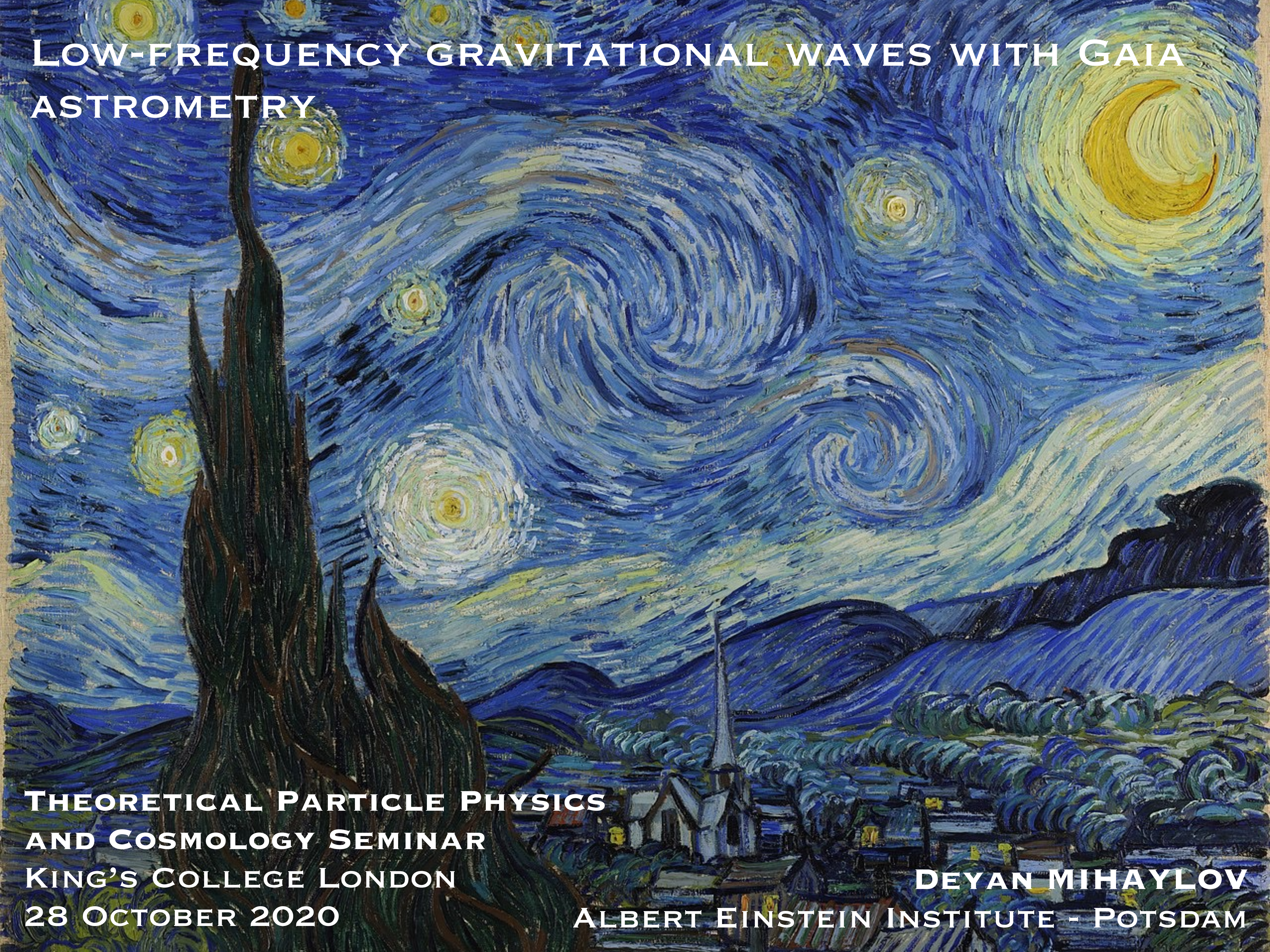
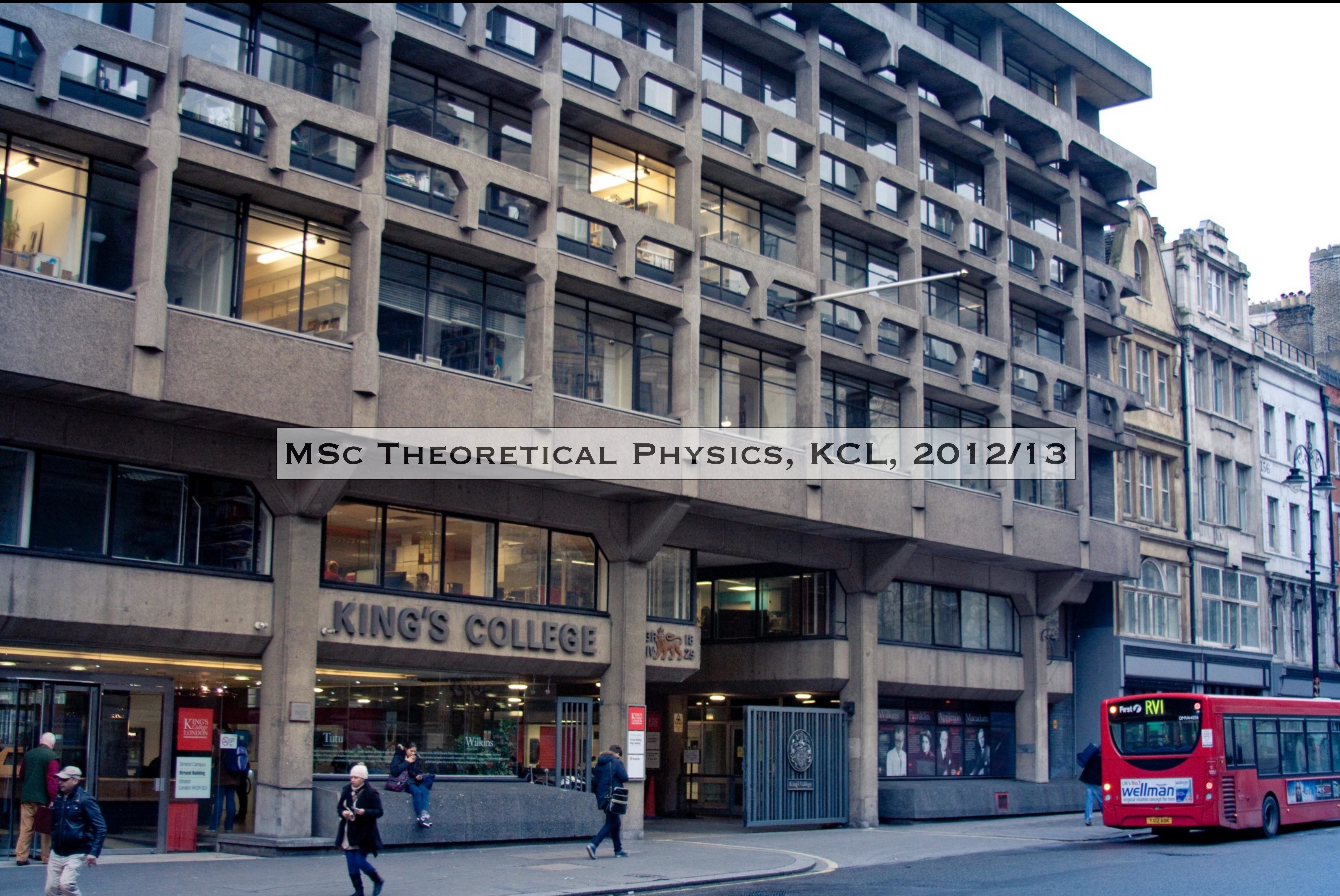


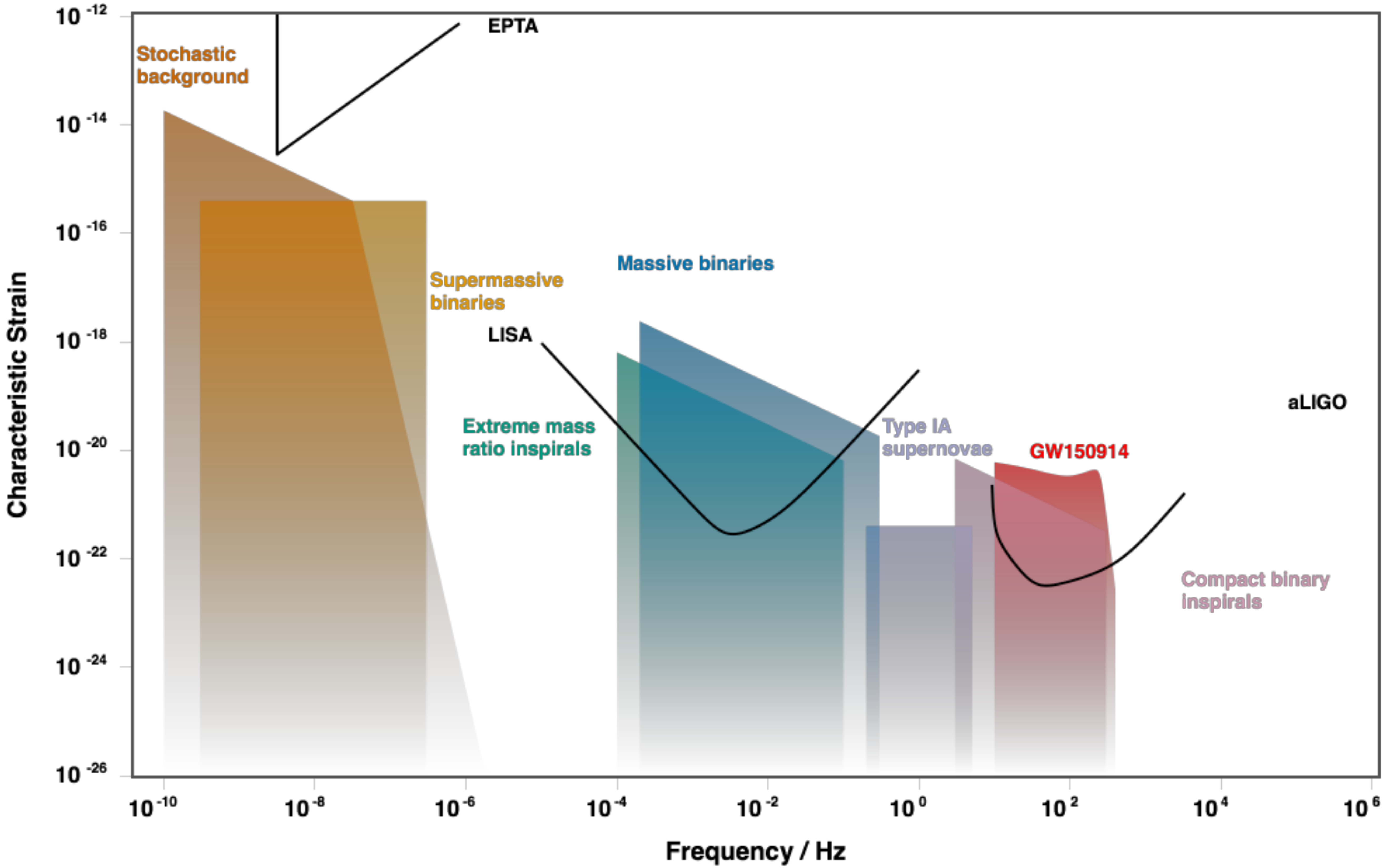
Low-frequency gravitational waves with Gaia astrometry

Theoretical Particle Physics and Cosmology Seminar
King's College London
28 October 2020

Deyan Mihaylov
Albert Einstein Institute - Potsdam

MSc Theoretical Physics, KCL, 2012/13





EPTA
Stochastic background
Supermassive binaries
Massive binaries
LISA
Extreme mass ratio inspirals
Type IA supernovae
GW150914
aLIGO
Compact binary inspirals
Characteristic Strain
Frequency / Hz
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
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10^{0}
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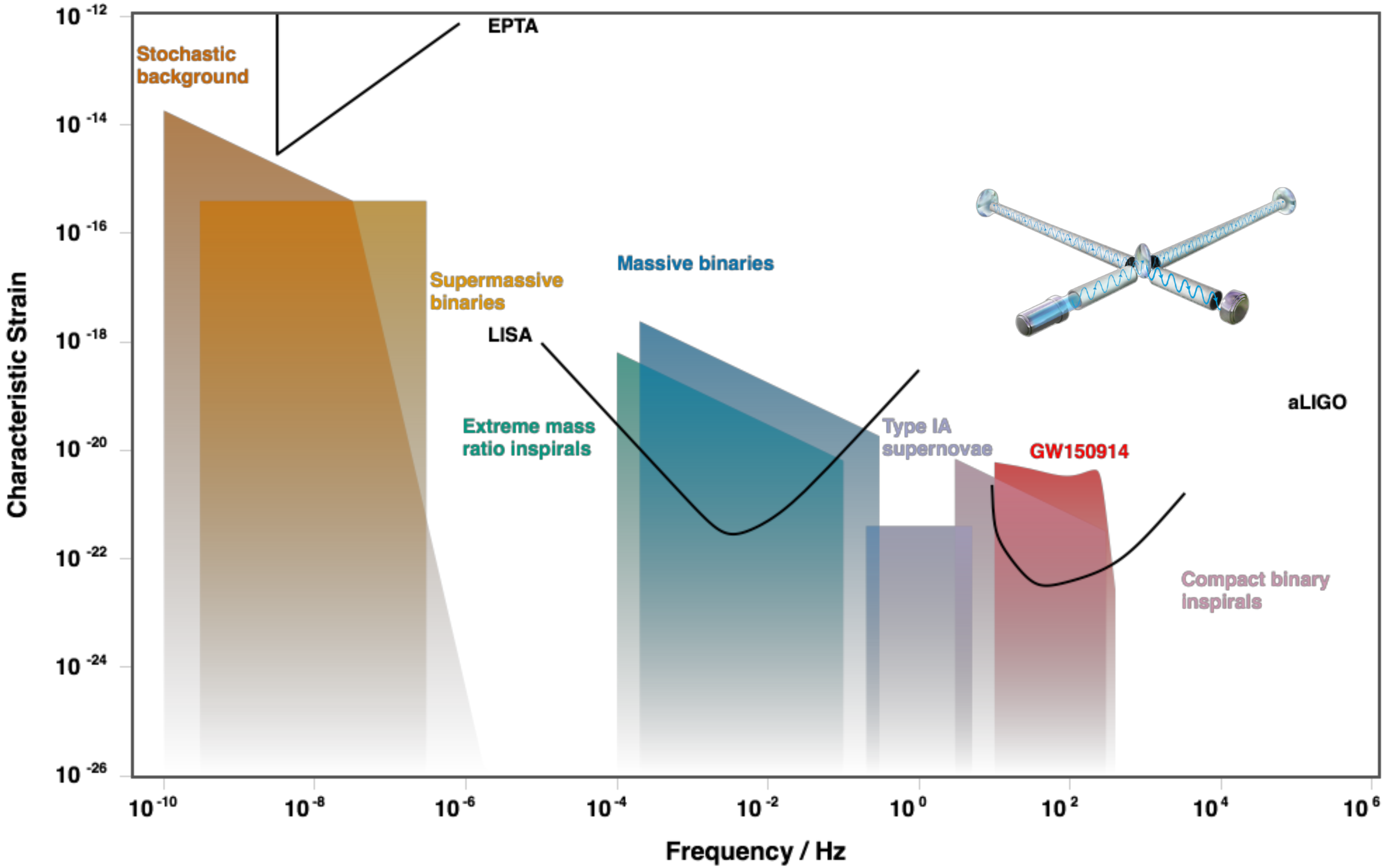
10^{4}
10^{6}



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10 -18
10 -20
10 -22
10 -24
10 -26
Frequency / Hz
10 -10
10 -8
10 -6
10 -4
10 -2
10 0
10 2
10 4
10 6
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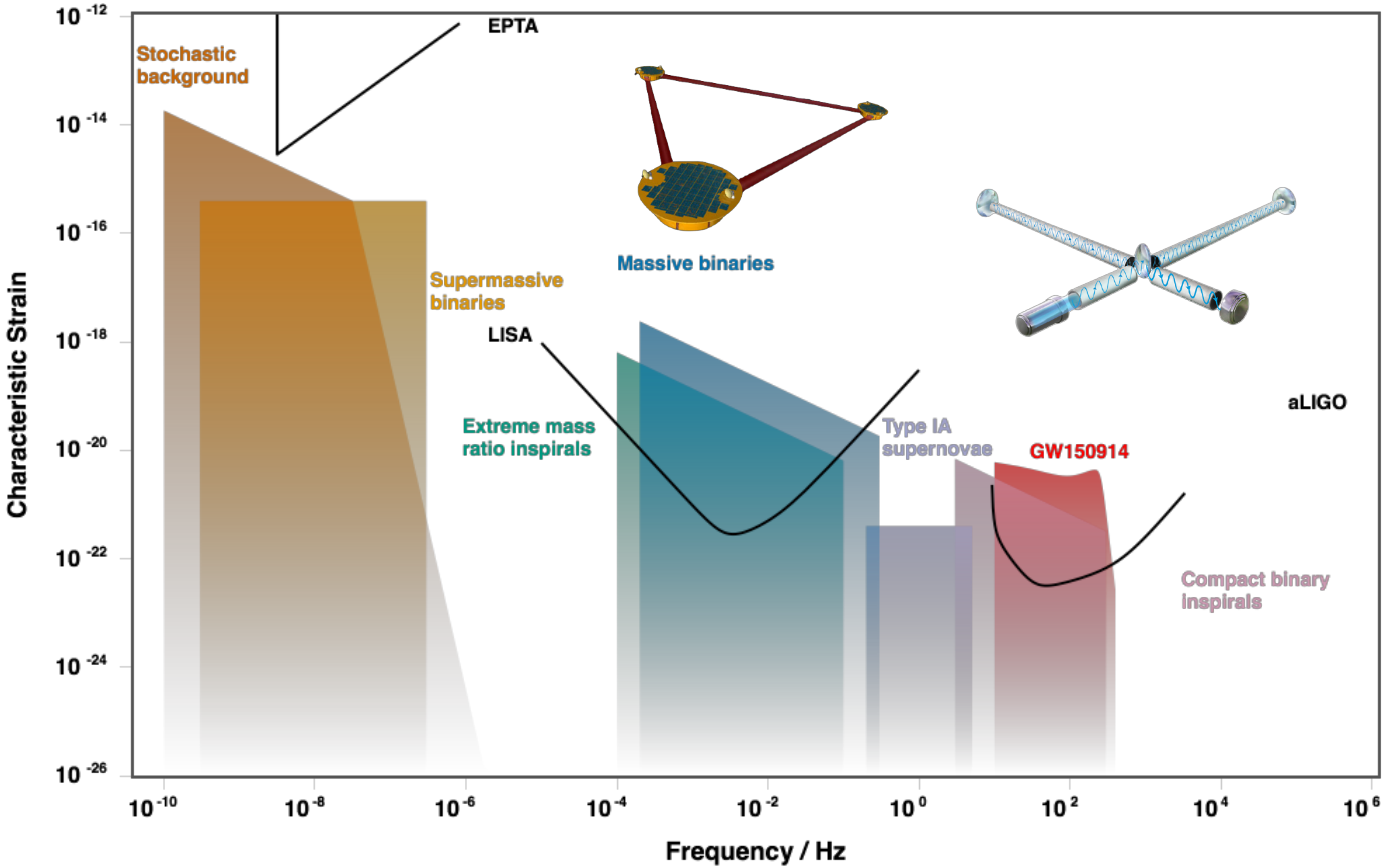
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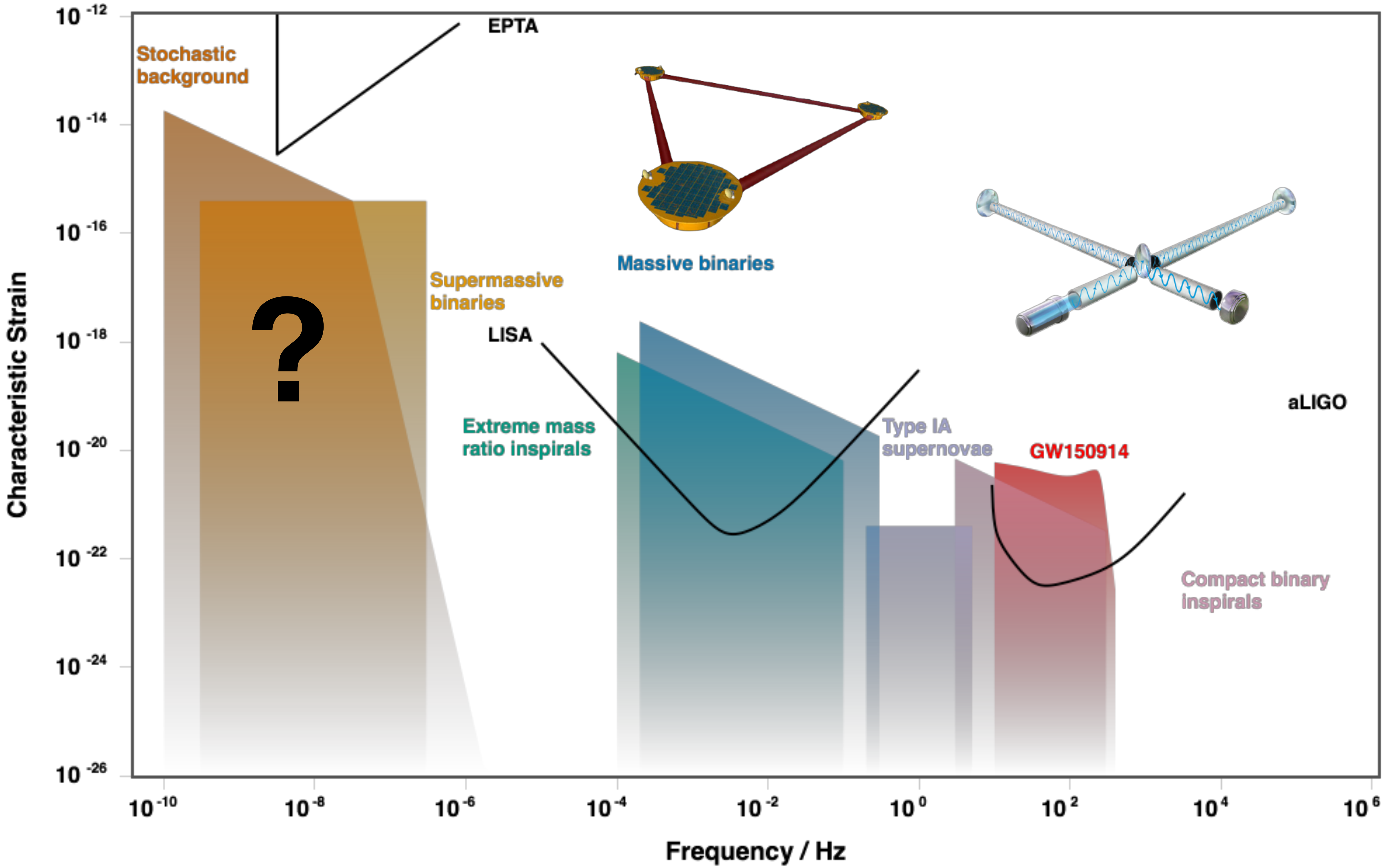

