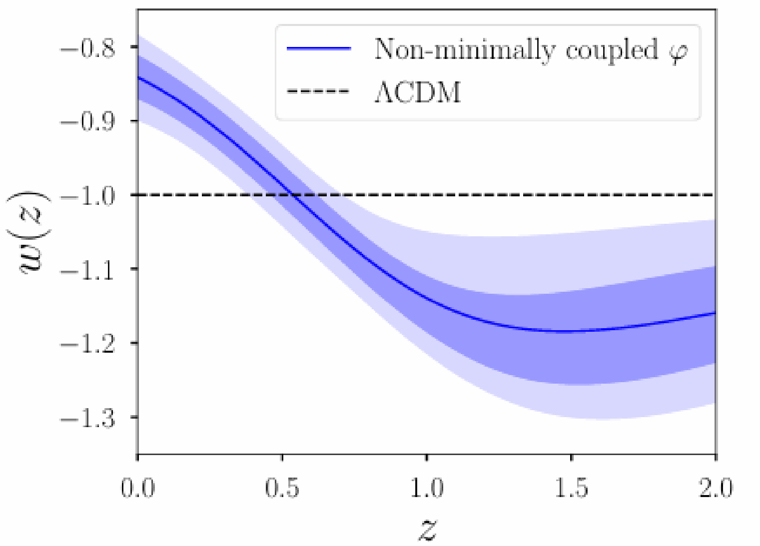
Simplest extension:

Effective dark energy action

$$S = \int d^4x\sqrt{-g}(\frac{m_{\rm Pl}^2}{2}(1+\Omega(t))R + \Lambda(t) + \dots)$$





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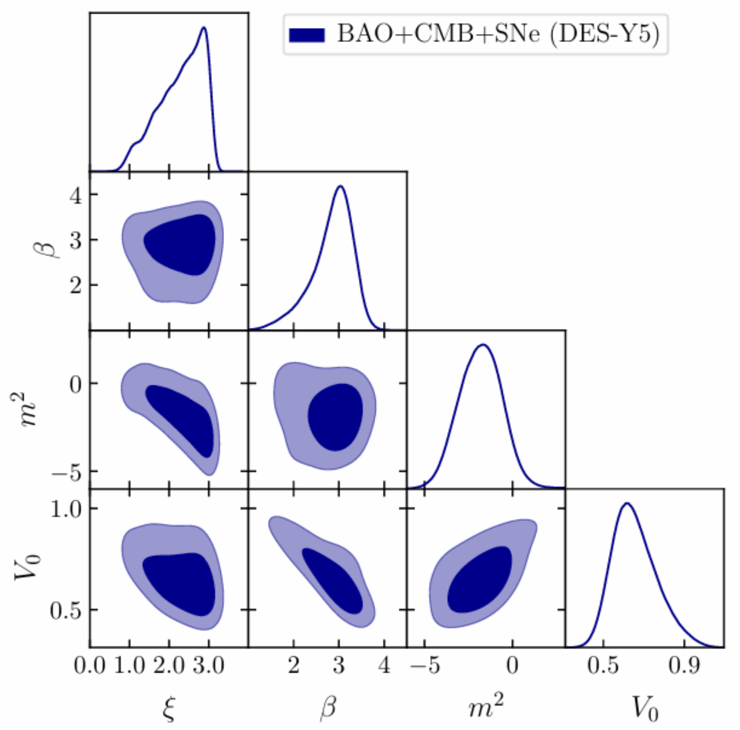


$$S = \int d^4x\sqrt{-g}\left(\frac{m_{\rm Pl}^2}{2}F(\phi)R - V(\phi)\right)$$

$$F(\phi) = 1 - \xi\frac{\phi^2}{m_{\rm Pl}^2}$$

$$V(\Phi) = V_0 + \beta\phi + \frac{1}{2}m^2\phi^2$$

Evidence for scalar-tensor models

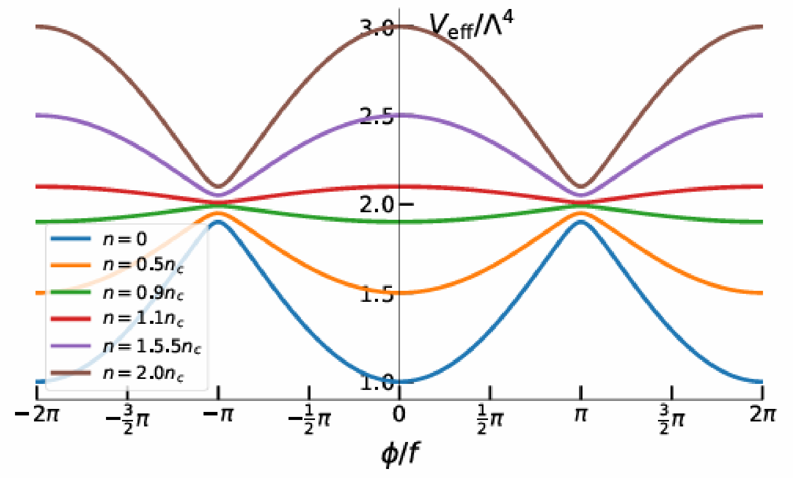
The models here are in the Jordan frame

QCD inspired model:

$$F = A^{-2}$$

$$V(\phi) = \Lambda^4(1 + v - \sqrt{1 - \xi \sin^2(\frac{\phi}{2f})})$$

$$A(\phi) = 1 + \frac{\sigma_N}{m_0}\sqrt{1 - \xi \sin^2(\frac{\phi}{2f})}$$



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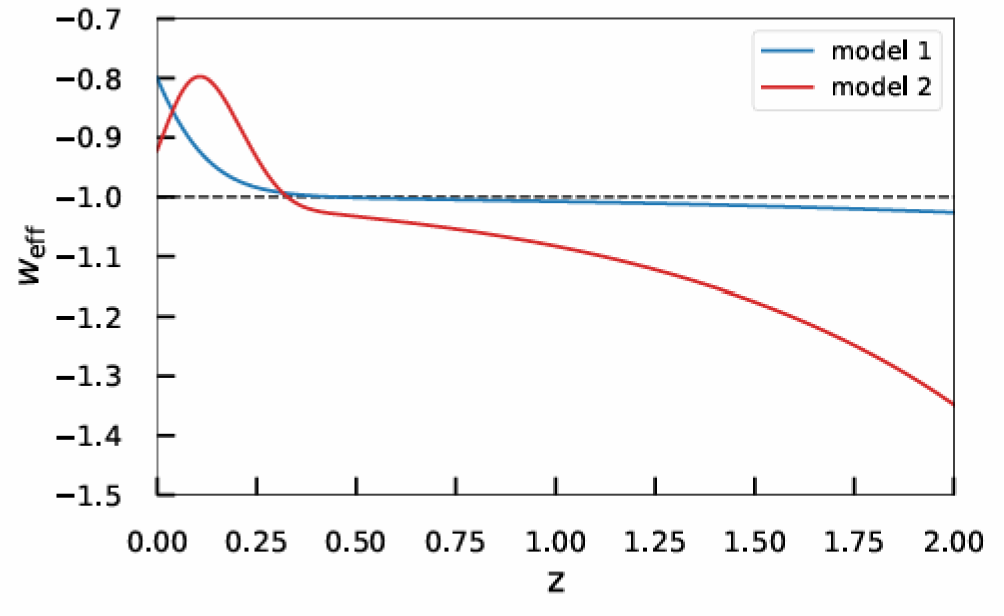


- **Model 1** has $\frac{\sigma_N}{m_0} = 0.01$, $f = 0.25\, M_{\rm Pl}$, and $v = 0$.
- **Model 2** has $\frac{\sigma_N}{m_0} = 0.02$, $f = 0.035\, M_{\rm Pl}$, and $v = 4$.