Characteristic Strain
10 -12
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10 -16
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10 -22
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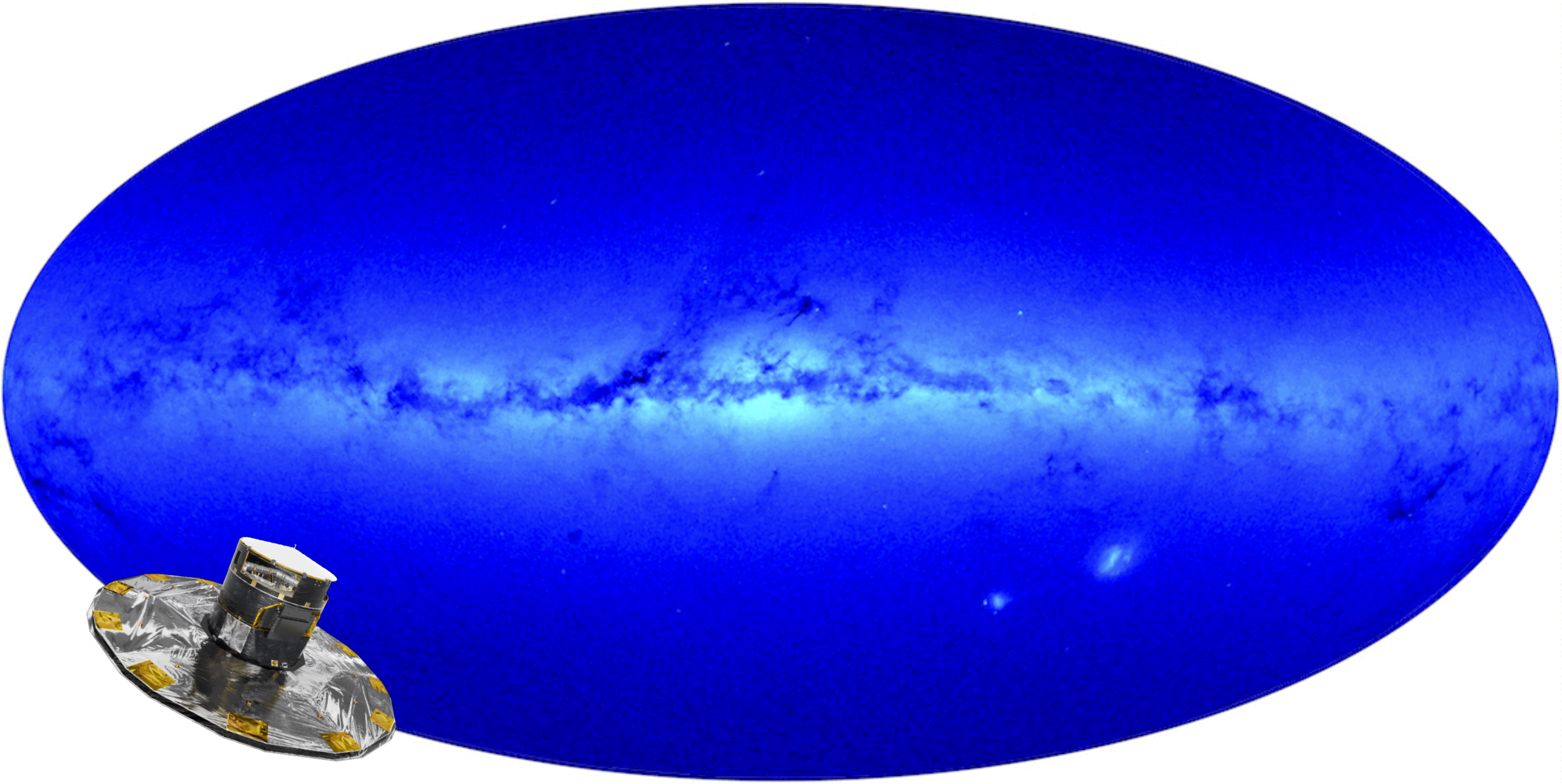
Frequency / Hz

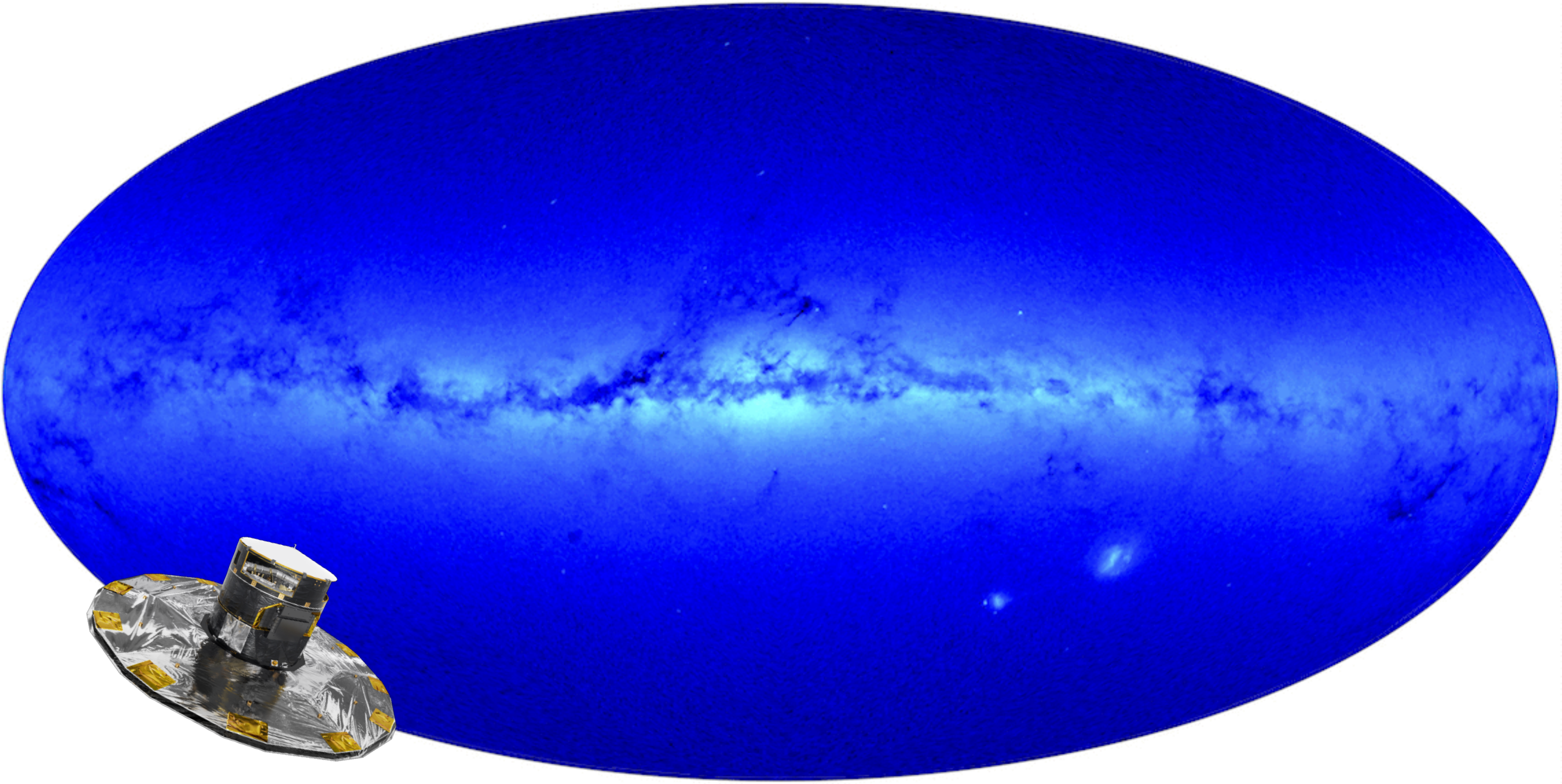


Gerry Gilmore

Anthony Lasenby

Jonathan Gair

Christopher Moore



GERRY GILMORE

ANTHONY LASENBY

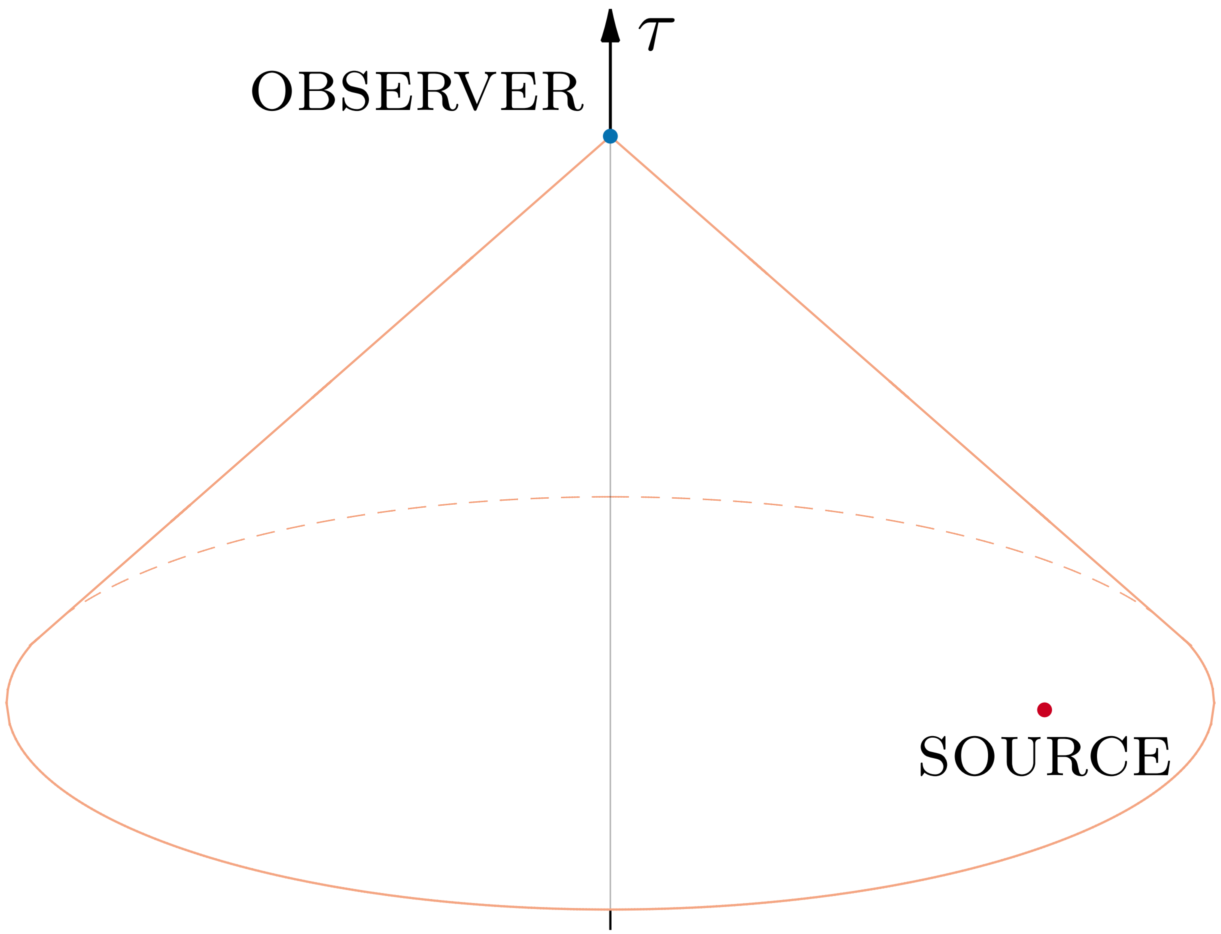
JONATHAN GAIR

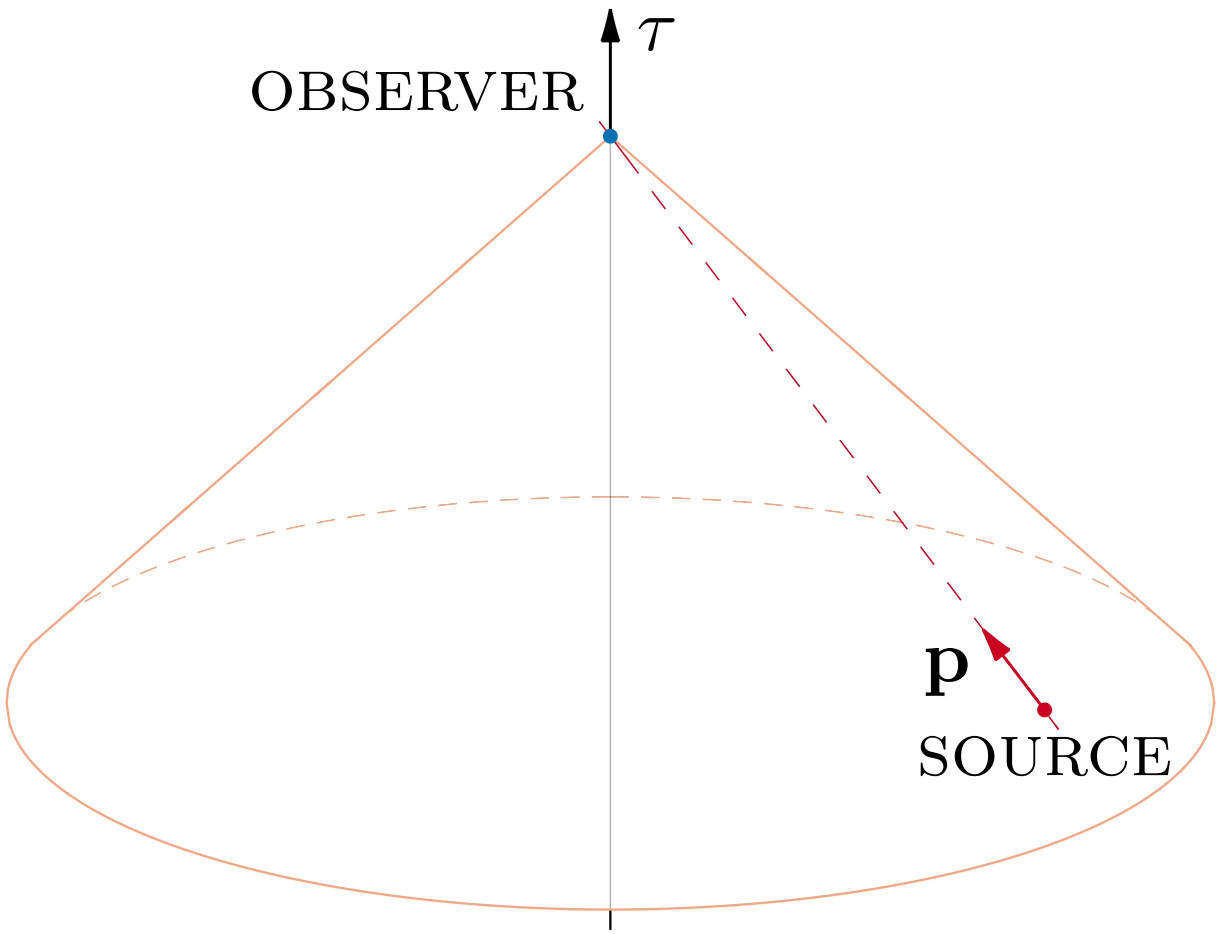
CHRISTOPHER MOORE

Contents

Astrometric response of a Gravitational Wave

Observer (Earth) and photon source (star) are at rest in flat space

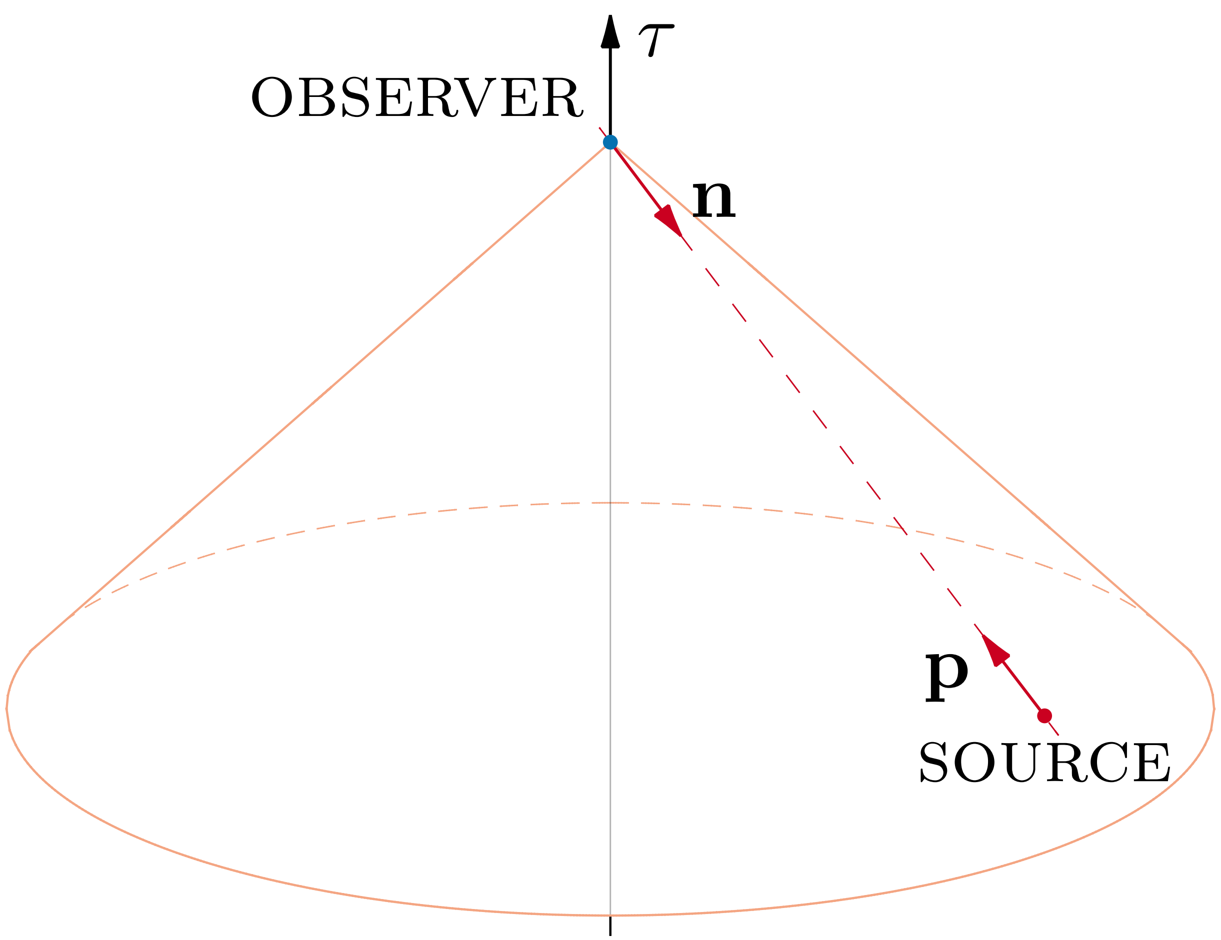




Observer (Earth) and photon source (star) are at rest in flat space

Joined by a null geodesic

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\, x^\mu(\lambda) = 0, \quad p^\mu = \frac{\mathrm{d}}{\mathrm{d}\lambda}\, x^\mu(\lambda) \equiv \text{const.}$$