The low energy description

On large scales, GR can be seen as the lowest order effective action involving the metric up to two derivatives:

$$S_{\mathrm{Einstein-Hilbert}} = \frac{m_{\mathrm{Pl}}^2}{2}\int d^4x\sqrt{-g}(R+\dots)$$

Incorporating a single scalar field, the effective action up to second order in derivatives:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{(\partial\phi)^2}{2} - V(\phi) + \mathcal{L}_m(\psi_i, \tilde{g}_{\mu\nu})\right)$$

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$$

Higher order terms in derivatives are suppressed unless breaking the expansion scheme by terms of order:

$$\partial/H = \mathcal{O}(1)$$

In these theories the effective potential takes into account the presence of matter

$$V_{\rm eff}(\varphi) = V(\varphi) + (\frac{A(\varphi)}{A(\varphi_c)} - 1)\rho_m$$

The conserved matter density is related to the Einstein frame density by

$$\rho_E = \frac{A(\varphi)}{A(\varphi_c)}\rho_m$$

The dark energy density becomes:

$$\rho_{\rm eff} = \frac{1}{2}\dot\varphi^2 + V_{\rm eff}(\varphi)$$

Calibration with the observed matter density is taken when

$$\varphi = \varphi_c$$

Conservation of dark energy reads:

$$\dot\rho_{\rm eff} + 3H(1+\omega_{\rm eff})\rho_{\rm eff} = 0$$

Calibration is usually taken to be now but in principle this has to be determined by comparison with data.

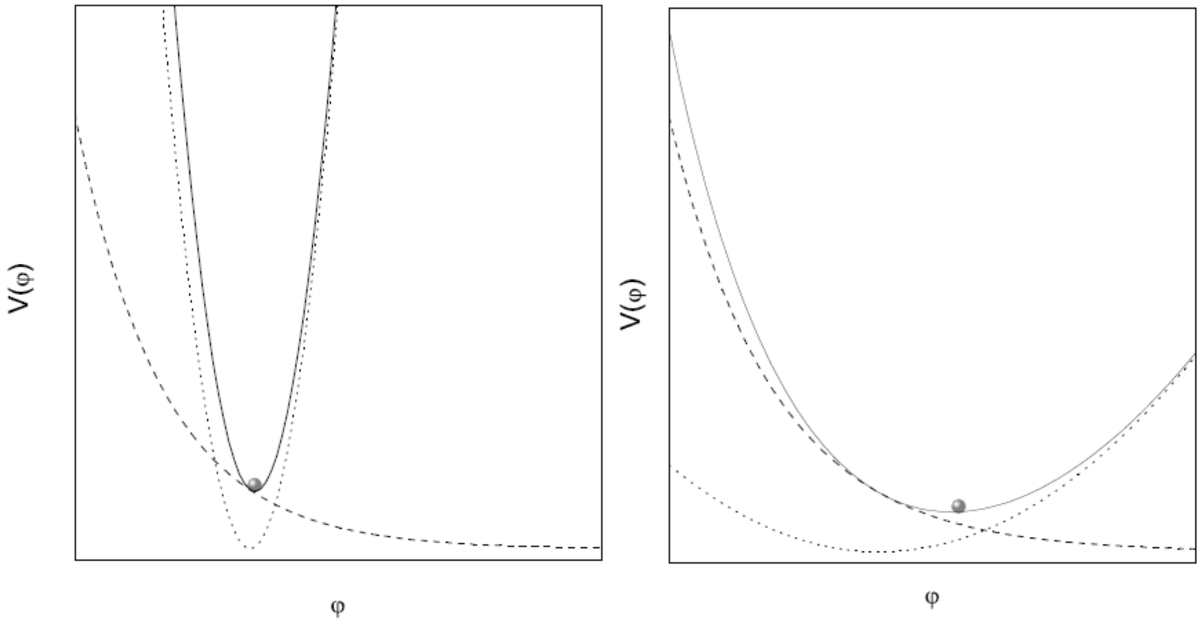
$$\omega_{\rm eff} = \frac{p_\varphi}{\rho_{\rm eff}}$$

$$p_\varphi = \frac{1}{2}\dot\varphi^2 - V(\varphi)$$

Assume that the effective potential has a minimum

$$\frac{dV}{d\phi} = -\frac{\beta(\phi)}{m_{\rm Pl}} \frac{A(\phi)}{A(\phi_c)} \rho_m$$

$$\beta(\phi) = m_{\rm Pl} \frac{d \ln A(\phi)}{d\phi}$$



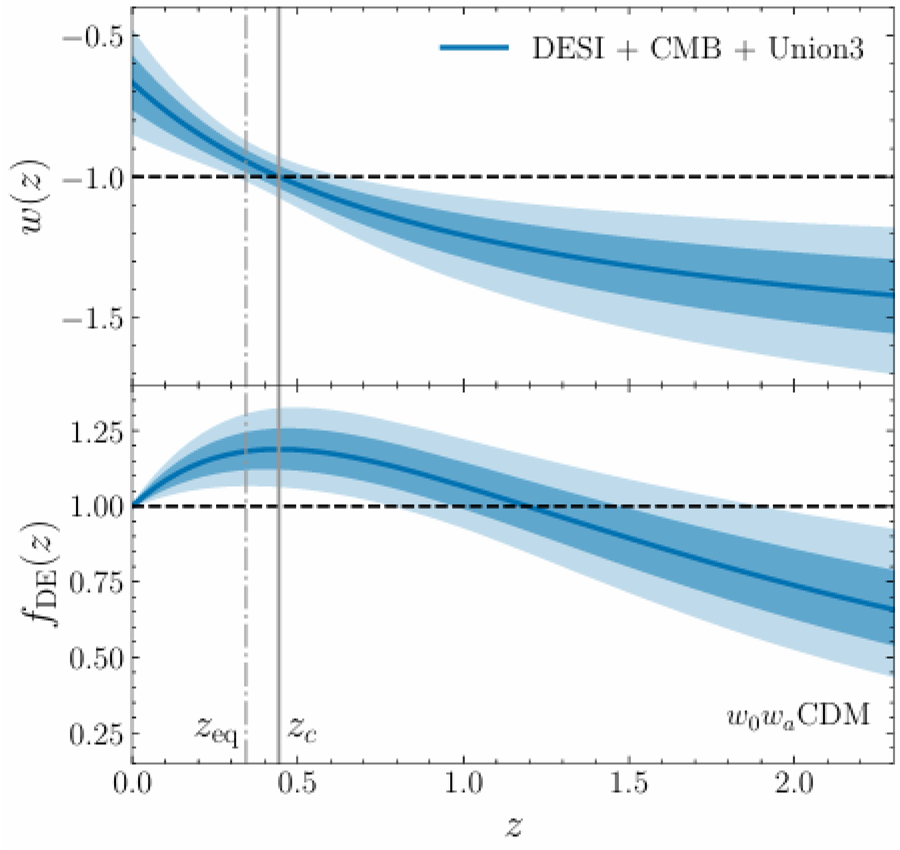
Concentrate on the evolution of the minimum close to calibration taken to be at low redshift:

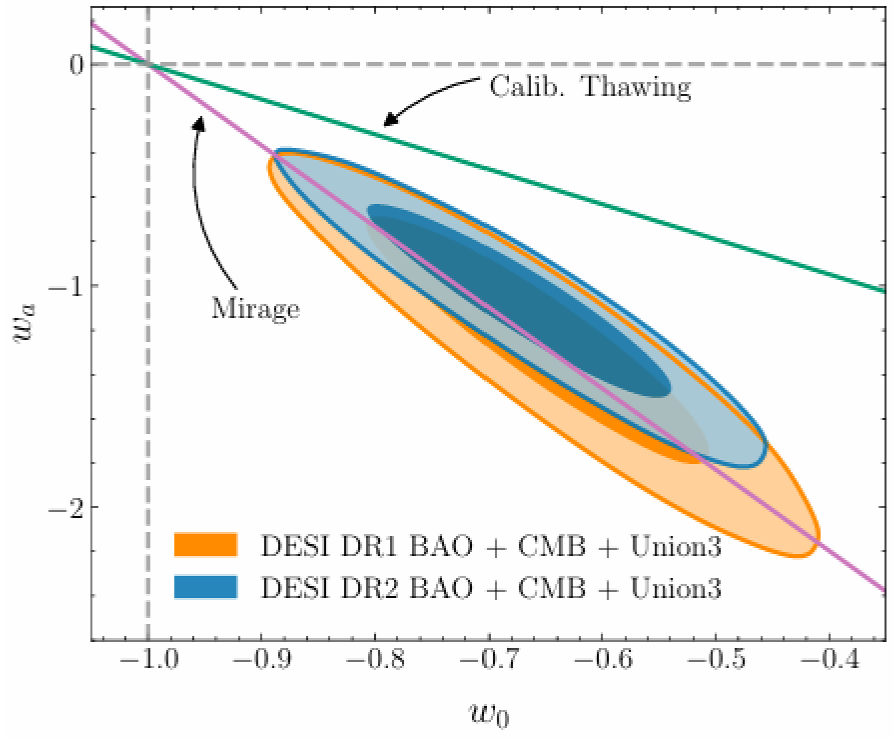
$$\phi - \phi_c = -\frac{\beta(\phi_c)}{m_{\rm eff}^2 m_{\rm Pl}} (\rho_m - \rho_c)$$

$$m_{\rm eff}^2 = \frac{d^2 V_{\rm eff}}{d\phi^2}|_{\phi=\phi_c}$$

$$\omega_{\rm eff} = -\frac{1}{1 - \beta^2(\phi_c) \frac{\rho_m}{V_{\rm DE}} \frac{(\rho_m - \rho_c)}{m_{\rm Pl}^2 m_{\rm eff}^2}}$$

$$V_{\rm DE} = V(\phi_c)$$





$$-1.6 \leq \omega_a \leq -0.8$$

$$\omega_0 = -1 + \frac{z_c}{z_c+1}\omega_a \Rightarrow z_c \sim 0.37$$

1/3.66

$$\frac{\beta^2(\phi_c)H_0^2}{m_{\rm eff}^2} = -\frac{\Omega_{\Lambda 0}}{9\Omega_{m0}^2(1+z_c)^5}\omega_a$$

$$0.047 \leq \frac{\beta^2(\phi_c)H_0^2}{m_{\rm eff}^2} \leq 0.093$$

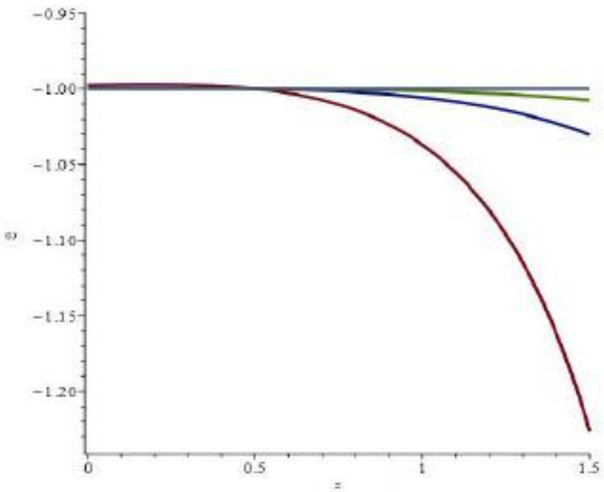
$$\omega_{\text{eff}} = -\frac{1}{1-\beta^2(\phi_c)\frac{\rho_m}{V_{\text{DE}}}\frac{(\rho_m-\rho_c)}{m_{\text{Pl}}^2 m_{\text{eff}}^2}}$$

- Crosses the phantom divide when: $\rho_m < \rho_c$

- Negligible variation unless: $m_{\text{eff}} \simeq H_0, \quad \beta(\phi_c) \sim 0.1$

Two major issues!

- ❑ For such a low mass, the minimum is not an attractor so undamped oscillations. Need to make comparison in Jordan frame.
- ❑ Long range forces with gravitational strength!



$z_c = 0.5$

$\beta(\phi_c) = 0.1, \frac{m_{\text{eff}}}{H_0} = 2, 5, 10, 10^3$

- Interactions between DE-DM $\beta \leq 10^{-2}$ CMB

$$\omega_{\rm eff} = -\frac{1}{1 - \beta^2(\phi_c)\frac{\rho_m}{V_{\rm DE}}\frac{(\rho_m - \rho_c)}{m_{\rm Pl}^2 m_{\rm eff}^2}}$$

- Crosses the phantom divide when: $\rho_m < \rho_c$
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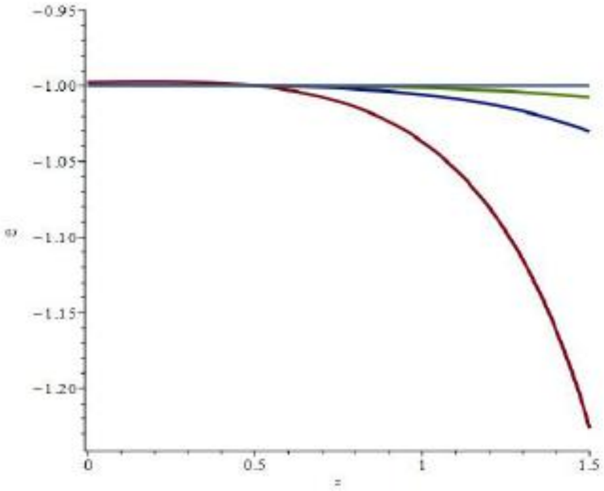

$z_c = 0.5$

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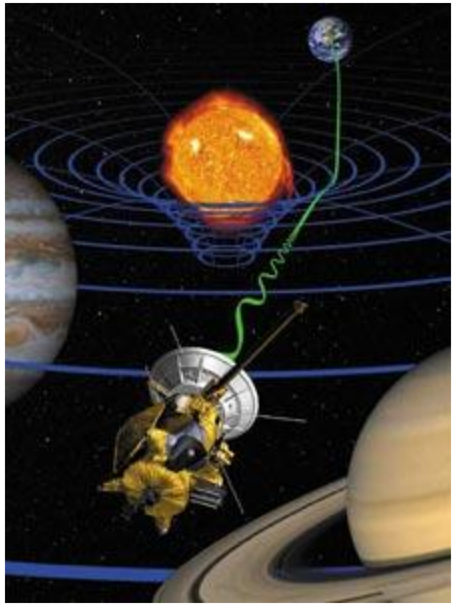
- Interactions between DE-DM $\beta \leq 10^{-2}$
- Interaction between DE-baryons

Interactions with baryons induce a change in Newton's law:

$$\Phi = -\frac{G_N M}{r}(1 + 2\beta^2 e^{-r/\lambda})$$



Bertotti et al. (2004)

For long range forces with large λ, the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay around a big object: the Sun):

$$\beta^2 \leq 2 \cdot 10^{-5}$$

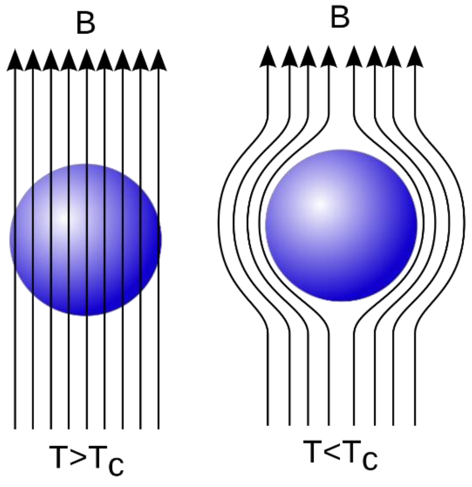
Fifth force: $\vec{a}_\phi = -\vec{\nabla}\phi$

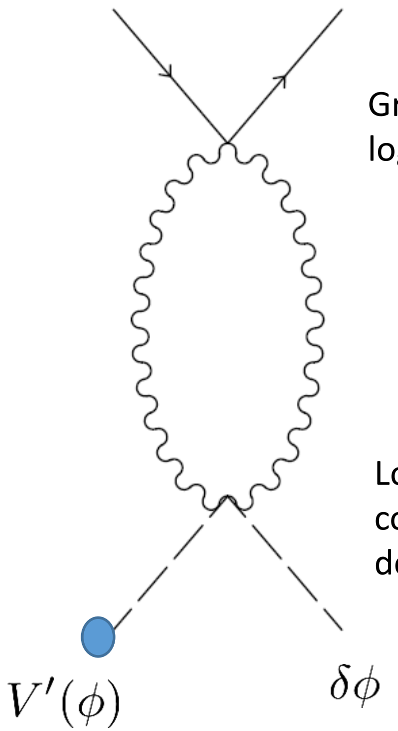
- Interaction between DE-baryon

Could be postulated to vanish…

Generically non-vanishing coupling becomes unobservable thanks to:

Screening

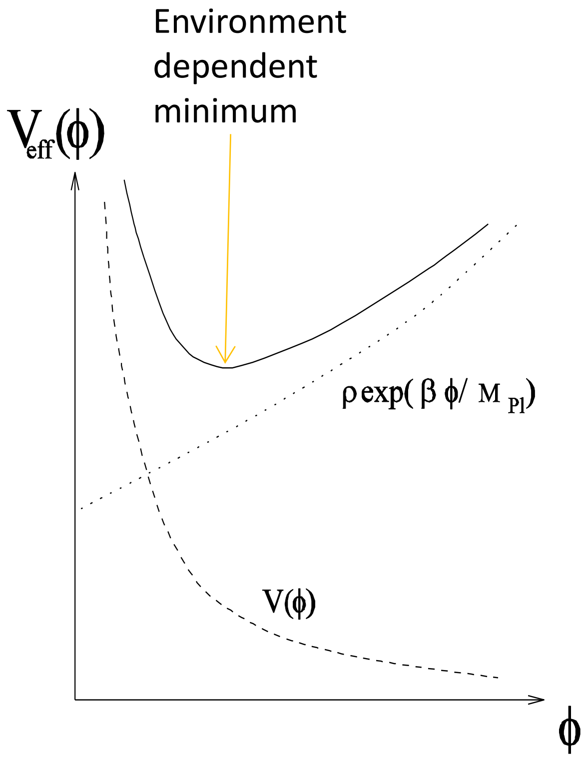


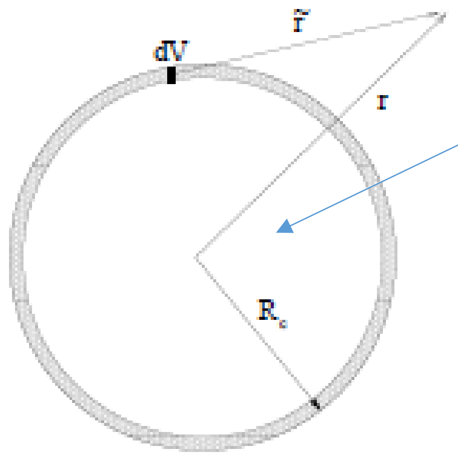


Chameleon Screening

When coupled to matter, scalar fields have a ***matter dependent effective potential***

$$V_{eff}(\phi) = V(\phi) + \rho_m\left(\frac{A(\phi)}{A(\phi_c)} - 1\right)$$





Large mass inside object

$$m_{\rm in} R \gg 1$$

The field generated from deep inside is Yukawa suppressed. Only a *thin shell* radiates outside the body. Hence suppressed scalar contribution to the fifth force.

In single field case, screening implies:

$$m_{\rm eff}/H_0 \gtrsim 10^3 \Rightarrow \omega_{\rm eff} \simeq -1$$

Multi-field dark energy sector

$$\mathcal{L} = -\frac{1}{2} g_{ij}(\phi^k)\partial_\mu \phi^i \partial^\mu \phi^j - V(\phi^i)$$

For light dark energy fields, screening and phantom crossing can happen with more than one field and a **non-trivial σ-model metric**.

$$\mathcal{L} \supset -\frac{1}{2}((\partial\phi)^2 + W^2(\phi)(\partial a)^2)$$

Dilaton

Axion

The axio-dilaton case corresponding to the volume modulus of compactifications from 10d to 4d:

$$K = -3m_{\rm Pl}^2 \ln(T + \bar{T}) \longrightarrow \beta = \frac{1}{\sqrt{6}}$$

The volume modulus can be decomposed in:

$$T = e^{\phi} + ia \longrightarrow e^{K/6m_{\rm Pl}^2} = e^{-\phi/2}$$

dilaton

axion

$$W(\phi) = e^{-\phi}$$

Dilaton coupling to matter of order one!