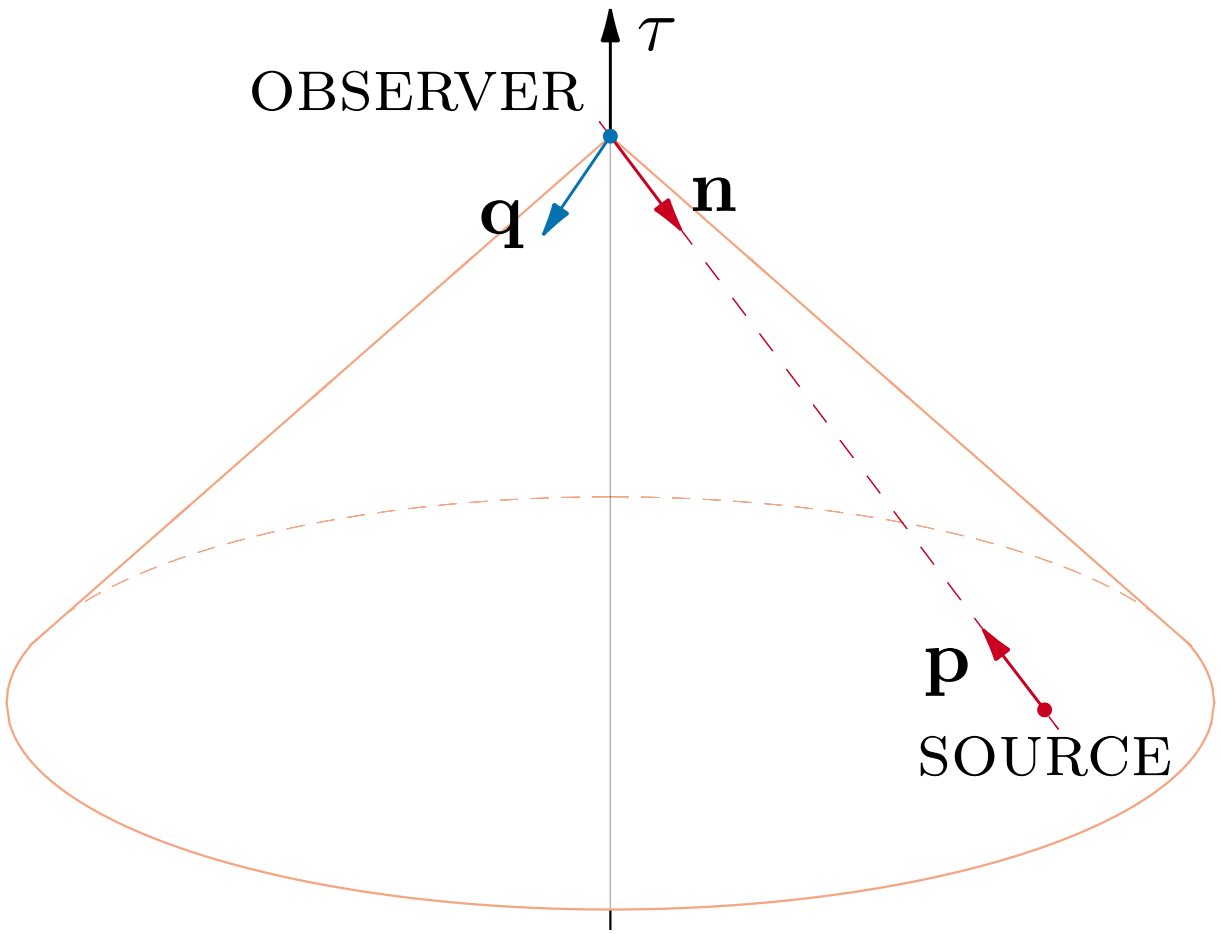
OBSERVER (EARTH) AND PHOTON SOURCE (STAR) ARE AT REST IN FLAT SPACE

JOINED BY A NULL GEODESIC

$$\frac{d^2}{d\lambda^2} x^\mu(\lambda) = 0, \quad p^\mu = \frac{d}{d\lambda} x^\mu(\lambda) \equiv \text{const.}$$

THE OBSERVER MEASURES ASTROMETRIC POSITION AND FREQUENCY

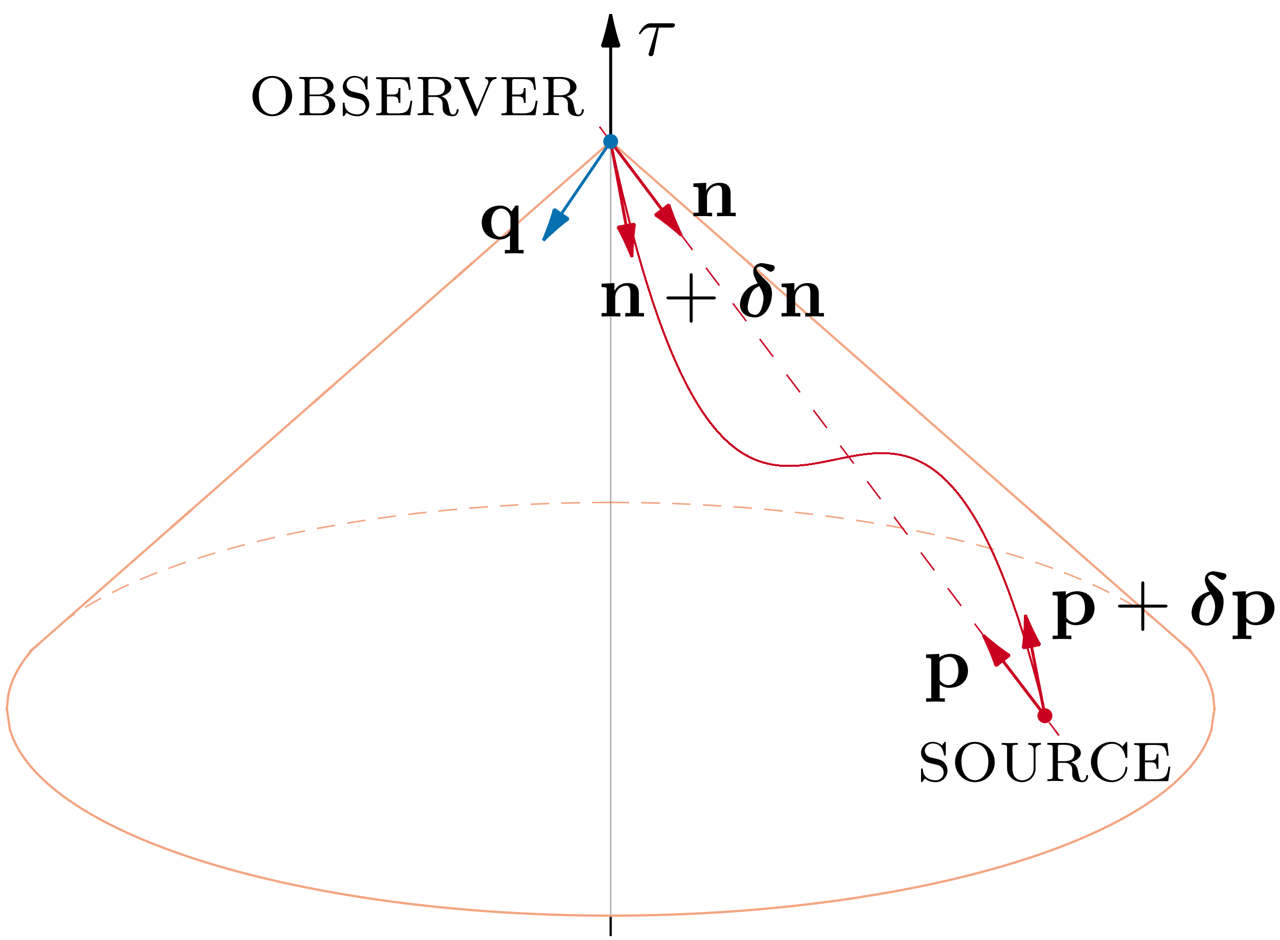
$n_{\hat{i}}, \quad \Omega$



Now consider perturbing the flat space-time with a GW

$$h_{\mu\nu}(t, x^i) = \Re\{H_{\mu\nu} \exp(\mathrm{i}k_\rho x^\rho)\},$$

$$k^\rho = \omega(1, -q^i)$$



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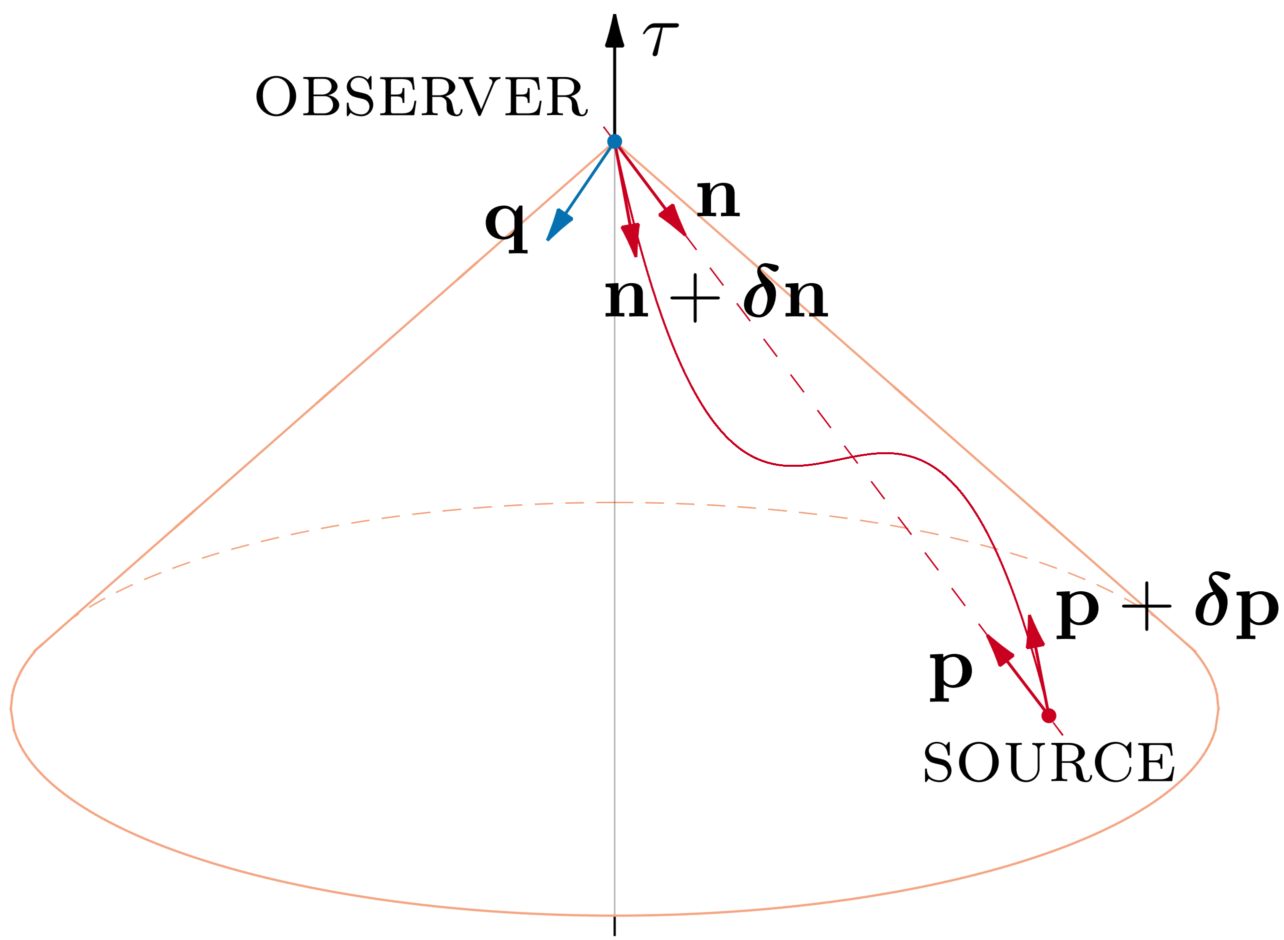
$$h_{\mu\nu}(t, x^i) = \Re\{H_{\mu\nu}\exp(\mathrm{i}k_\rho x^\rho)\}\,,$$
$$k^\rho = \omega\,(1, -q^i)$$

Worldlines of **observer** and **source** are unaffected

Photon worldline is a geodesic in both metrics

$$x^\mu(\lambda) \mapsto x^\mu(\lambda) + \delta x^\mu(\lambda)$$



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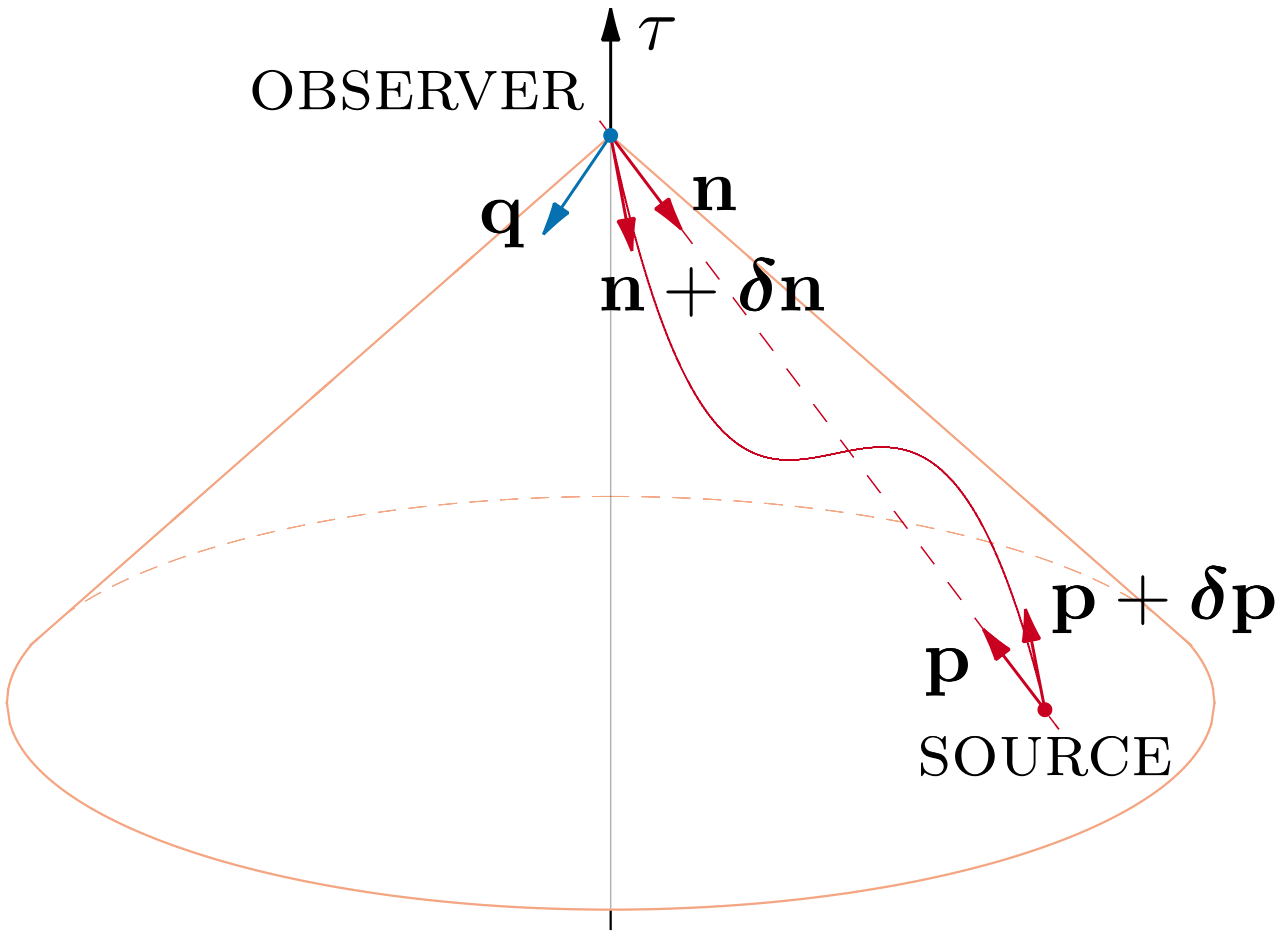
Worldlines of **observer** and **source** are unaffected

Photon worldline is a geodesic in both metrics

$$x^\mu(\lambda) \mapsto x^\mu(\lambda) + \delta x^\mu(\lambda)$$

Evolves according to the parallel transport equation

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\,\delta x^\mu_{t_0}(\lambda) = -\Gamma^\mu_{\nu\rho}\,p^\nu p^\rho$$

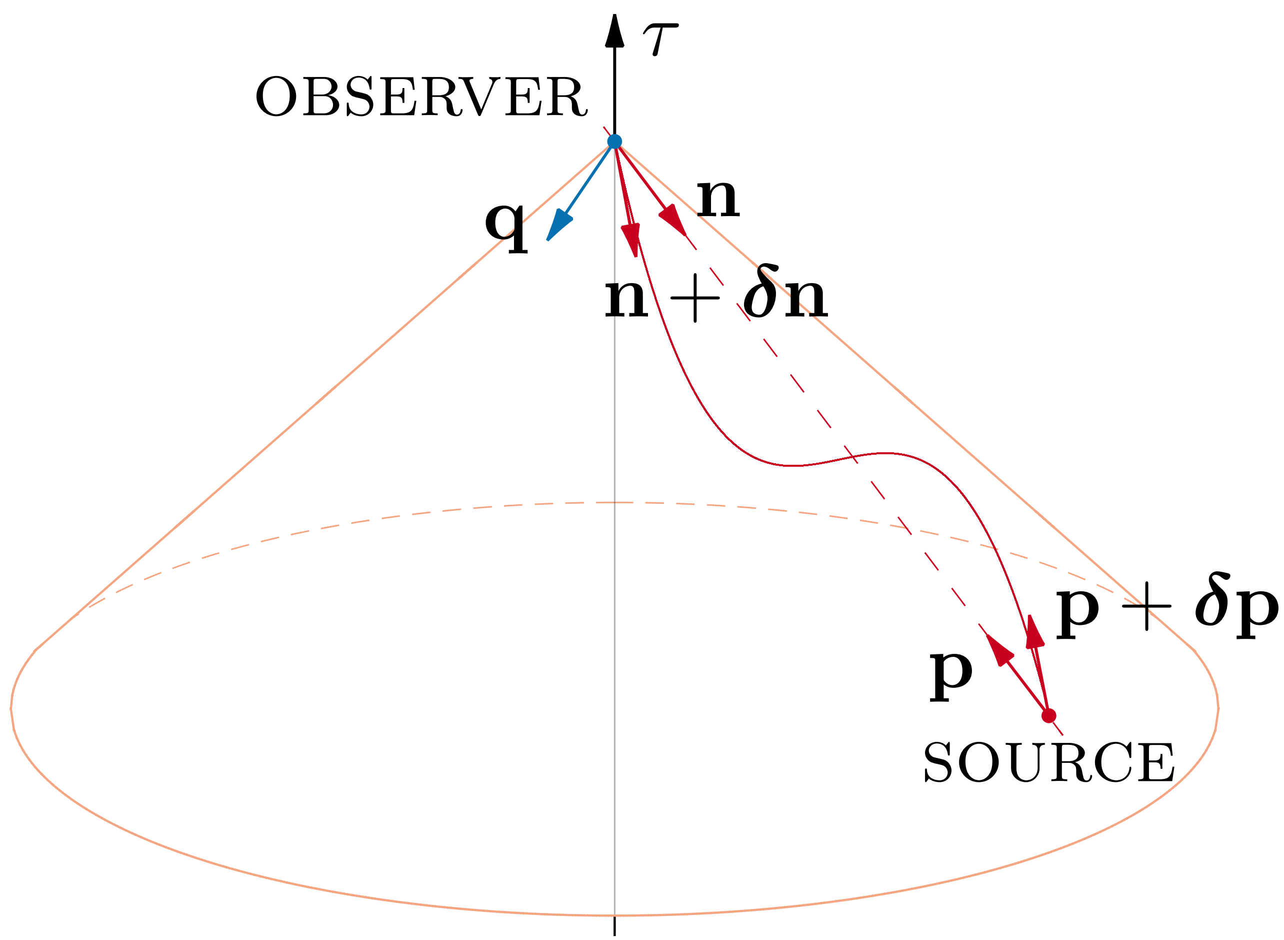


Integrate along the worldline of the photon

Boundary conditions:

A. Photon path is null

B. Photon path intersects source and observer worldlines



Integrate along the worldline of the photon

Boundary conditions:

A. Photon path is null

B. Photon path intersects source and observer worldlines

Perturbed astrometric position and frequency

$n_{\hat{\imath}} + \delta n_{\hat{\imath}}, \quad \Omega_{\rm obs}$

Redshift

$$z = \frac{n^i n^j}{2\left(1 - n_k q^k\right)} \left[h_{ij}(\text{OBS}) - h_{ij}(\text{SOURCE})\right]$$

Redshift

$$z = \frac{n^i n^j}{2\left(1 - n_k q^k\right)} \left[h_{ij}(\textsc{obs}) - h_{ij}(\textsc{source})\right]$$

Astrometric shift

$$\delta n_{\hat{i}} = \Bigg[\Bigg(\Bigg\{1 + \frac{\mathrm{i}(2 - \vec{q}\cdot\vec{n})}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right]\Bigg\} n_{\hat{i}}$$
$$- \Bigg\{1 + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right]\Bigg\} q_{\hat{i}}\Bigg) \frac{H_{jk} n^j n^k}{2(1 - \vec{q}\cdot\vec{n})}$$
$$- \Bigg\{\frac{1}{2} + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1 - \vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n})\right)\right]\Bigg\} H_{\hat{i}j} n^j\Bigg] \exp(-\mathrm{i}\omega t_0)\,.$$