Screening depends on the large mass of the axion inside the object:

$$m_{\text{in}} R \gg 1$$

It also requires a sharp jump of the axion between vacuum and matter:

$$V(a) = \frac{1}{2} m_a^2 (a - a_+)^2 + \rho_m \frac{(a - a_-)^2}{2\Lambda_a^2}$$

The Klein-Gordon for the dilaton is:

Coupling constant to matter.

$$\Box\phi = WW'(\partial a)^2 + \frac{\beta}{m_{\rm Pl}^2}\rho$$

Axion driven “potential”

Far away from the body we expect:

$$\phi = \phi_\infty - \frac{L m_{\rm Pl}}{r}$$

$$L = 2\beta G_N M$$

In the absence of screening.

Local value of the field in the environment

The delta function as a source term implies a jump of the derivative of the dilaton:

$$\phi'_{\rm out}(R) - \phi'_{\rm in}(R) = \left(\frac{WW'}{2\ell}\right)_{r=R}(a_+ - a_-)^2$$

$$Lm_{\rm Pl} = R^2\phi'_{\rm out}(R)$$

Needs to be negative to reduce the scalar charge of the object

The scalar charge L is determined by the competition between the different energy sources for the scalar field profile. This competition will select the **local value of the field** and determine the scalar charge.

In the axio-dilaton case, the “solar” system is small enough that the local energy is dominated by the local dynamics which differ from the cosmological one.

$$E_{\rm kin} = 2\pi \int_0^\infty dr\; r^2 (W^2 (a')^2 + (\phi')^2)$$

Gives a surface contribution

Dependent on the local value

$$E_{\rm kin,a} = \frac{\pi}{\ell} R^2 W^2 (r=R)(a_+^2 - a_-^2)$$

$$E_{\rm kin} = E_0 + \frac{L^2}{4 G_N R}$$

$$L = 2\beta G_N M + R^2 (\frac{W W'}{2\ell})_{r=R} \frac{(a_+ - a_-)^2}{m_{\rm Pl}}$$

Exponential Screening:

Screening takes place when:

$$W^2(\phi) = e^{-\xi\phi/m_{\rm Pl}}$$

The coupling to matter becomes:

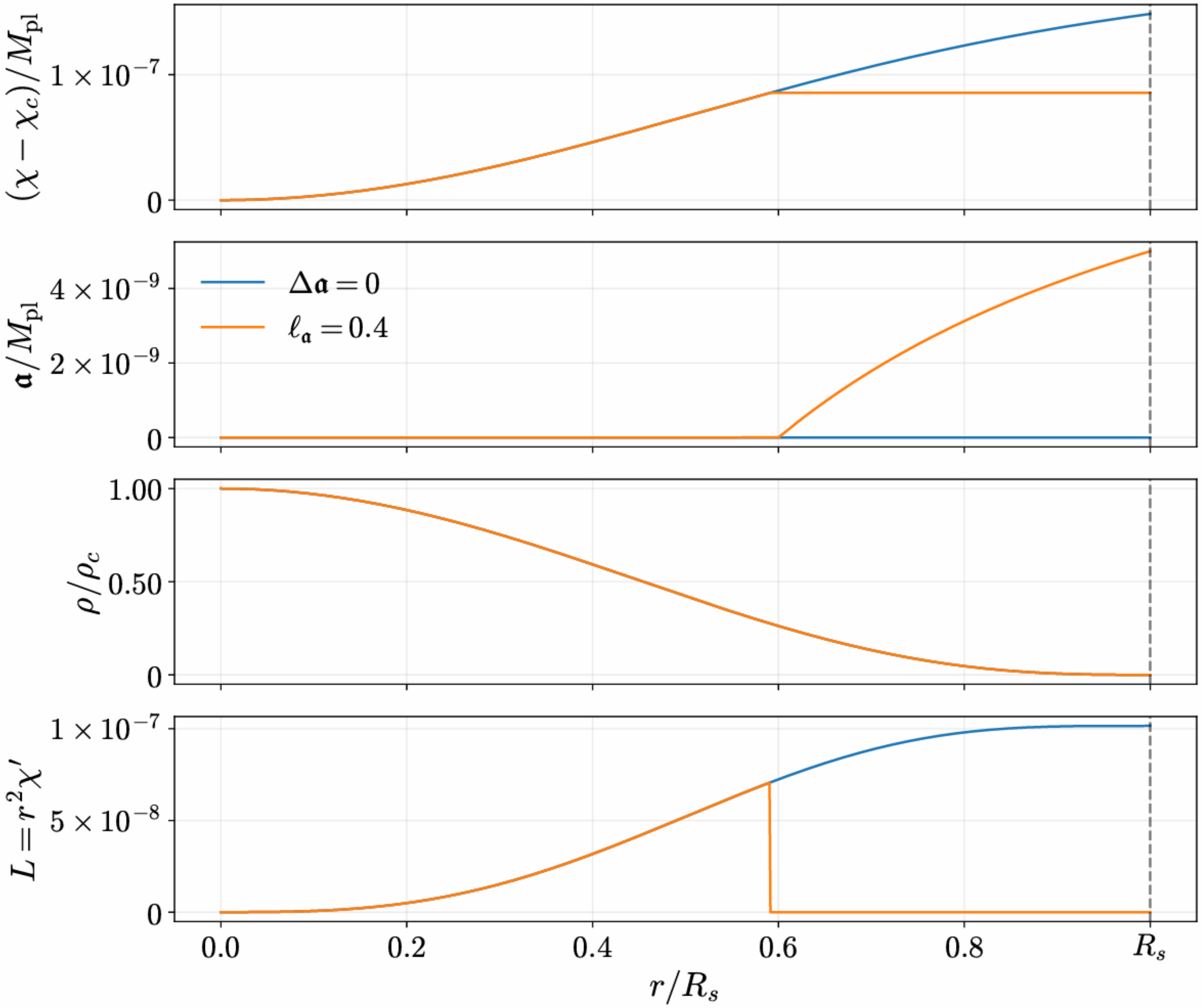
$$\beta_{\rm eff} = \beta \frac{R}{2\xi G_N M}$$

Screening in solar system:

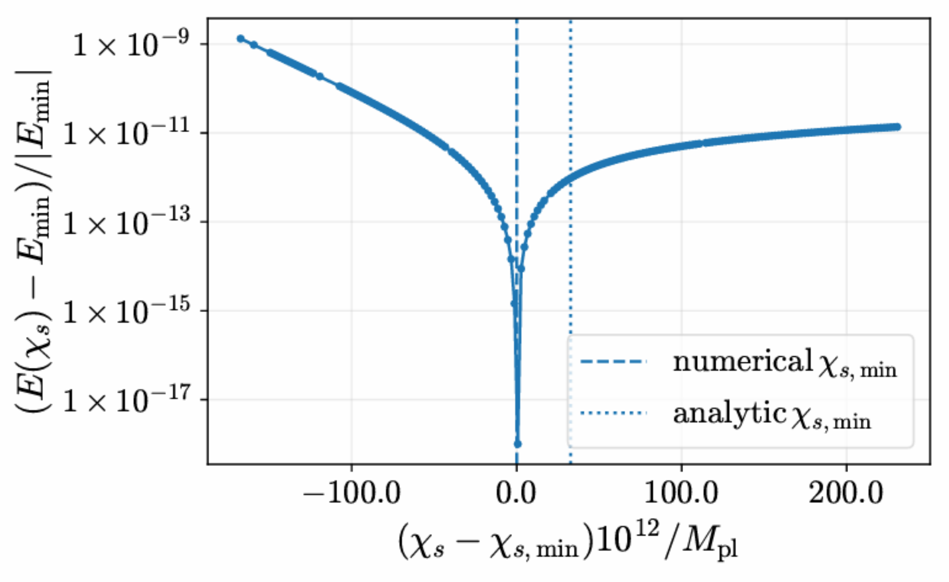
$$\xi = \sqrt{\frac{2}{3}}$$

$$\xi \gtrsim 10^9 \Rightarrow \frac{m_{\rm Pl}}{\xi} \lesssim 10^9 \rm GeV$$

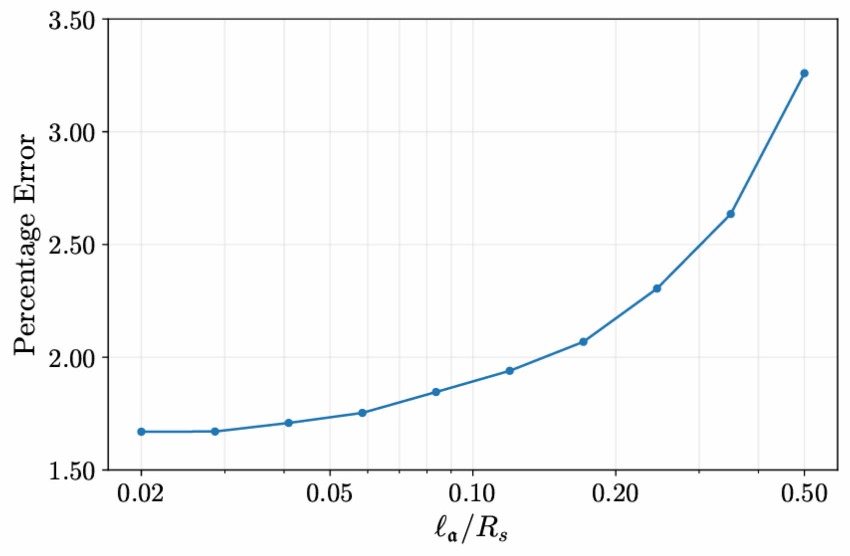
$$\Lambda_\phi = \frac{m_{\rm Pl}}{\xi}$$



$(\chi-\chi_c)/M_{\rm pl}$
1×10^{-7}
0
$\mathfrak{a}/M_{\rm pl}$
4×10^{-9}
2×10^{-9}
0
$\Delta\mathfrak{a}=0$
$\ell_{\mathfrak{a}}=0.4$
ρ/ρ_c
1.00
0.50
0
$L=r^2\chi'$
1×10^{-7}
5×10^{-8}
0
0.0
0.2
0.4
0.6
0.8
R_s
r/R_s



1×10^{-9}
1×10^{-11}
1×10^{-13}
1×10^{-15}
1×10^{-17}
$(E(\chi_s) - E_{\rm min})/|E_{\rm min}|$
numerical $\chi_{s,\,\rm min}$
analytic $\chi_{s,\,\rm min}$
-100.0
0.0
100.0
200.0
$(\chi_s - \chi_{s,\,\rm min})10^{12}/M_{\rm pl}$



3.50
3.00
2.50
2.00
1.50
Percentage Error
0.02
0.05
0.10
0.20
0.50
$\ell_{\mathfrak{a}}/R_s$

Early dark energy for free!

$$V(a) = \frac{1}{2}m_a^2(a - a_+)^2 + \rho_m \frac{(a - a_-)^2}{2\Lambda_a^2}$$

$$V(a_-) = \frac{1}{2}m_a^2(a_+ - a_-)^2$$

Early dark energy

A fraction of added matter

$$V(a_+) = \rho_m \frac{(a_+^2 - a_-)^2}{2\Lambda_a^2}$$

Early dark energy | Dark matter like

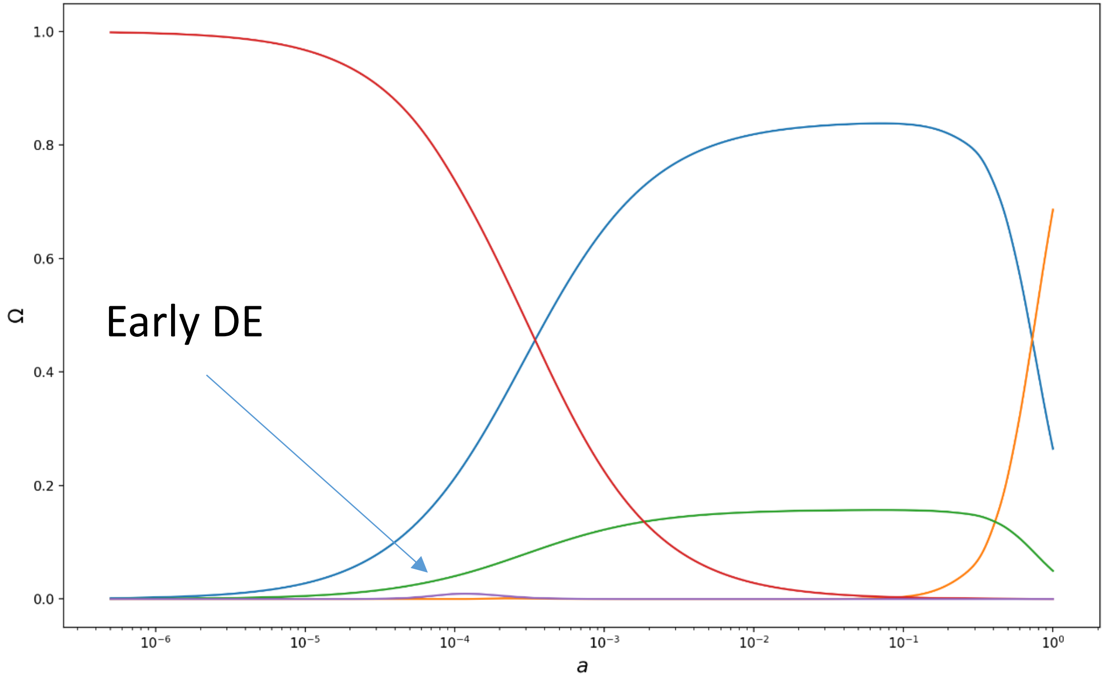
$m_a^2\Lambda_a^2$

For specific applications, we take an Albrecht-Skordis potential (also called Yoga for idiosyncratic reason) :

$$V(\phi) = U(\phi)e^{-\lambda\phi/m_{\rm Pl}}$$

Quadratic with non-trivial minimum. Only purpose is to reproduce the amount of dark energy.