Astrometric shift

$$\delta n_{\hat{\imath}} = \Bigg[\Bigg(\Bigg\{ 1 + \frac{\mathrm{i}(2 - \vec{q}\cdot\vec{n})}{\omega \lambda_{\mathrm{S}} \Omega (1 - \vec{q}\cdot\vec{n})} \left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n}) \right) \right] \Bigg\} n_{\hat{\imath}}$$

$$- \Bigg\{ 1 + \frac{\mathrm{i}}{\omega \lambda_{\mathrm{S}} \Omega (1 - \vec{q}\cdot\vec{n})} \left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n}) \right) \right] \Bigg\} q_{\hat{\imath}} \Bigg) \frac{H_{jk} n^j n^k}{2(1 - \vec{q}\cdot\vec{n})}$$

$$- \Bigg\{ \frac{1}{2} + \frac{\mathrm{i}}{\omega \lambda_{\mathrm{S}} \Omega (1 - \vec{q}\cdot\vec{n})} \left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1 - \vec{q}\cdot\vec{n}) \right) \right] \Bigg\} H_{\hat{\imath}j} n^j \Bigg] \exp(-\mathrm{i}\omega t_0)\,.$$

$$= \delta n_{\hat{\imath}} \left(h(\textsc{obs}), h(\textsc{source}) \right)$$

Astrometric shift

$$\delta n_{\hat{\imath}} = \Bigg[\Bigg(\Bigg\{1 + \frac{\mathrm{i}(2 - \vec{q}\cdot\vec{n})}{\omega\lambda_{\mathrm{S}}\Omega(1-\vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1-\vec{q}\cdot\vec{n})\right)\right]\Bigg\} n_{\hat{\imath}}$$

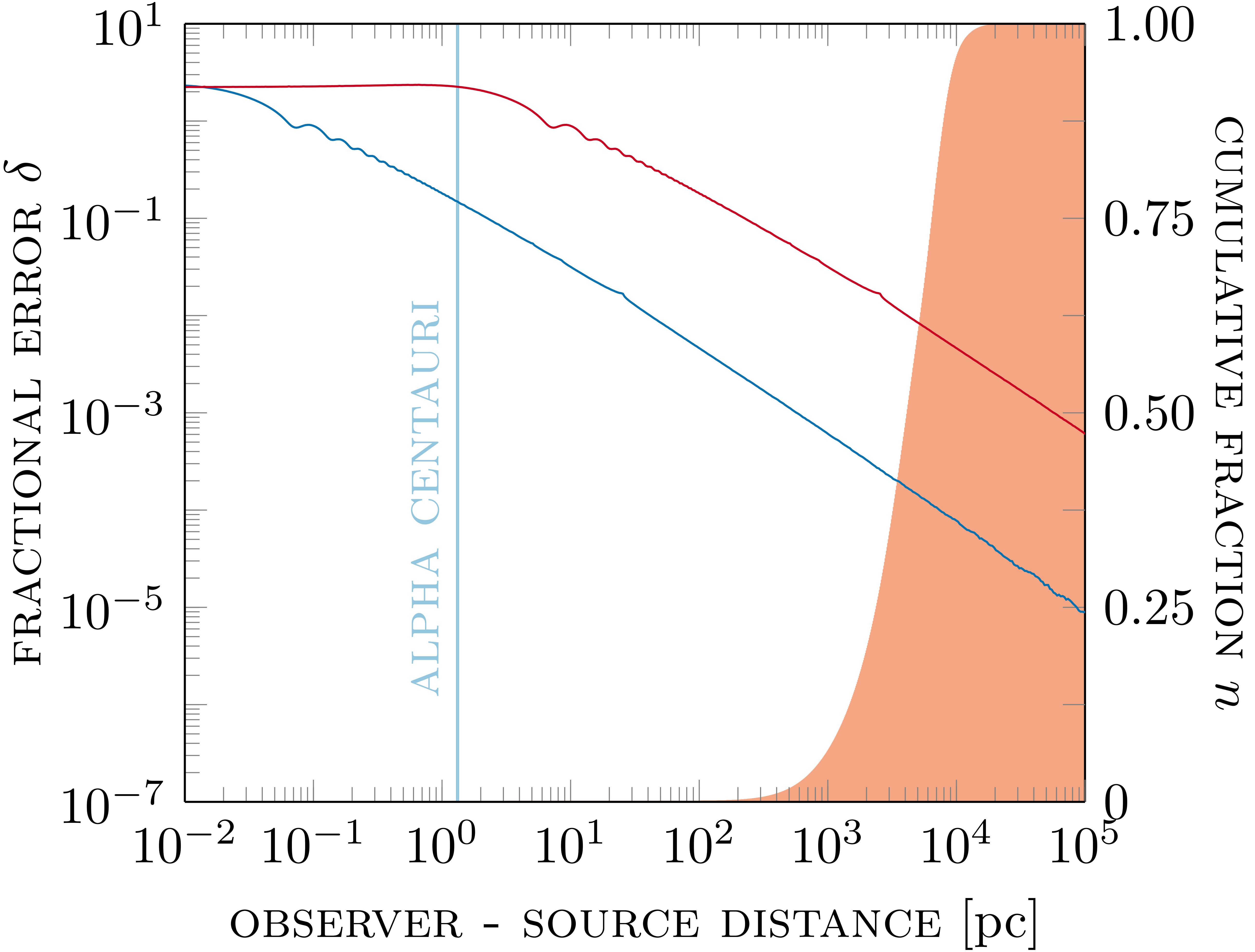
$$- \Bigg\{1 + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1-\vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1-\vec{q}\cdot\vec{n})\right)\right]\Bigg\} q_{\hat{\imath}}\Bigg) \frac{H_{jk} n^j n^k}{2(1-\vec{q}\cdot\vec{n})}$$

$$- \Bigg\{\frac{1}{2} + \frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1-\vec{q}\cdot\vec{n})}\left[1 - \exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1-\vec{q}\cdot\vec{n})\right)\right]\Bigg\} H_{\hat{\imath}j} n^j \Bigg] \exp(-\mathrm{i}\omega t_0)\,.$$

$$= \delta n_{\hat{\imath}}\left(h(\textsc{obs}), h(\textsc{source})\right)$$

in the distant source limit

$$\delta n_{\hat{\imath}} = \frac{n_{\hat{\imath}} - q_{\hat{\imath}}}{2(1-\vec{q}\cdot\vec{n})} h_{\hat{\jmath}\hat{k}}(\textsc{obs}) n^{\hat{\jmath}} n^{\hat{k}} - \frac{1}{2} h_{\hat{\imath}\hat{\jmath}}(\textsc{obs}) n^{\hat{\jmath}}\,.$$



10^1
10^{-1}
10^{-3}
10^{-5}
10^{-7}
FRACTIONAL ERROR δ
ALPHA CENTAURI
1.00
0.75
0.50
0.25
0
CUMULATIVE FRACTION n
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
10^4
10^5
OBSERVER - SOURCE DISTANCE [pc]

Effect on the sky

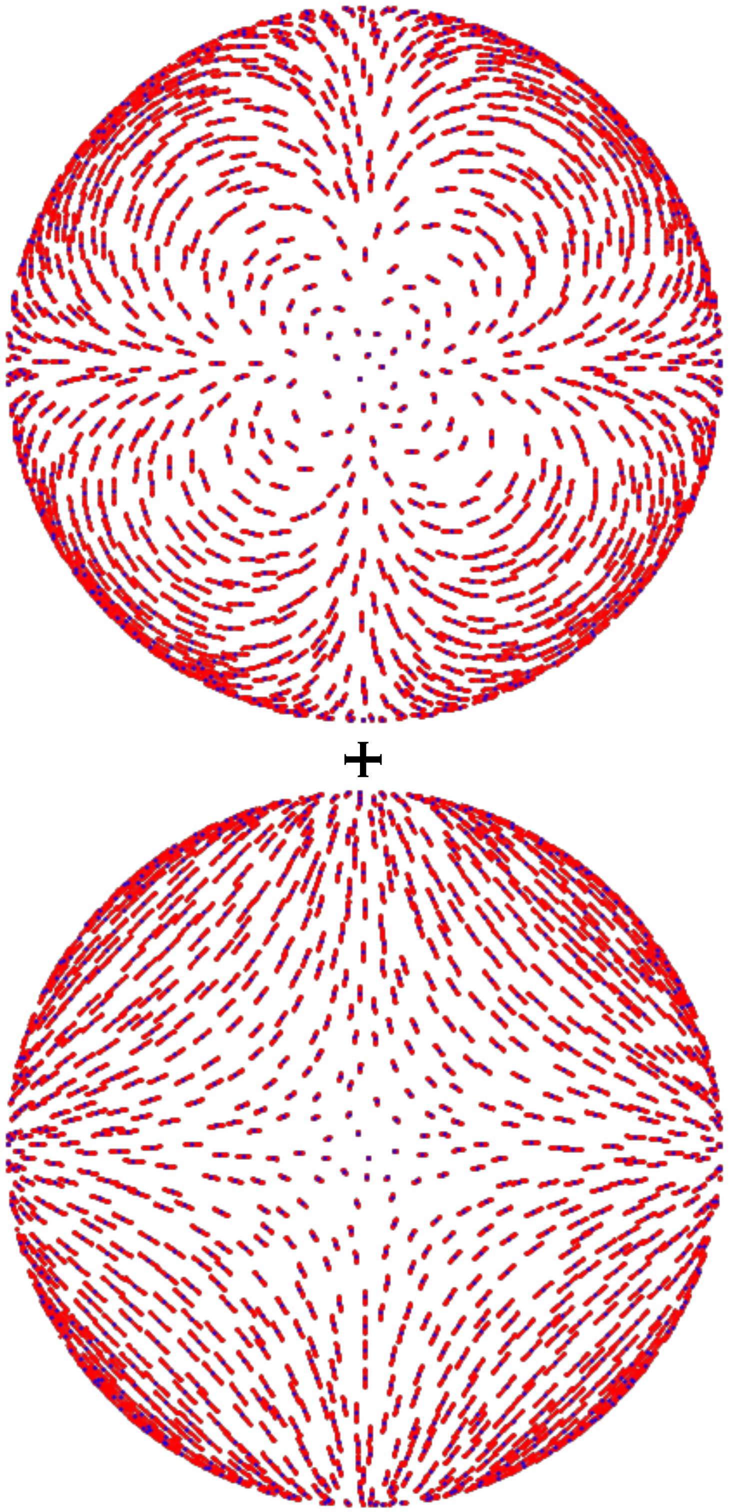
Astrometric shift

$$
\begin{aligned}
\delta n_{\hat{\imath}} =& \Bigg[\Bigg(\Bigg\{1+\frac{\mathrm{i}(2-\vec{q}\cdot\vec{n})}{\omega\lambda_{\mathrm{S}}\Omega(1-\vec{q}\cdot\vec{n})}\left[1-\exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1-\vec{q}\cdot\vec{n})\right)\right]\Bigg\}n_{\hat{\imath}} \\
&\qquad -\Bigg\{1+\frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1-\vec{q}\cdot\vec{n})}\left[1-\exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1-\vec{q}\cdot\vec{n})\right)\right]\Bigg\}q_{\hat{\imath}}\Bigg)\frac{H_{jk}n^j n^k}{2(1-\vec{q}\cdot\vec{n})} \\
&-\Bigg\{\frac{1}{2}+\frac{\mathrm{i}}{\omega\lambda_{\mathrm{S}}\Omega(1-\vec{q}\cdot\vec{n})}\left[1-\exp\left(-\mathrm{i}\omega\Omega\lambda_{\mathrm{S}}(1-\vec{q}\cdot\vec{n})\right)\right]\Bigg\}H_{\hat{\imath}j}n^j\Bigg]\exp(-\mathrm{i}\omega t_0)\,.
\end{aligned}
$$

$$
= \delta n_{\hat{\imath}}\left(h(\text{OBS}), h(\text{SOURCE})\right)
$$

in the distant source limit

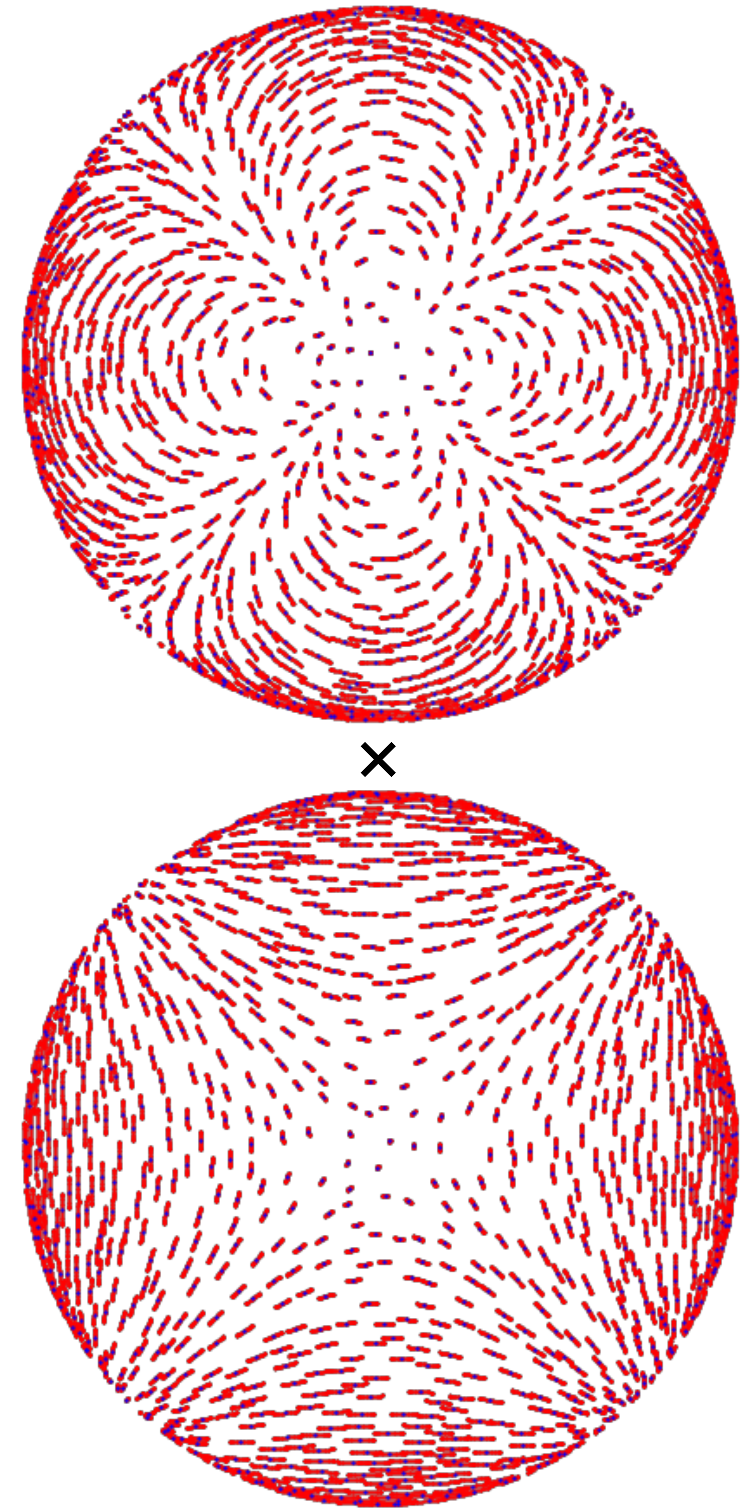
$$
\delta n_{\hat{\imath}} = \frac{n_{\hat{\imath}} - q_{\hat{\imath}}}{2(1-\vec{q}\cdot\vec{n})} h_{\hat{\jmath}\hat{k}}(\text{OBS}) n^{\hat{\jmath}} n^{\hat{k}} - \frac{1}{2} h_{\hat{\imath}\hat{\jmath}}(\text{OBS}) n^{\hat{\jmath}}\,.
$$

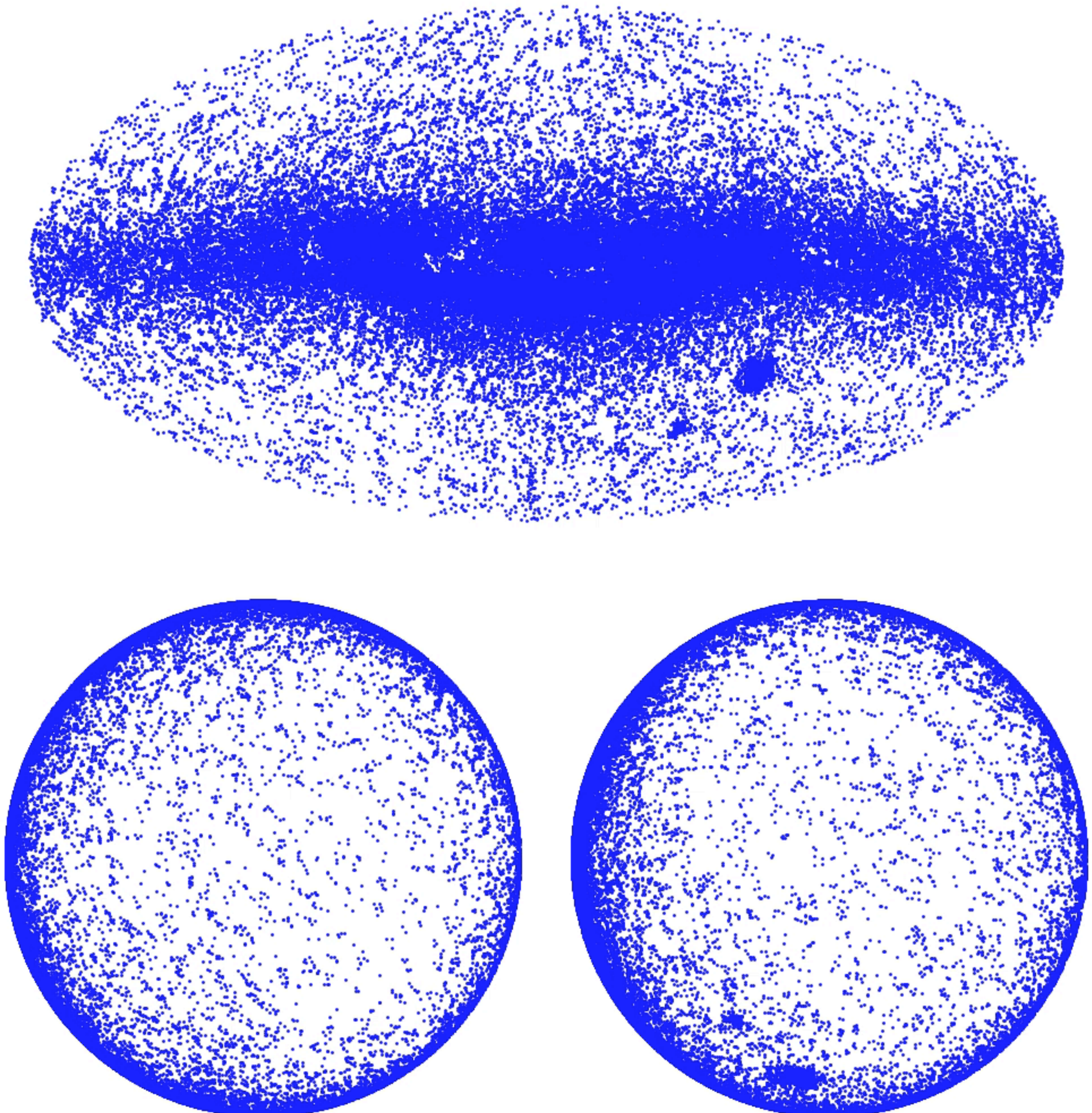


Northern hemisphere

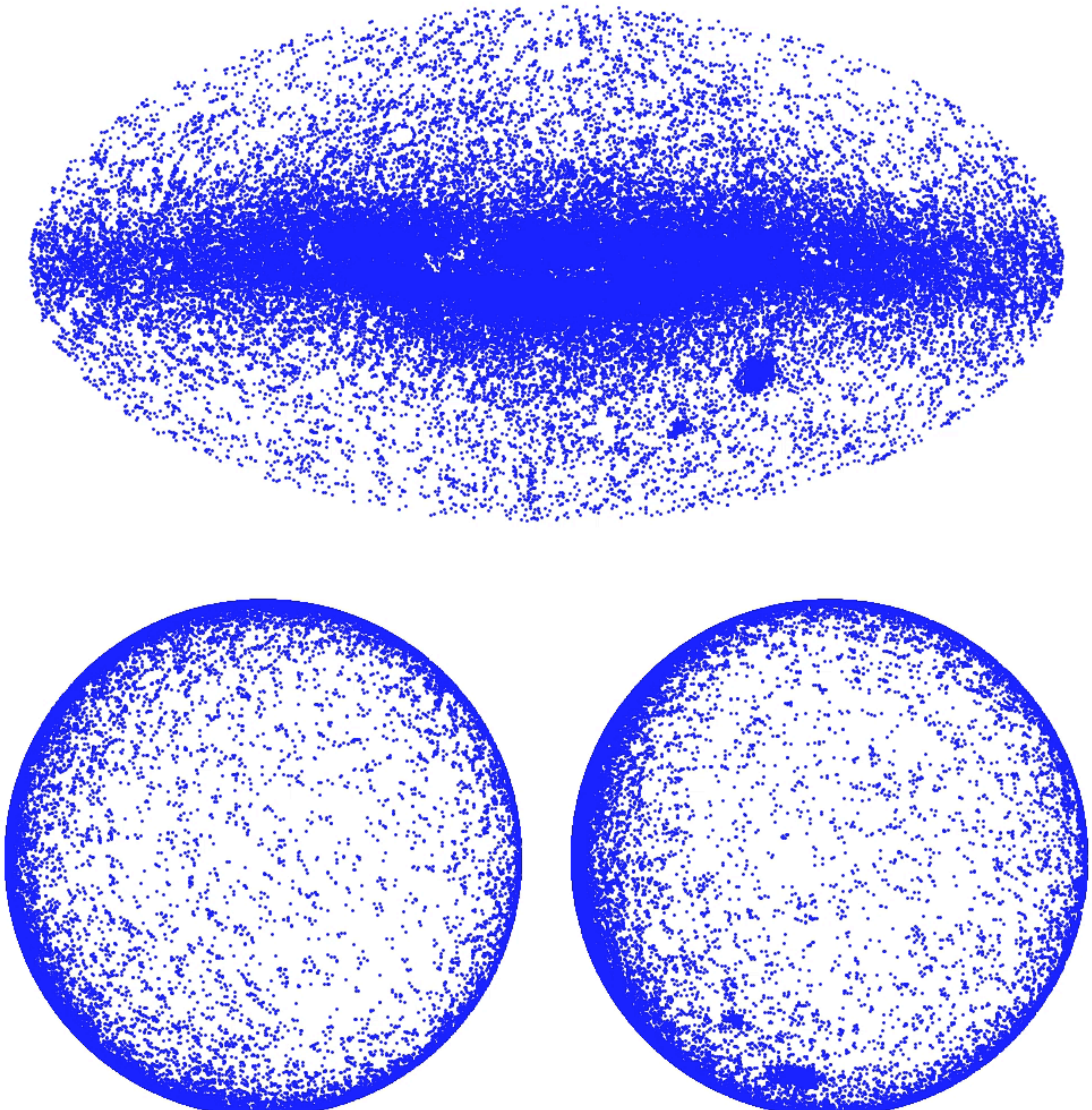
Southern hemisphere

Enhanced 10^{13} times





Enhanced
10^{13} times

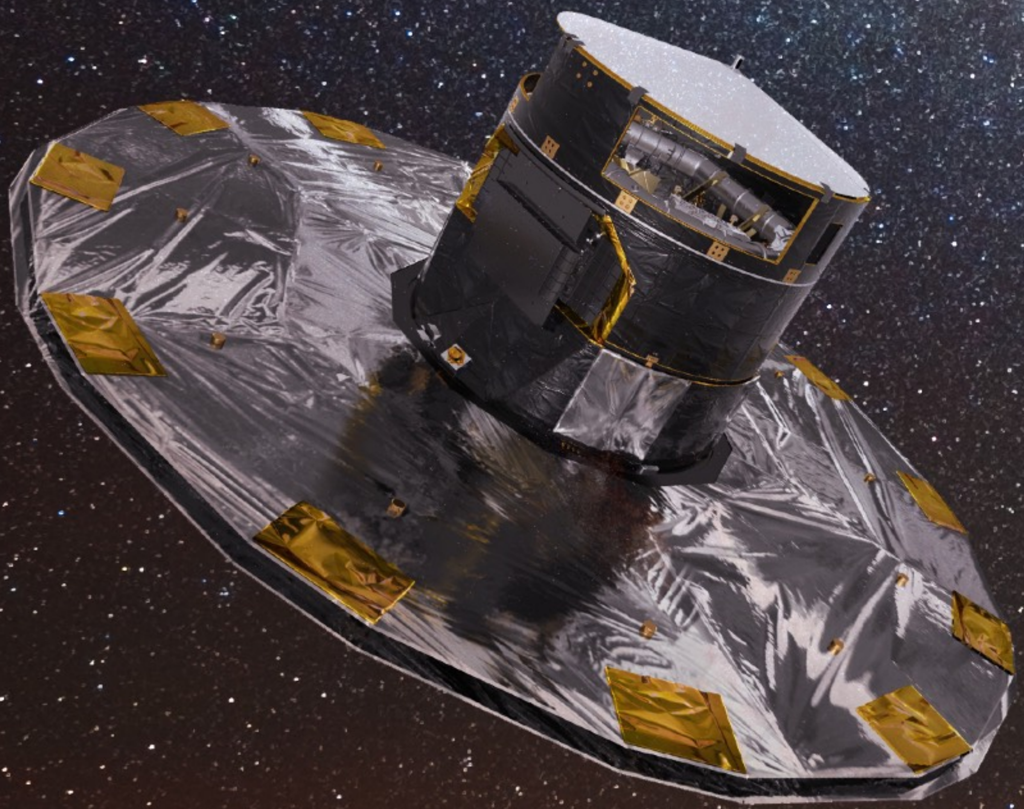


Enhanced
10^{13} times

Is this effect detectable?

Gaia

ESA mission for astrometry
in the Milky Way

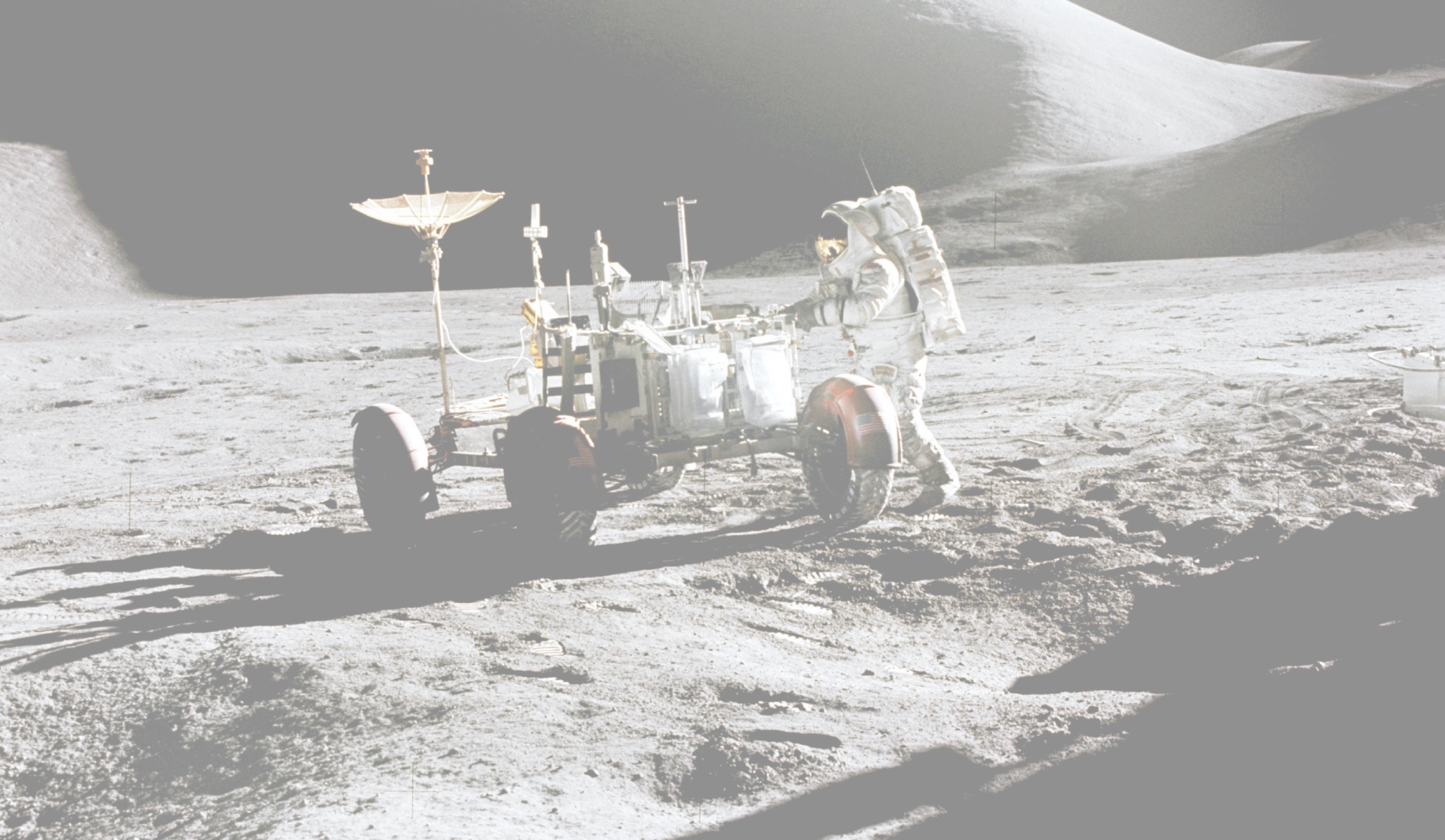


Objectives:

Map ~10^9 objects ~70 times each

Astrometric accuracy
~10 micro arc seconds

"A micro arc second is about the size of a period at the end of a sentence in the Apollo mission manuals left on the Moon as seen from Earth."



Sensitivity to individual events

~100 measurements of each object

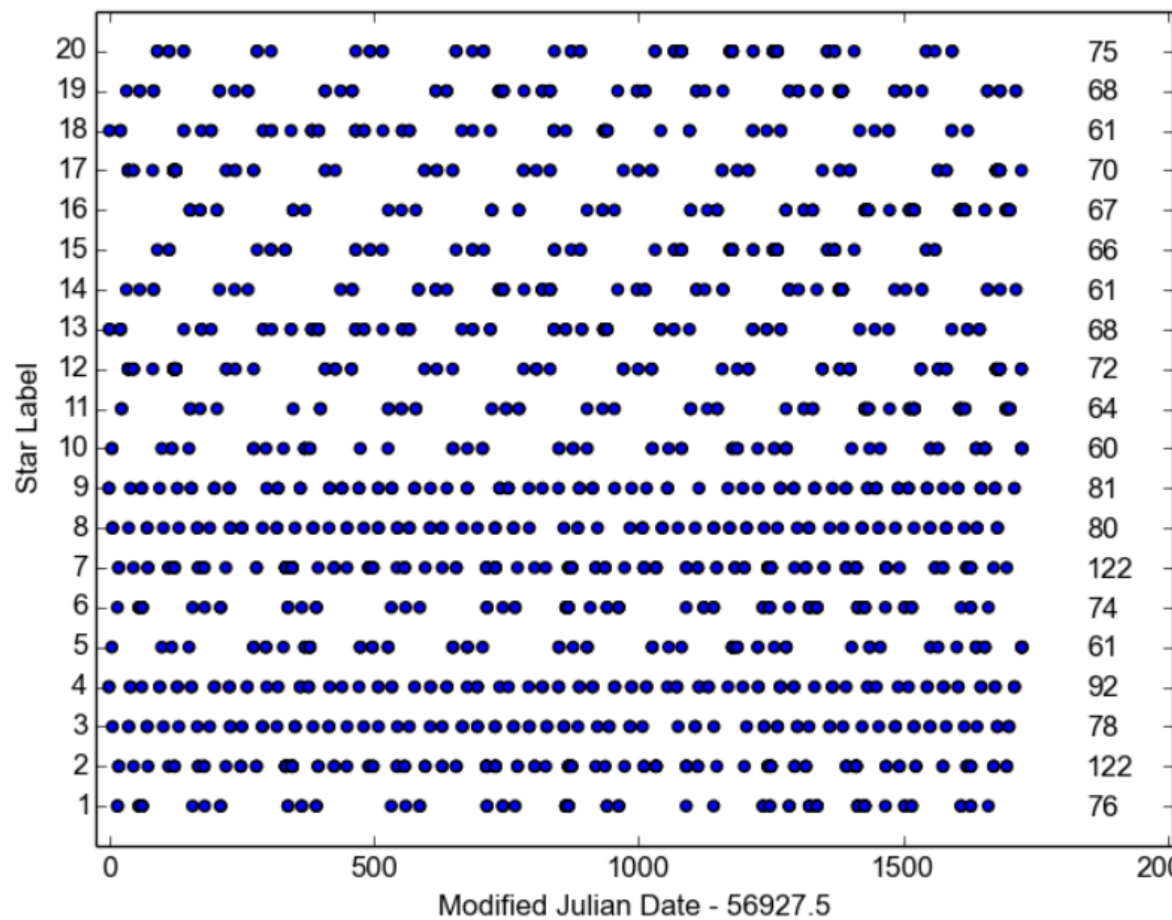
Mission duration 5-10 years

Sensitive in the range 10^{-8} - 3×10^{-7} Hz

Black hole binaries in the early PN inspiral, 10^{8} - 10^{10} solar masses

Caveats

Astrometric measurements are spread over 5 years



Noise + large scale systematics at this level

Gaia has a deadline of 5 years, PTA surveys continually improve

Detecting GWs with Gaia

It is only being seriously considered now, in the Gaia era

Detecting GWs with Gaia

It is only being seriously considered now, in the Gaia era

Biggest challenge is the size of the data set

Detecting GWs with Gaia

It is only being seriously considered now, in the Gaia era

Biggest challenge is the size of the data set

Data Releases 1 & 2 do not feature individual astrometric measurements, working with simulated data

Computational pipeline

Simulated data

Computational pipeline

Simulated data

Proper motion	+	Noise	+	GW signal

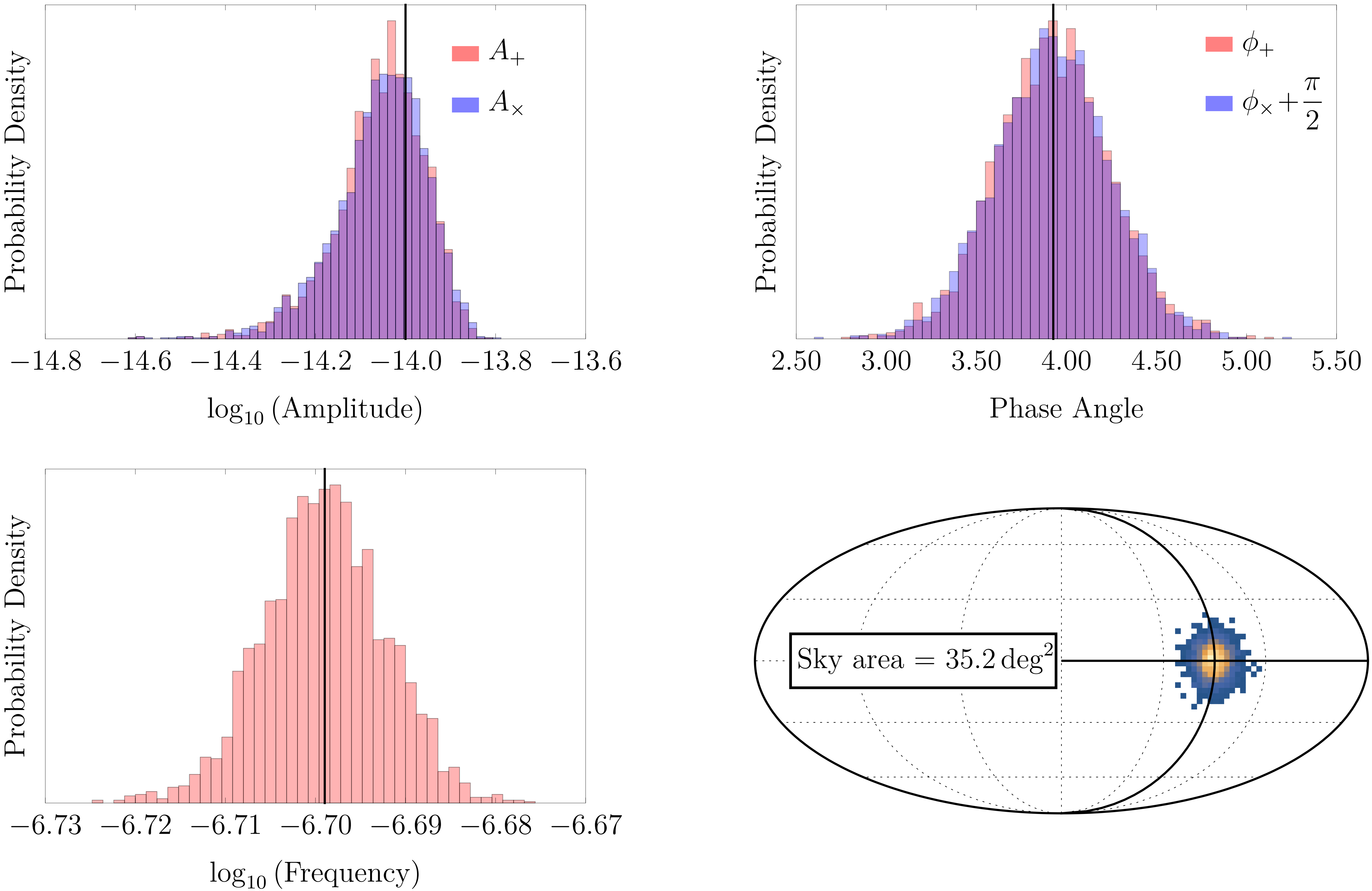
Computational pipeline

Simulated data

Noise + GW signal

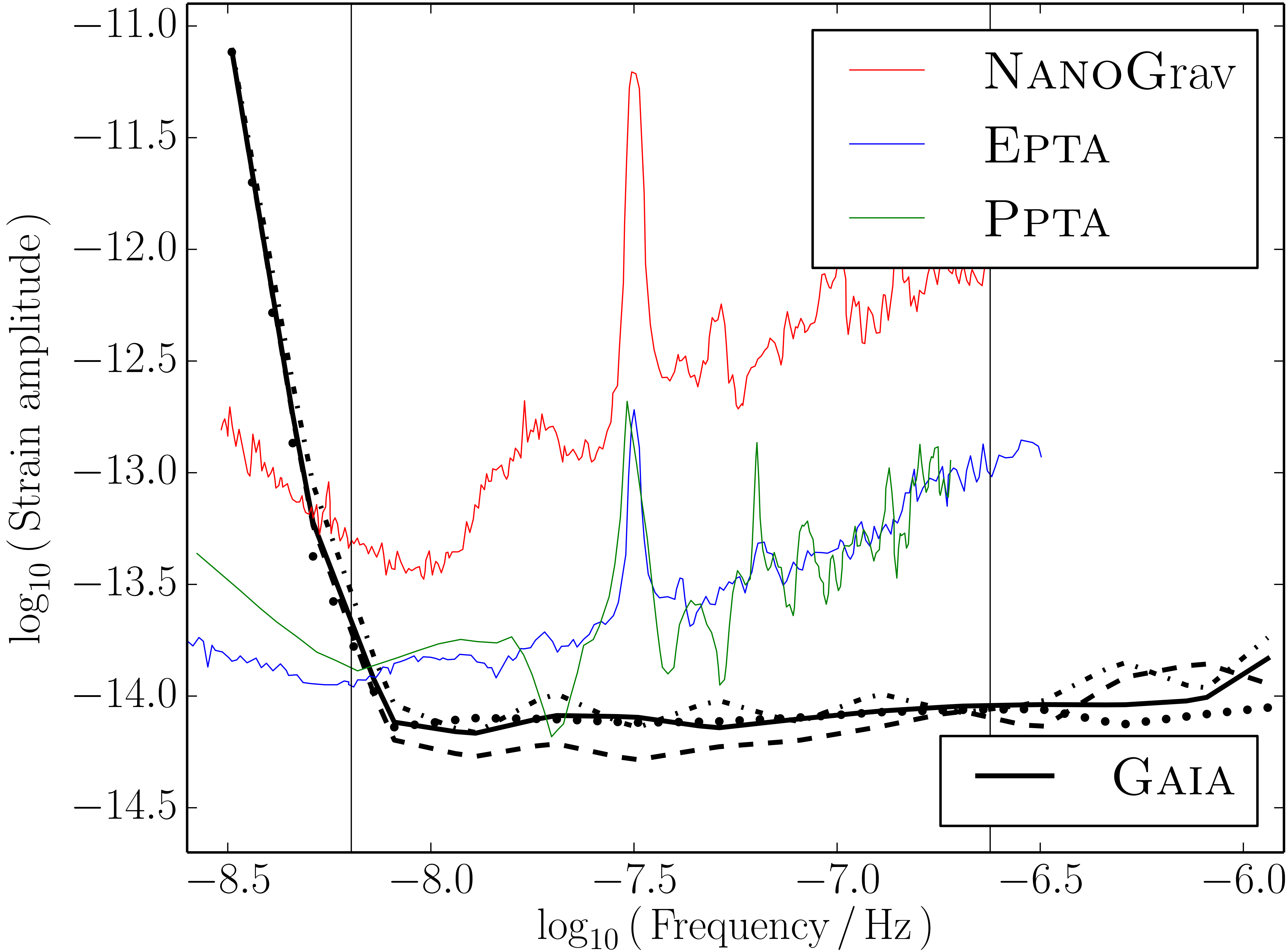
Pipeline for individual detections:

Bayesian inference on the parameter grid

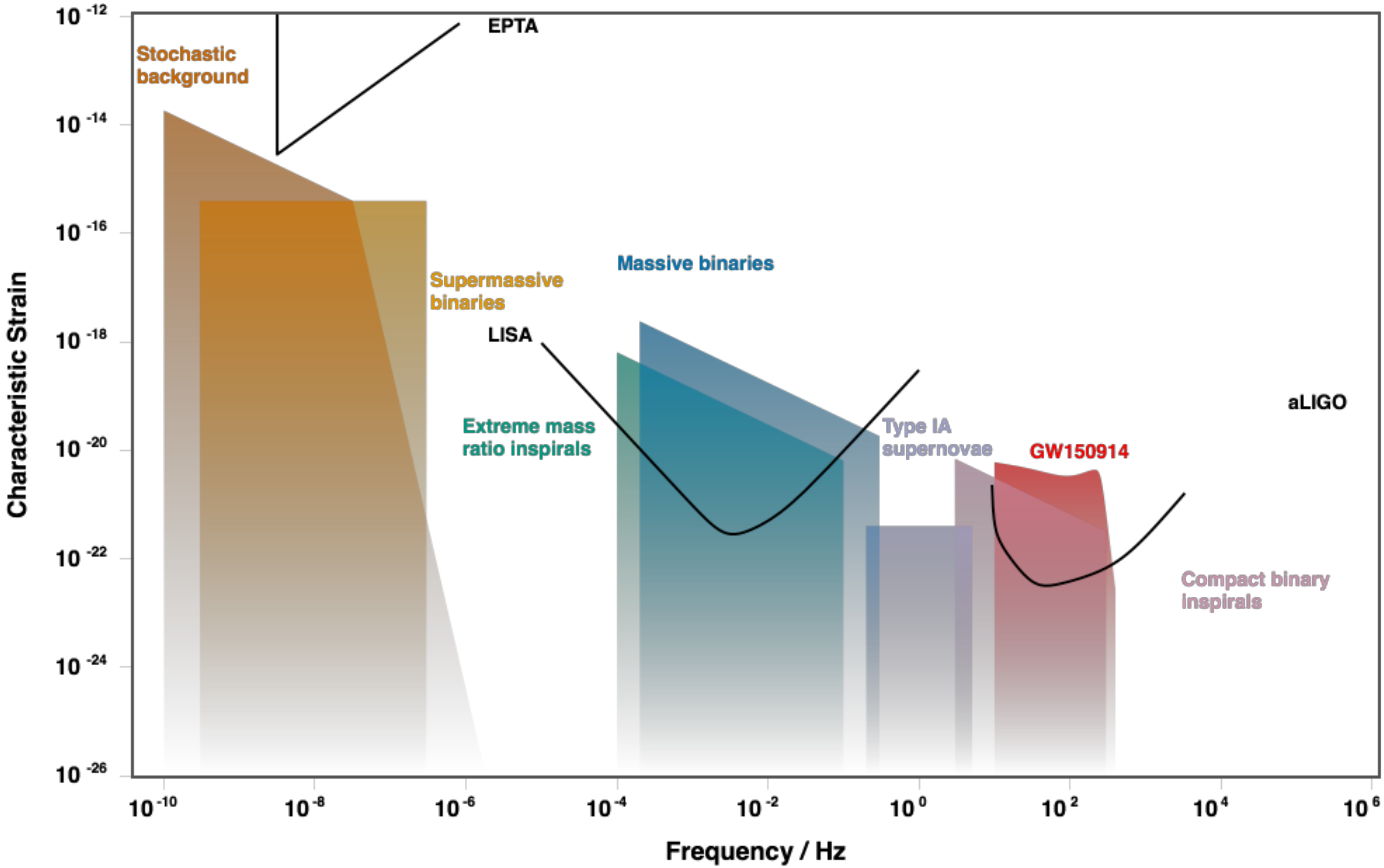


10^{10} SOLAR MASS BINARY 20 MPc AWAY

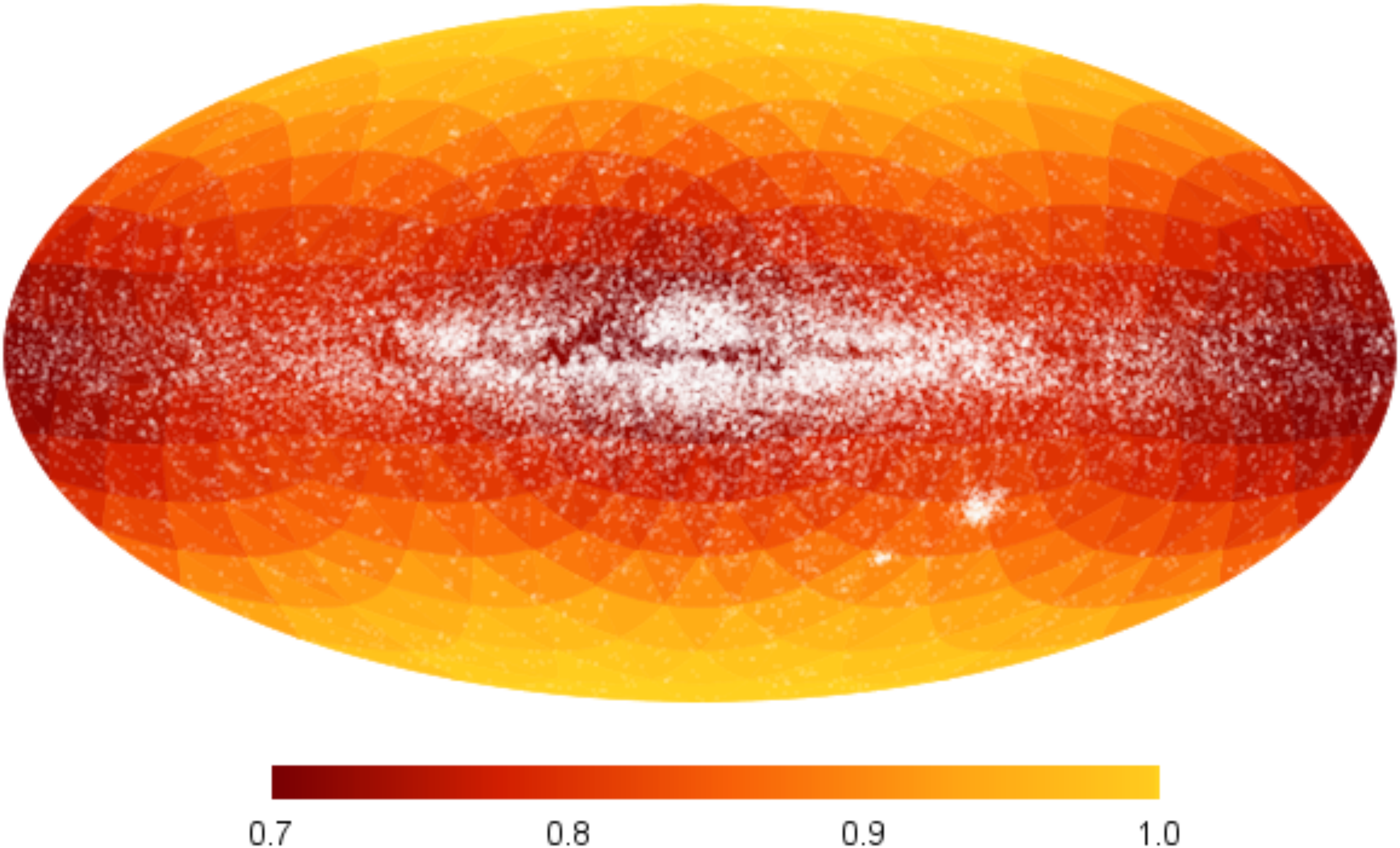
Frequency Sensitivity of Gaia



−11.0
−11.5
−12.0
−12.5
−13.0
−13.5
−14.0
−14.5
log10 (Strain amplitude)
−8.5
−8.0
−7.5
−7.0
−6.5
−6.0
log10 (Frequency / Hz)
NANOGrav
EPTA
PPTA
GAIA


Characteristic Strain
Frequency / Hz
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
EPTA
Stochastic background
Supermassive binaries
Massive binaries
LISA
Extreme mass ratio inspirals
Type IA supernovae
GW150914
aLIGO
Compact binary inspirals

Directional sensitivity of Gaia



30% VARIATION ACROSS THE SKY

Correlations of a Stochastic Background

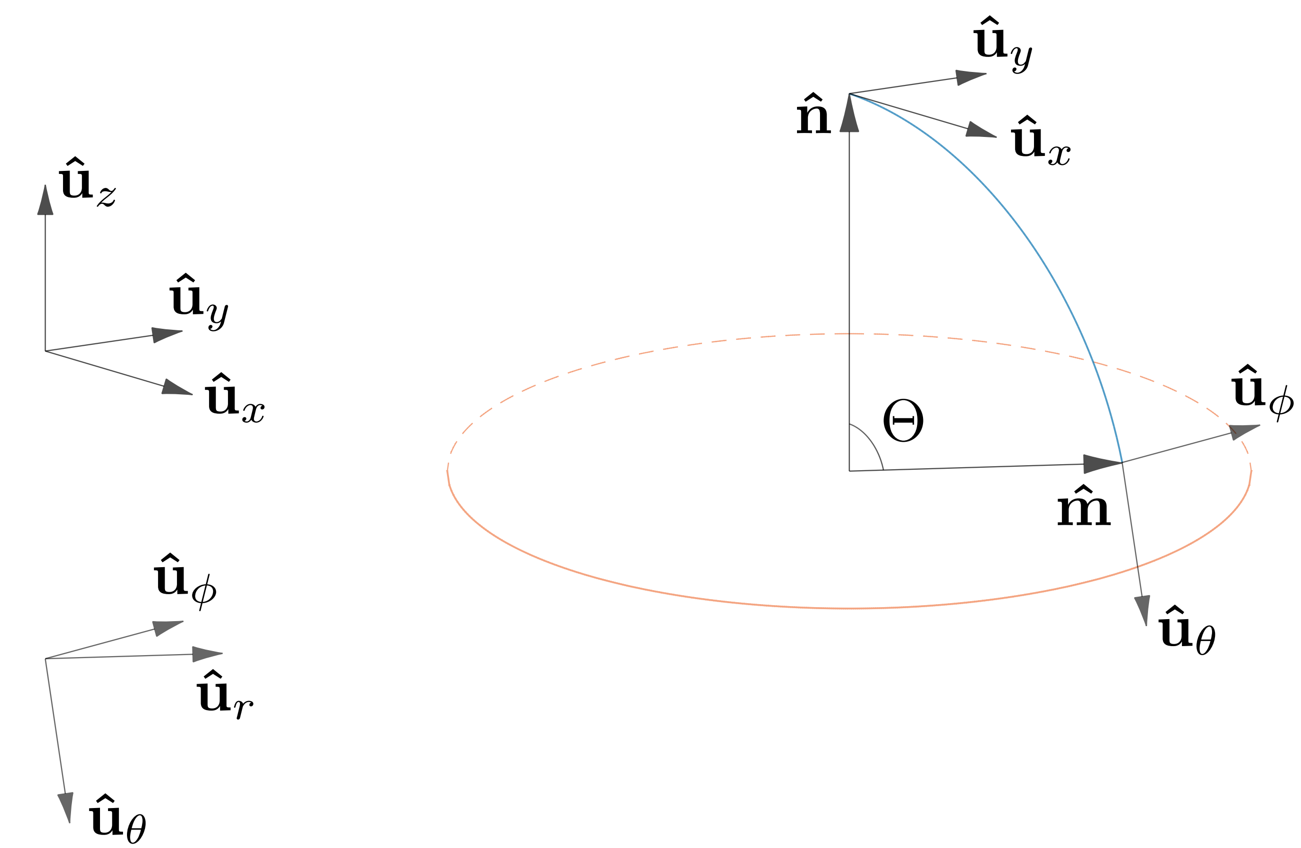
Stochastic GW background

$$\delta n_{\hat{\imath}} = \frac{n_{\hat{\imath}} - q_{\hat{\imath}}}{2(1 - \vec{q} \cdot \vec{n})} h_{\hat{\jmath}\hat{k}}(\mathrm{OBS}) n^{\hat{\jmath}} n^{\hat{k}} - \frac{1}{2} h_{\hat{\imath}\hat{\jmath}}(\mathrm{OBS}) n^{\hat{\jmath}} \,.$$

$$h_{ij}(t) = \Re \left\{ \sum_{P} \int_0^\infty \mathrm{d}f \int_{S^2} \mathrm{d}\Omega_{\mathbf{q}} \, A_P(\mathbf{q}, f) \, e^{-2\pi \mathrm{i} f t} \epsilon_{ij}^P(\mathbf{q}) \right\},$$

Investigate correlations of stars on the sky

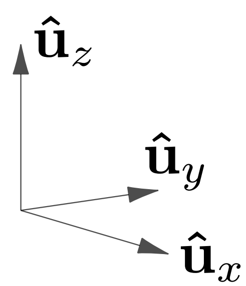
$$\Gamma^{P}_{ij}(\Theta) \propto \int_{S^2} \mathrm{d}\Omega_{\mathbf{q}}\, \delta n_i(n_k, t)\, \delta m_j(m_\ell, t)$$

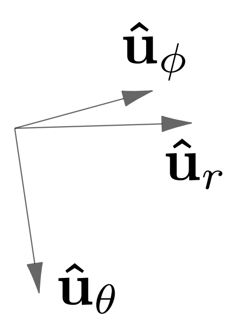


$$C = \left(\begin{array}{c|c} \Gamma_{x\theta} & \Gamma_{x\phi} \\ \hline \Gamma_{y\theta} & \Gamma_{y\phi} \end{array} \right)$$

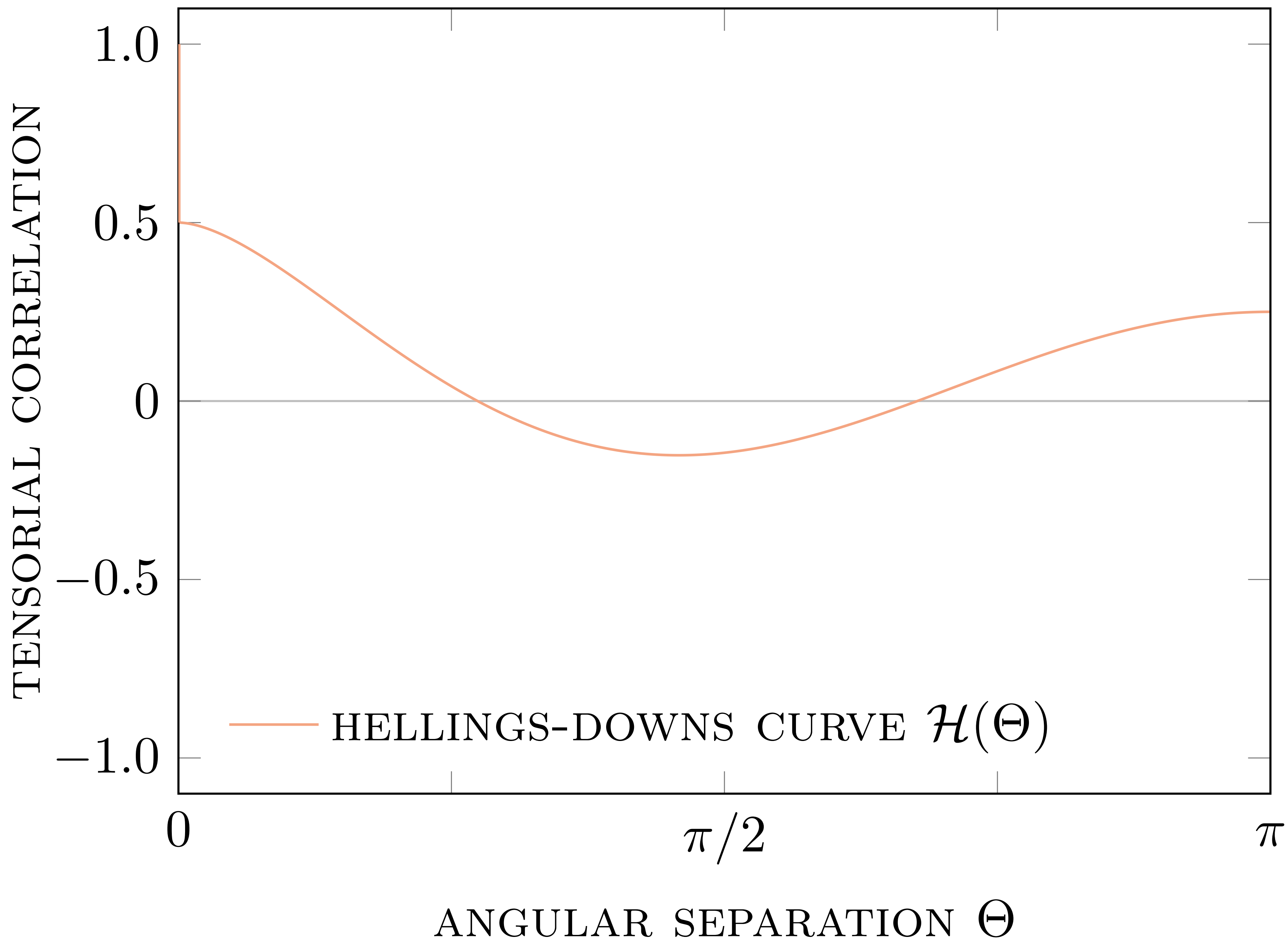
Investigate correlations of stars on the sky

$$\Gamma_{ij}^{P}(\Theta) \propto \int_{S^2} \mathrm{d}\Omega_{\mathbf{q}}\, \delta n_i(n_k, t)\, \delta m_j(m_\ell, t),$$