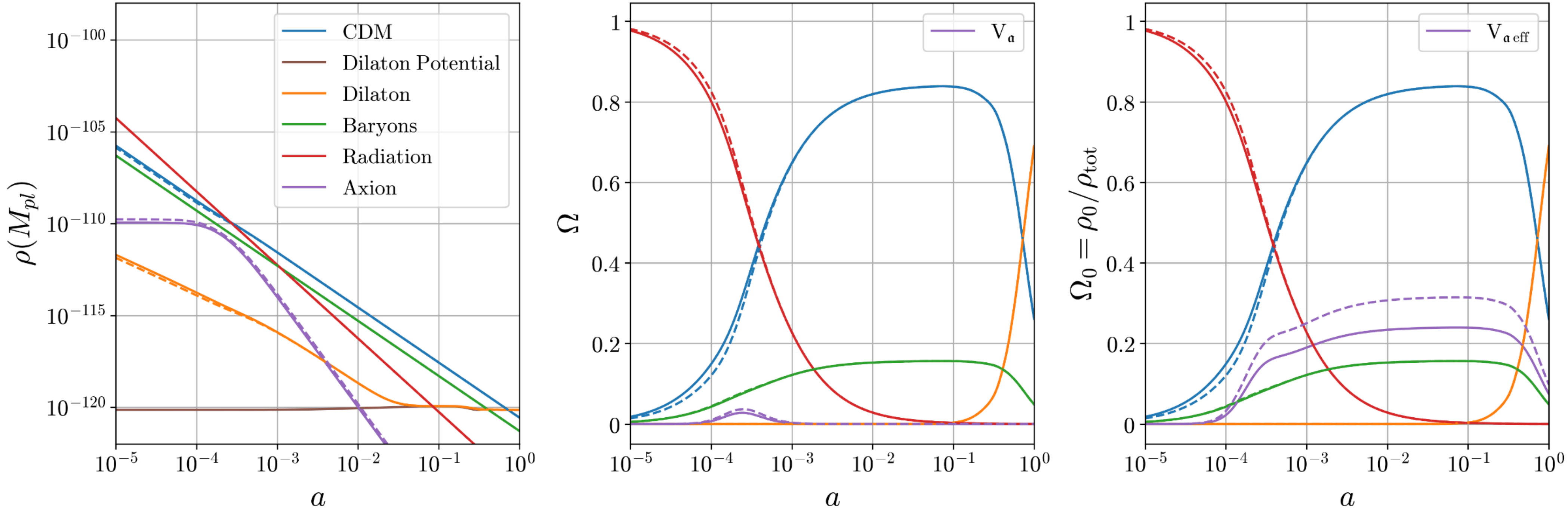
$$A(\phi) = e^{-\beta\frac{\phi}{m_{\rm Pl}}}$$



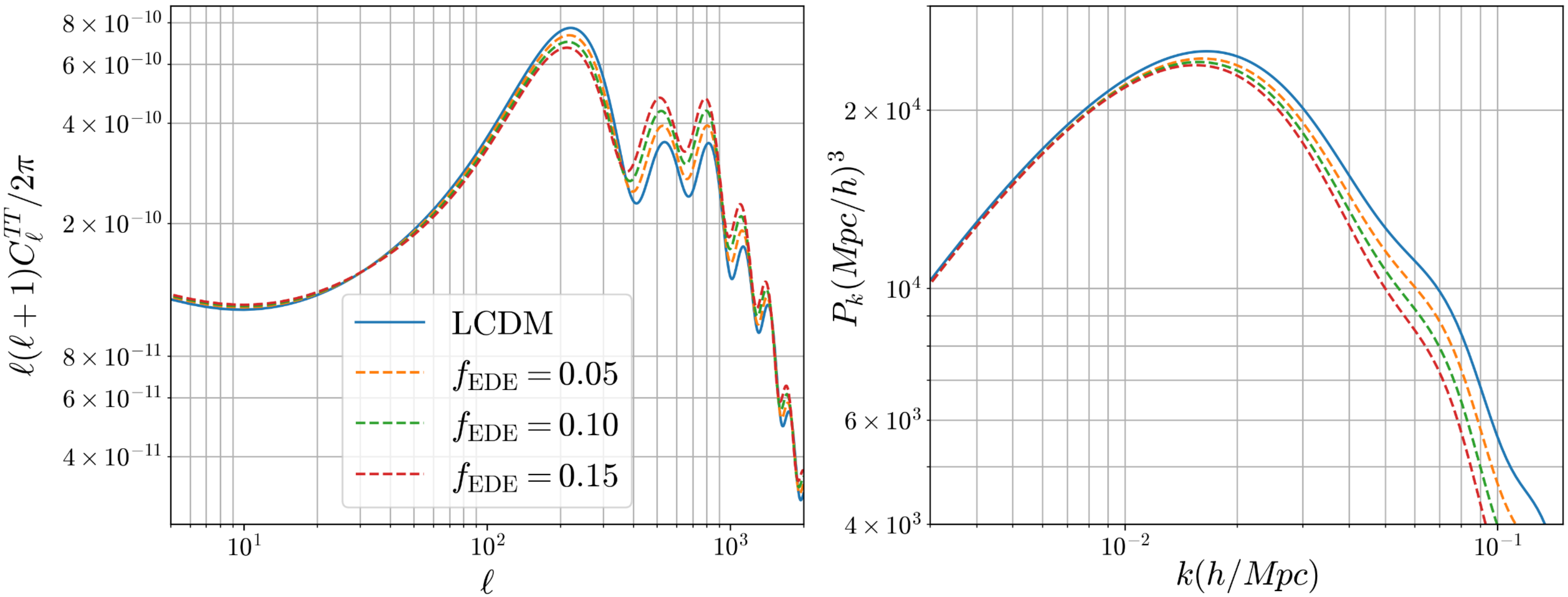
$$\lambda = 4\beta$$

Conformal rescaling can violate the swampland bound.

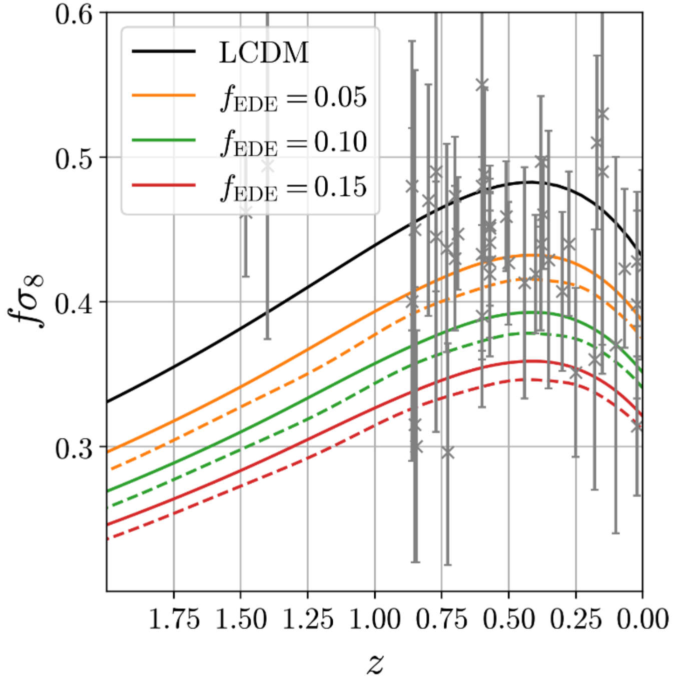
$$m_a = 2.10^{-15}\text{eV},\ \Lambda_\phi = 2.10^{8}\text{GeV}, \Lambda_a = 5.10^{5}\text{GeV}$$