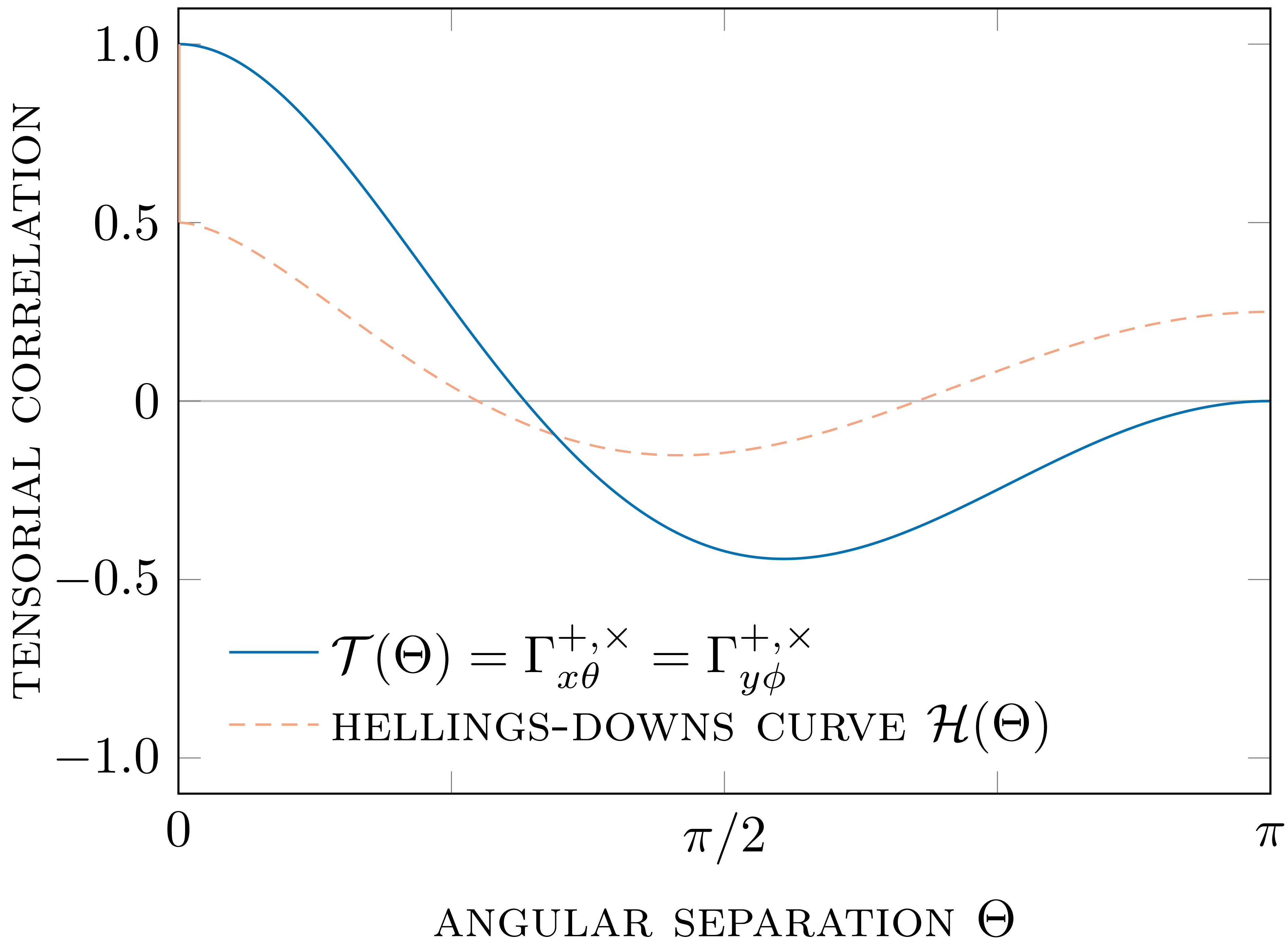


GR modes



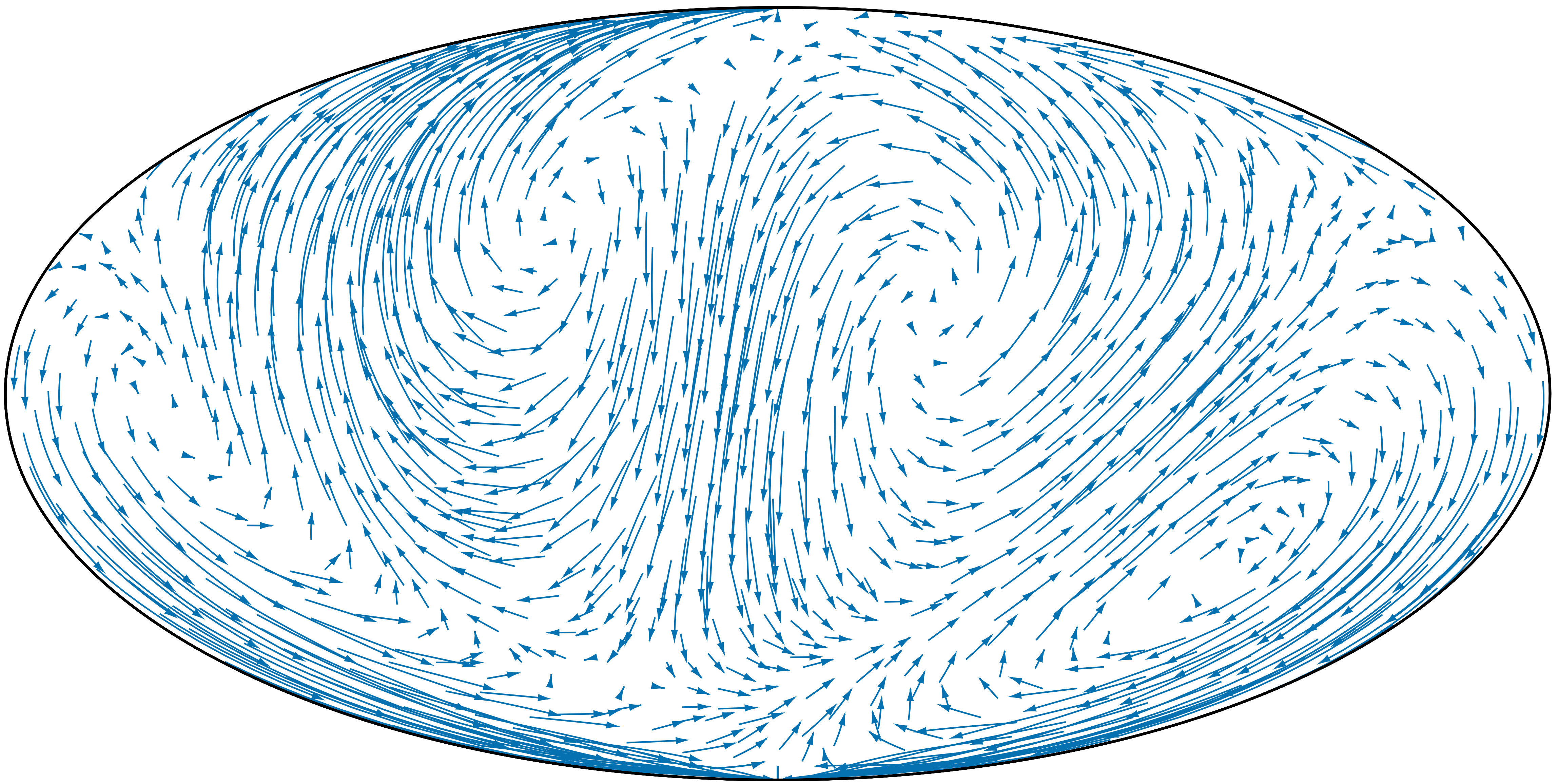
Cf. Book and Flanagan, 2001

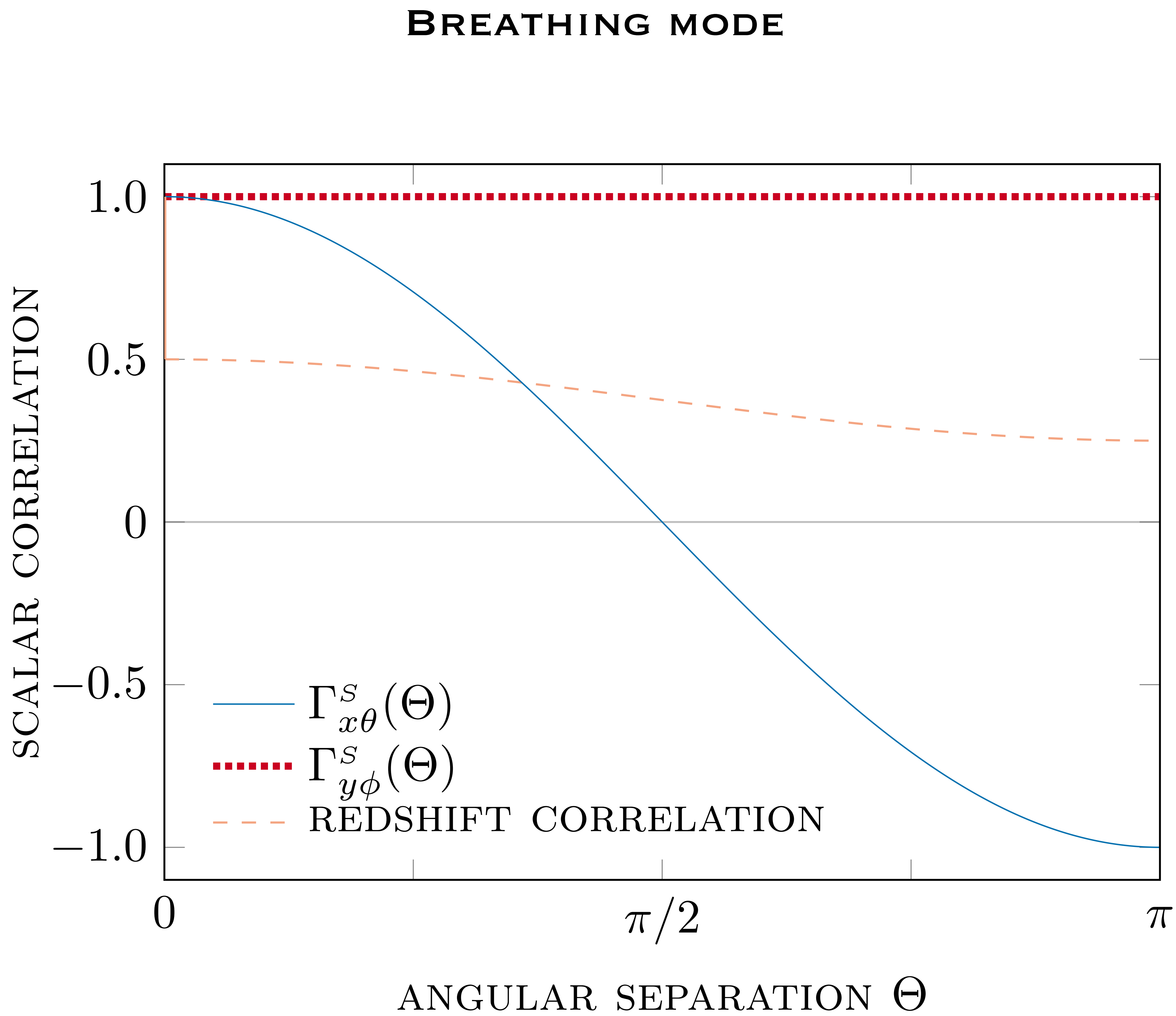
GR modes



Cf. Book and Flanagan, 2001

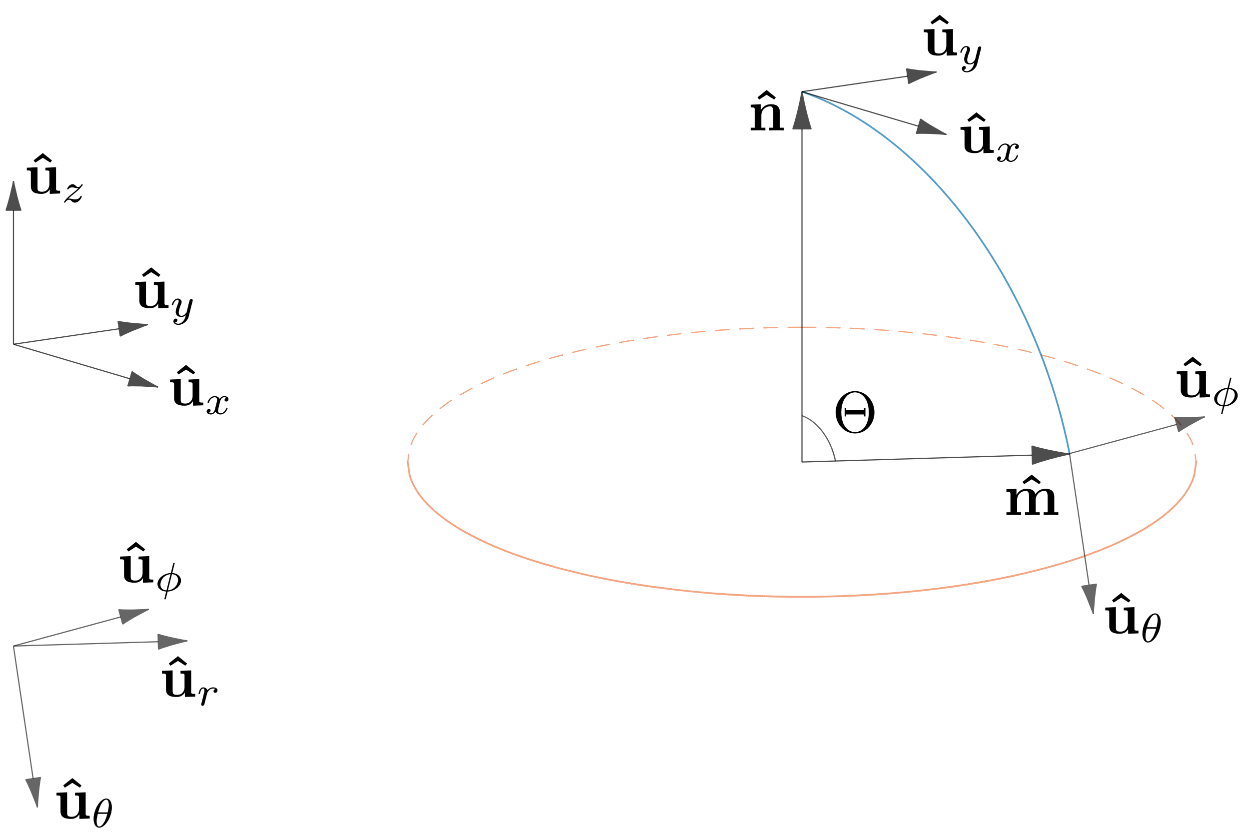
GR MODES



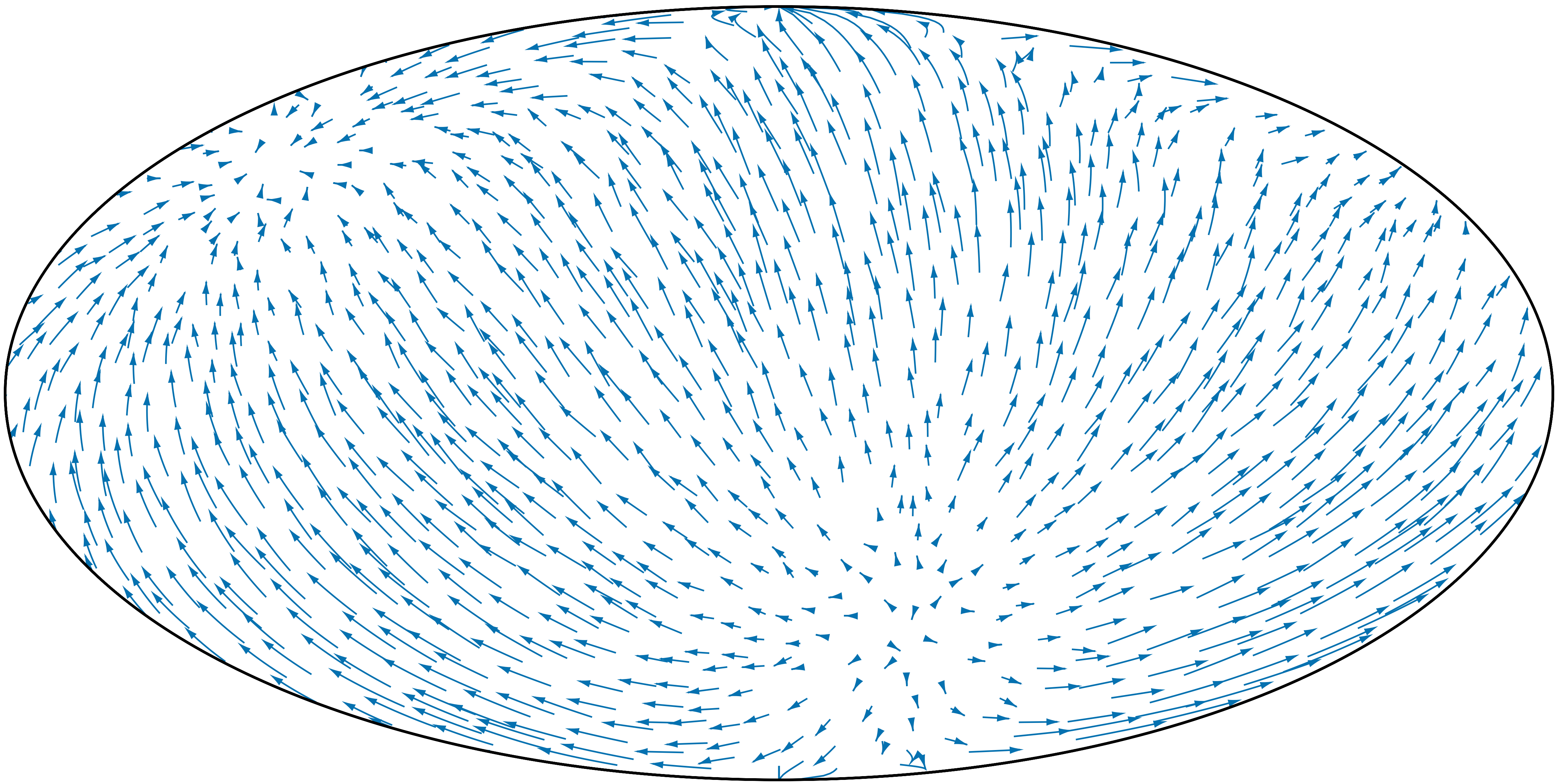


Breathing mode
1.0
0.5
0
−0.5
−1.0
0
π/2
π
scalar correlation
angular separation Θ
$\Gamma^S_{x\theta}(\Theta)$
$\Gamma^S_{y\phi}(\Theta)$
redshift correlation