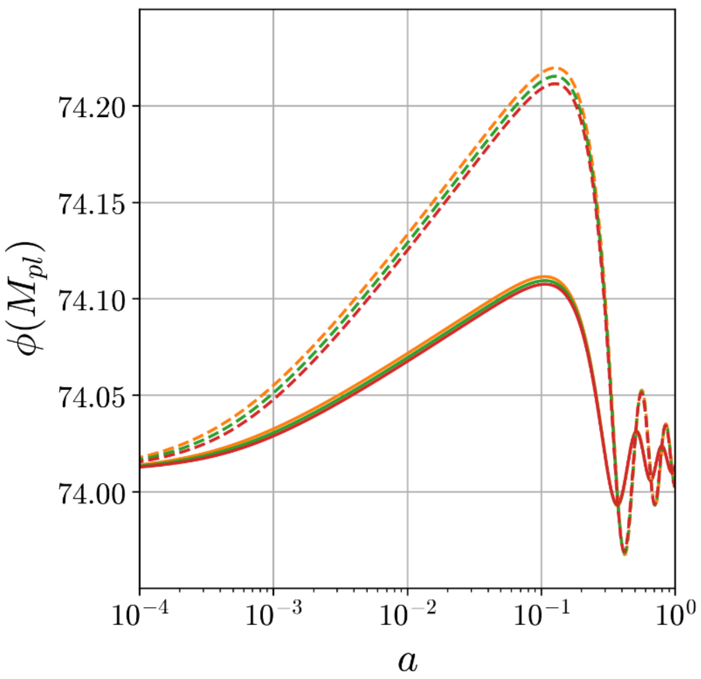
Early dark energy shows its face in the form of the axion potential.



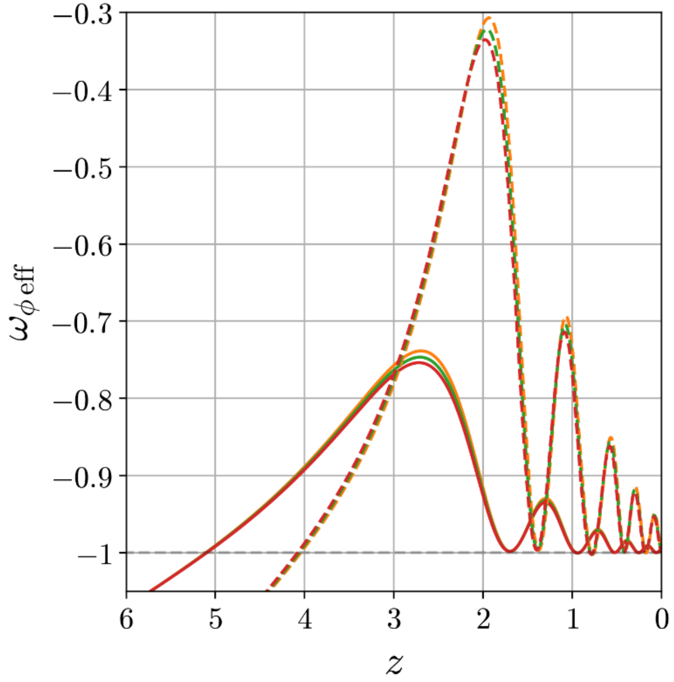
Increasing the early dark energy fraction increases the deviation from LCDM.



Dotted lines: the coupling to matter increases and growth reduces



The dilaton is light and oscillates



The effective equation states oscillates and crosses the phantom divide.

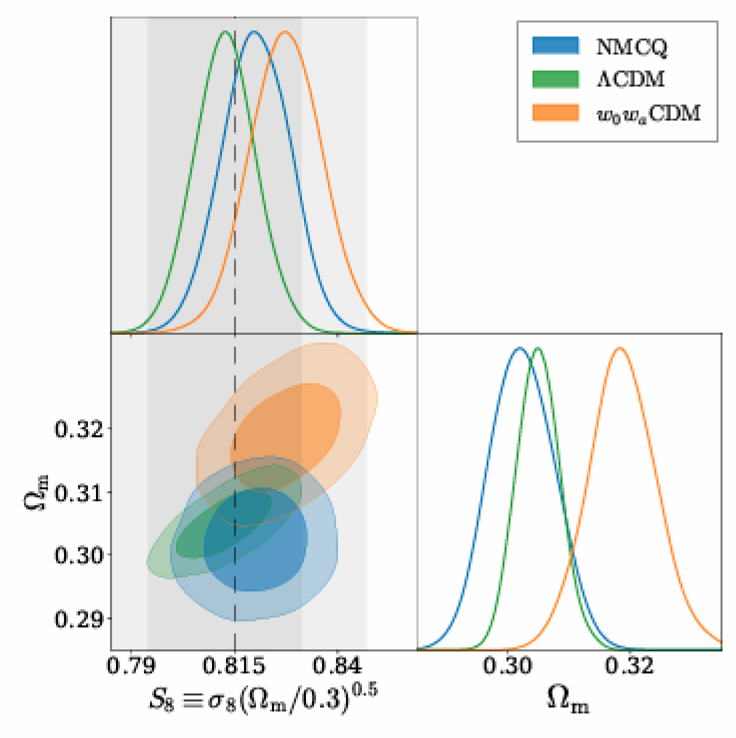
Summary

Multi-field dark energy models coupled to matter can have a number of interesting features:

- ✓ Can accommodate a fraction of early dark energy whilst screening.
- ✓ Varying equation of state crossing phantom divide and oscillations.
- ✓ Have less growth than LCDM despite long range fifth forces.

Model Building?

Prospects



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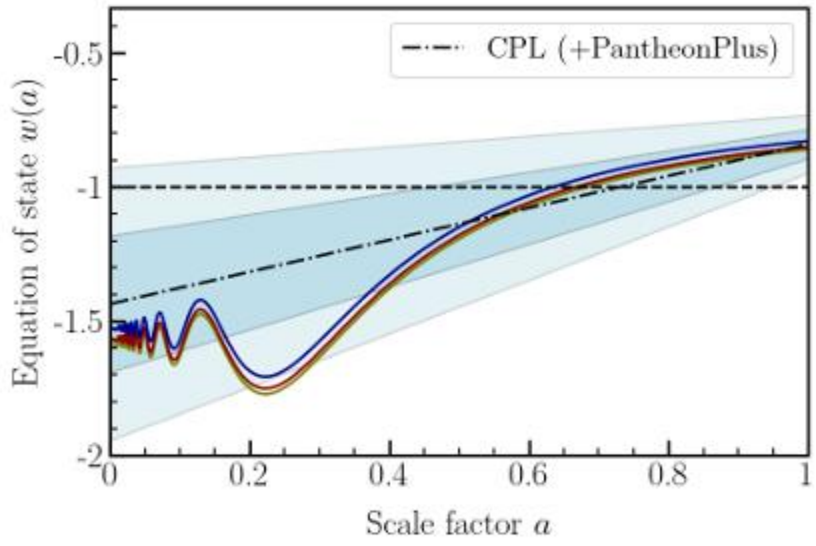

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