Breathing mode



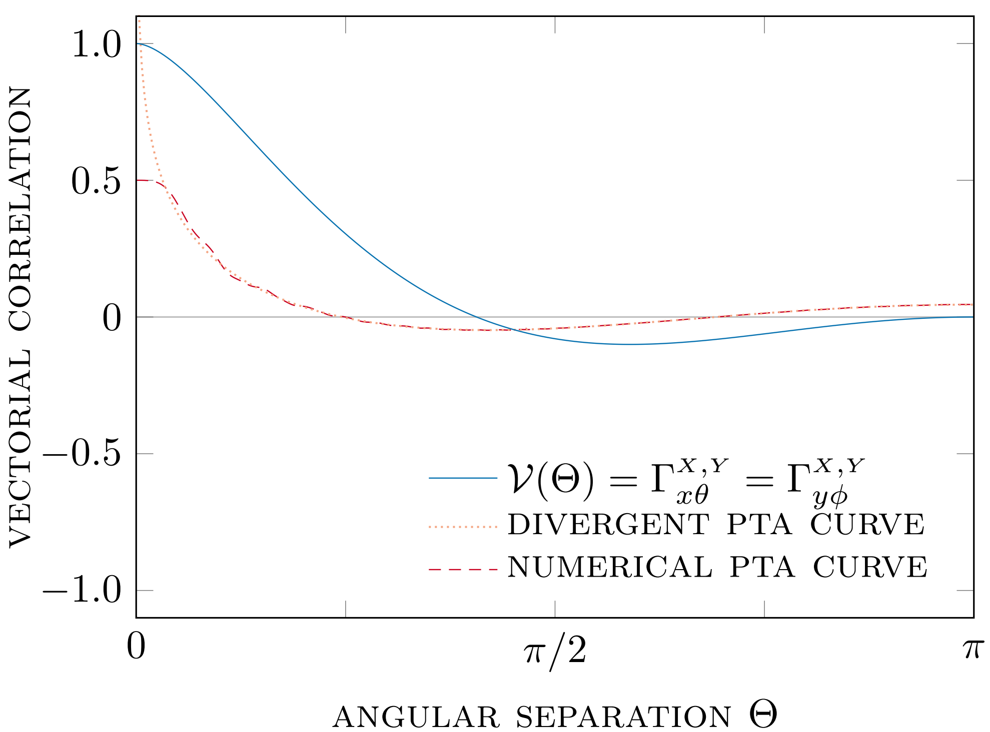
Breathing mode

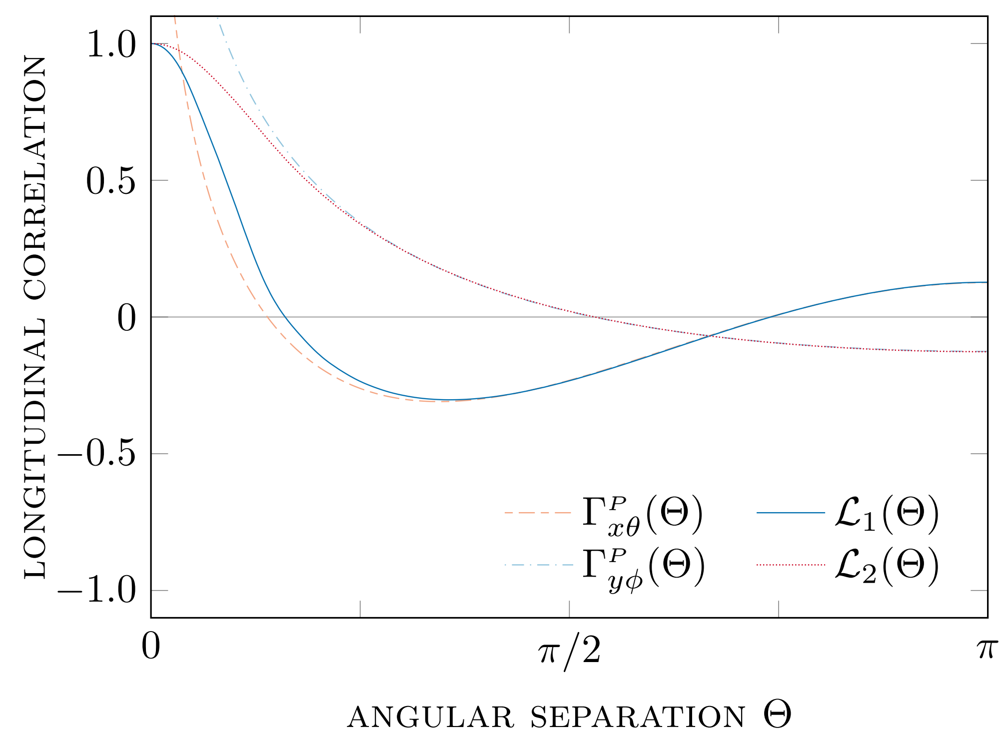


Vectorial modes

+

Scalar longitudinal mode

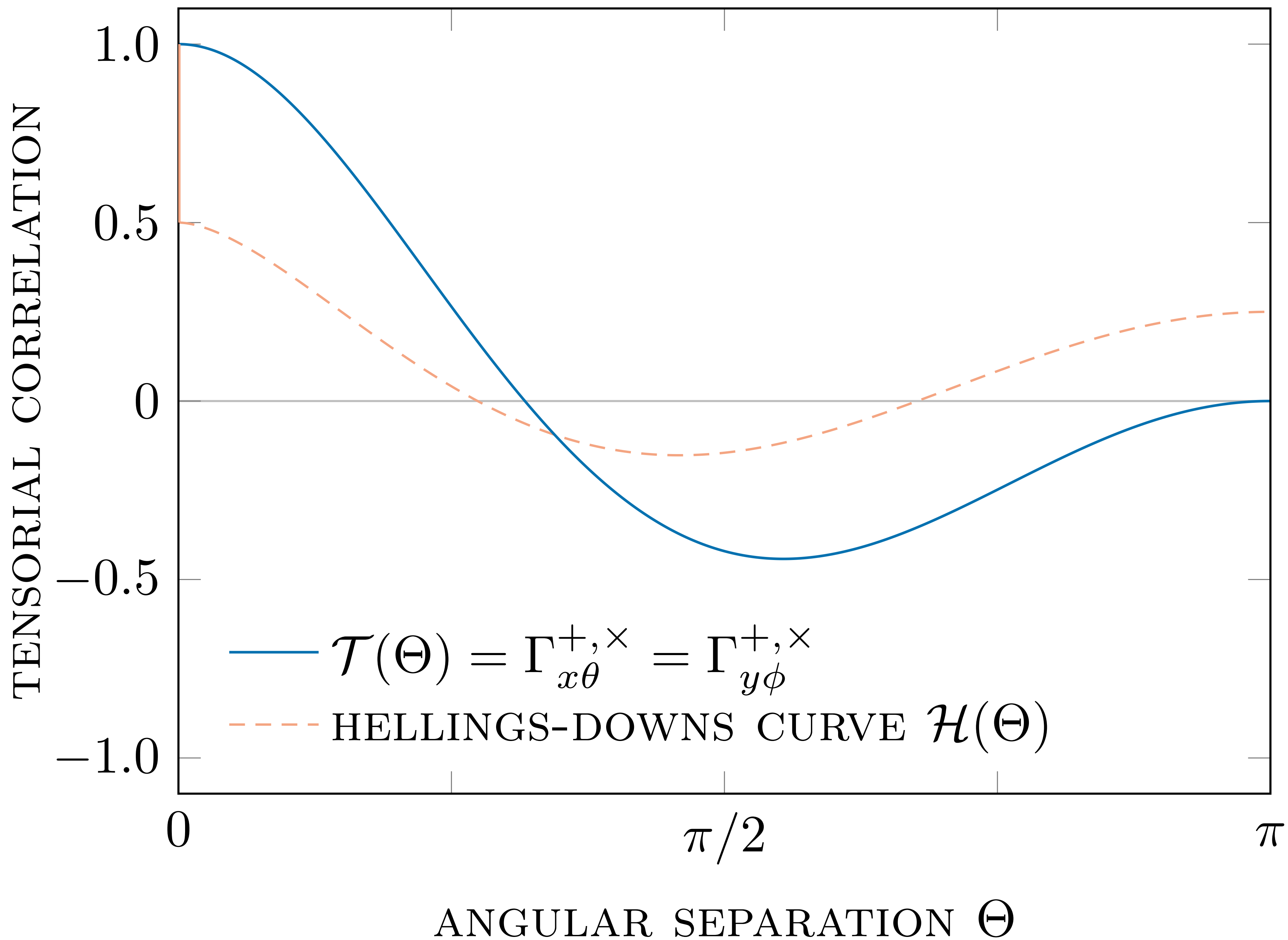




arXiv:1804.00660

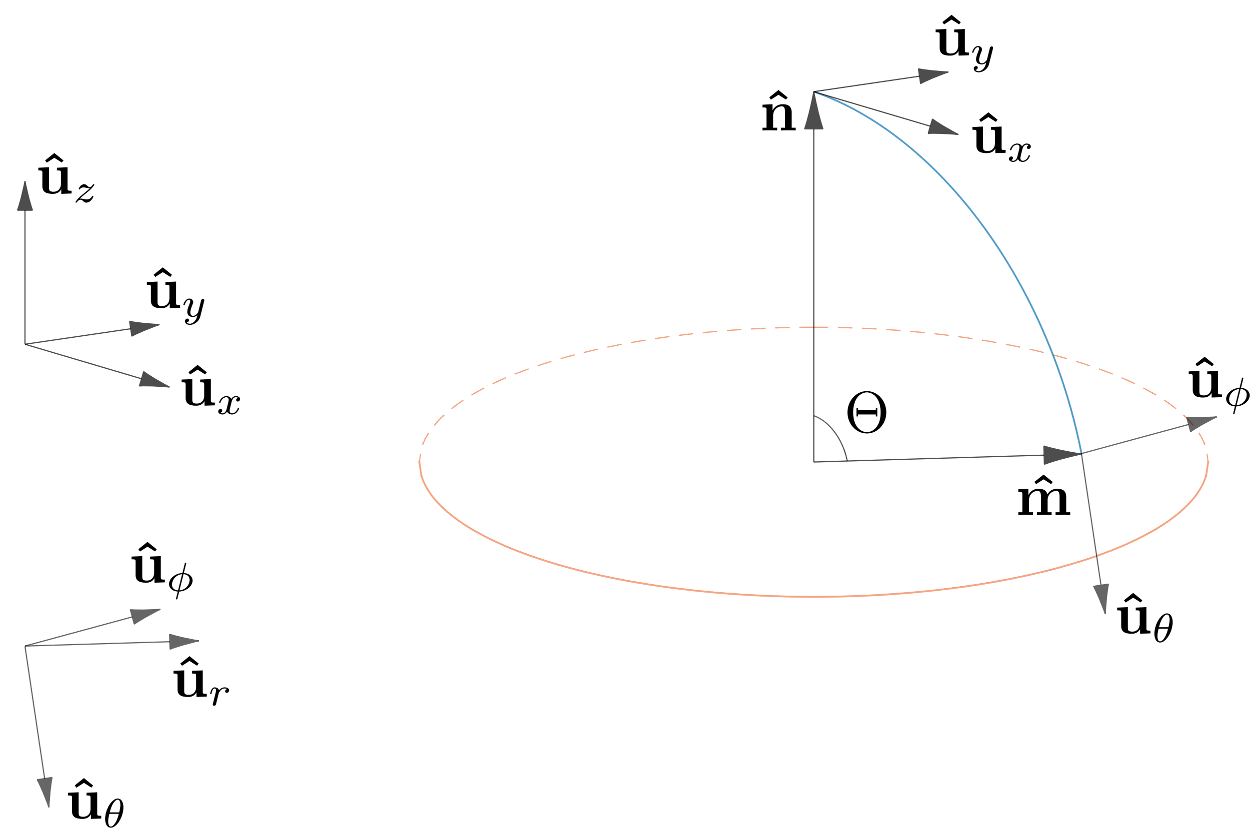
Redshift-astrometry Correction

GR modes

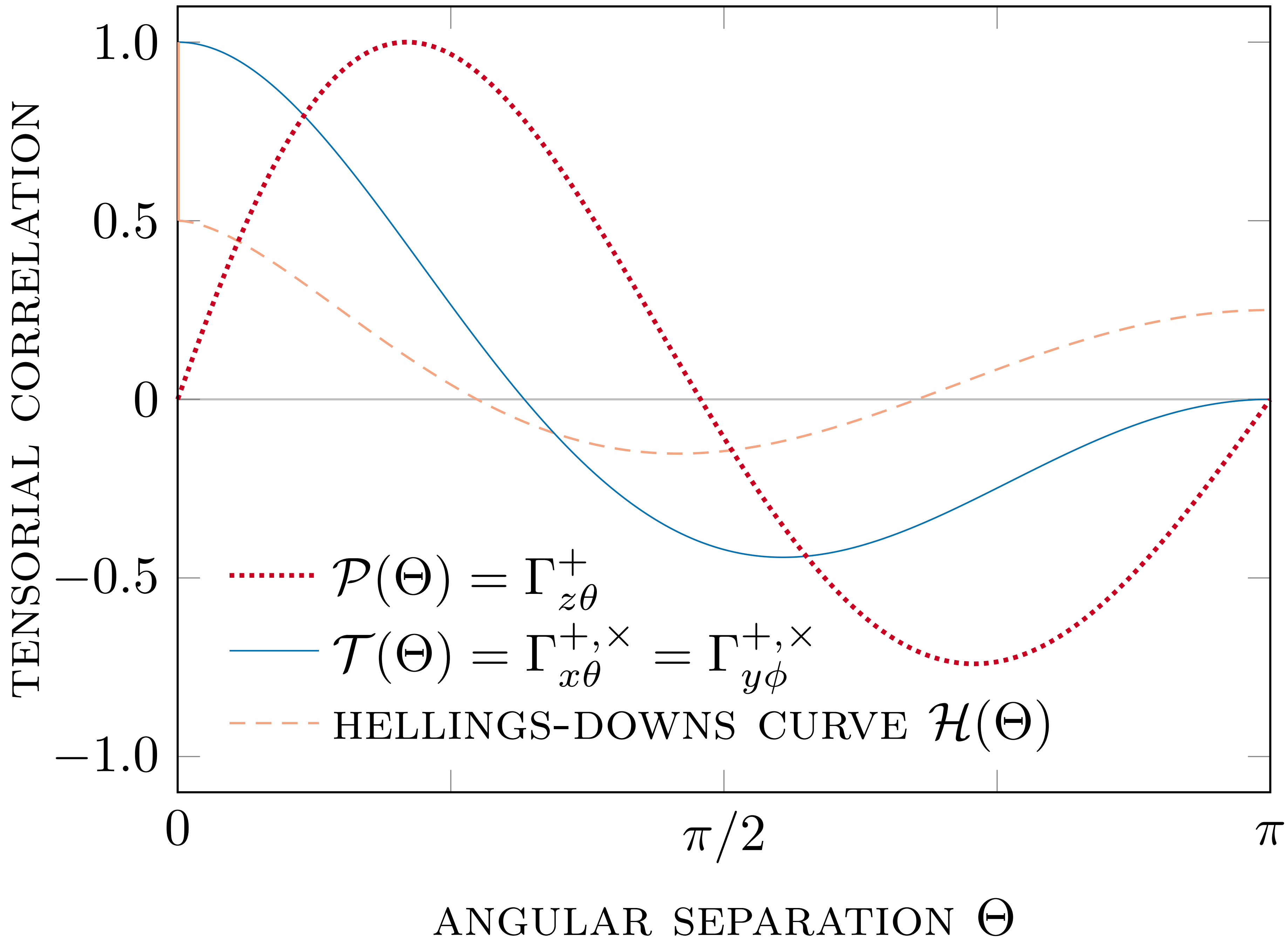


Cf. Book and Flanagan, 2001

Breathing mode



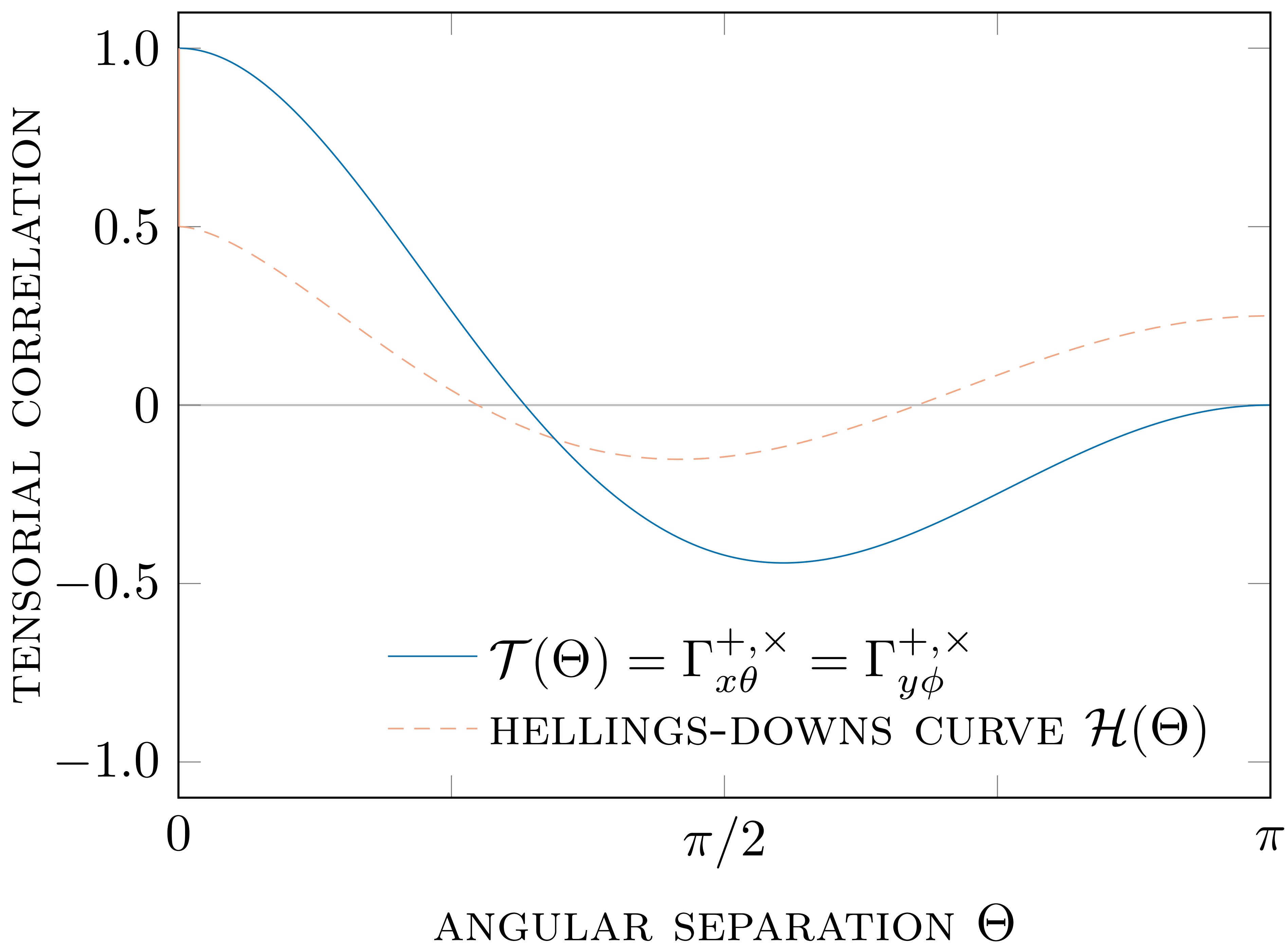
GR modes



1.0
0.5
0
−0.5
−1.0
tensorial correlation
$\mathcal{P}(\Theta) = \Gamma^{+}_{z\theta}$
$\mathcal{T}(\Theta) = \Gamma^{+,\times}_{x\theta} = \Gamma^{+,\times}_{y\phi}$
hellings-downs curve $\mathcal{H}(\Theta)$
0
$\pi/2$
π
angular separation Θ

Massive Graviton Correction

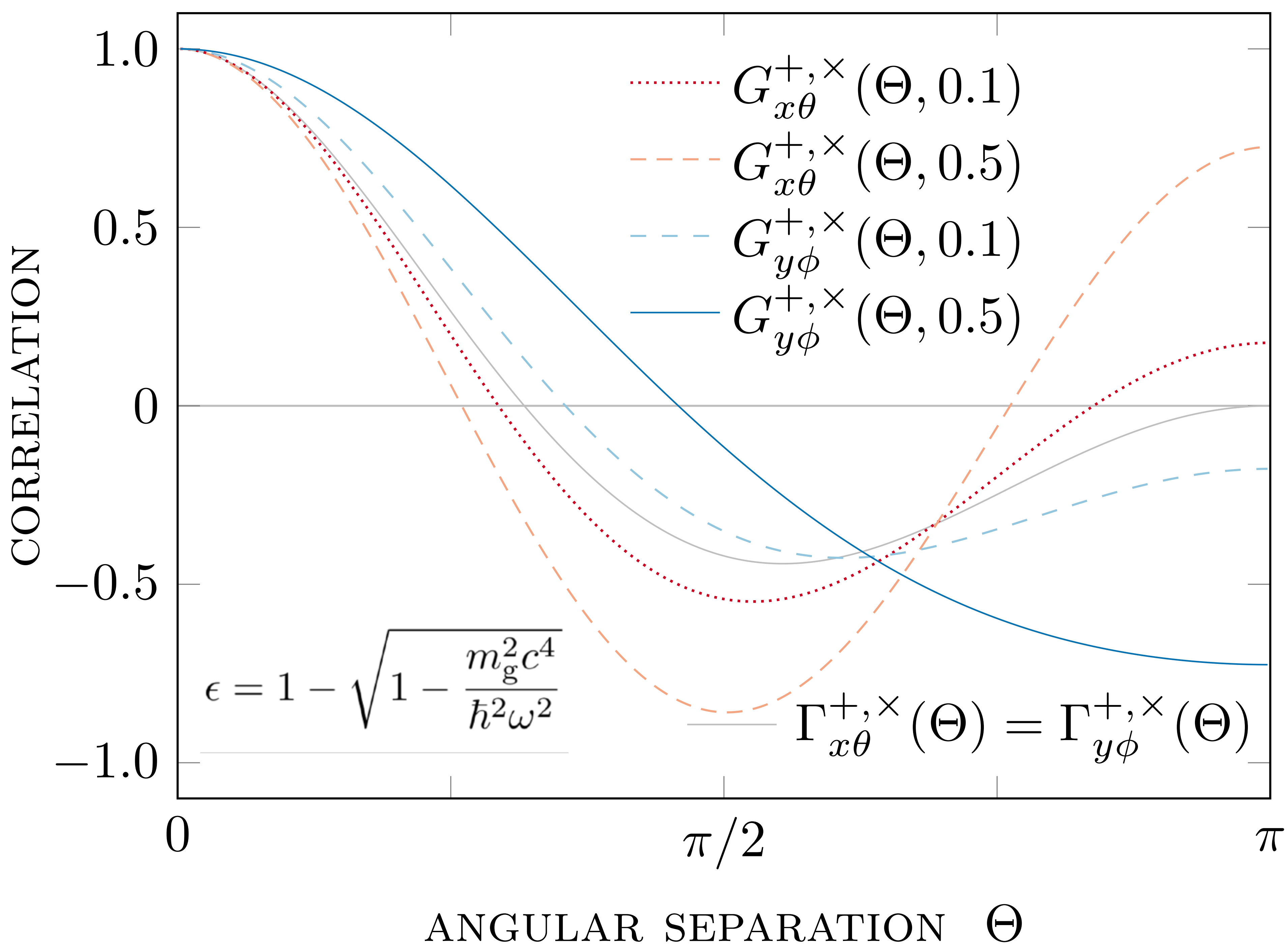
GR modes



Cf. Book and Flanagan, 2001

GR MODES WITH MASSIVE GRAVITON CORRECTIONS

$$\Gamma^{\mathrm{P}}_{ij}(\Theta, \epsilon)$$



Conclusions

1. GWs induce periodic perturbations in the astrometric measurements of stars

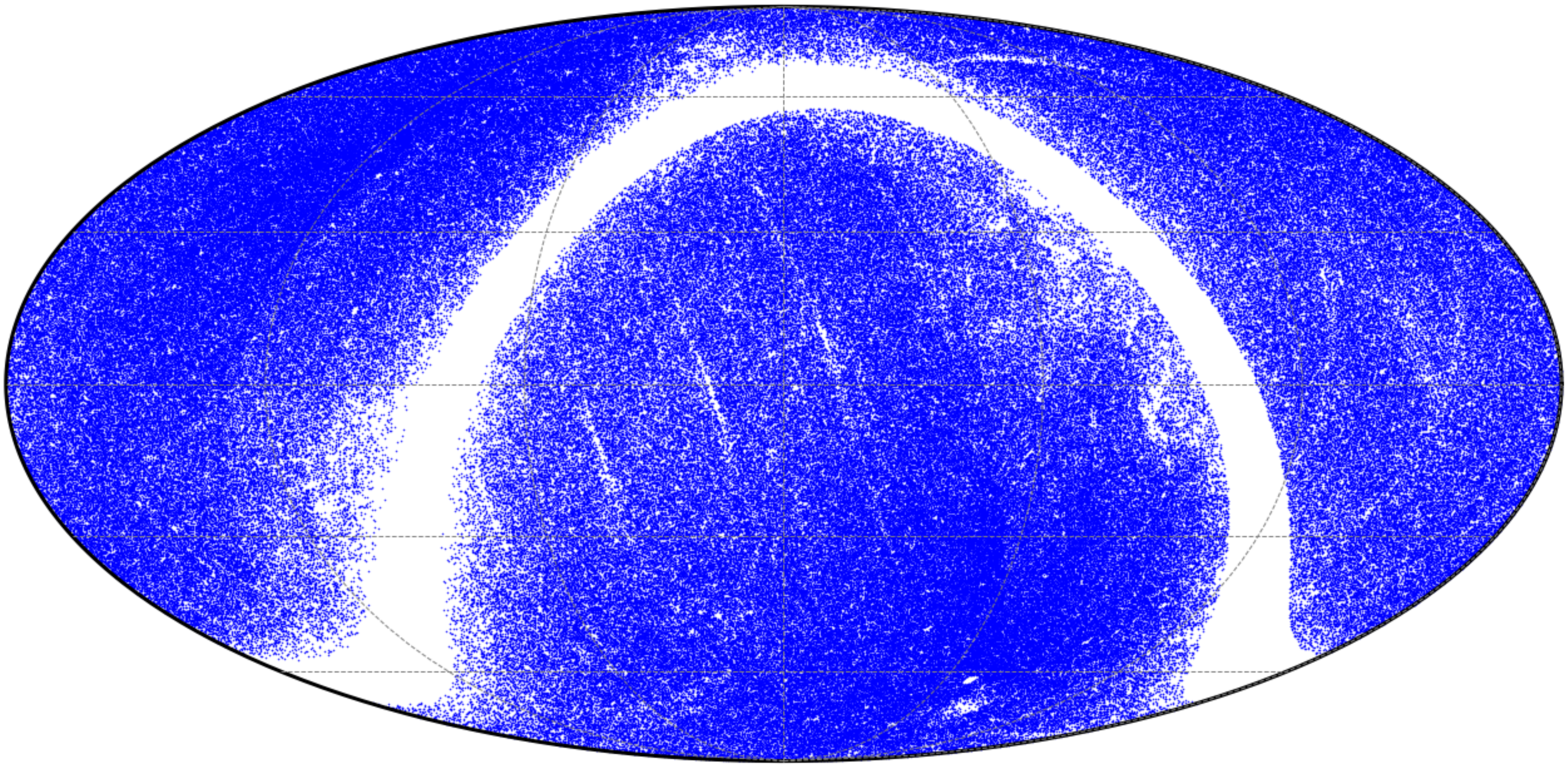
2. Gaia is the ideal tool to study this effect

3. We have developed a data analysis pipeline

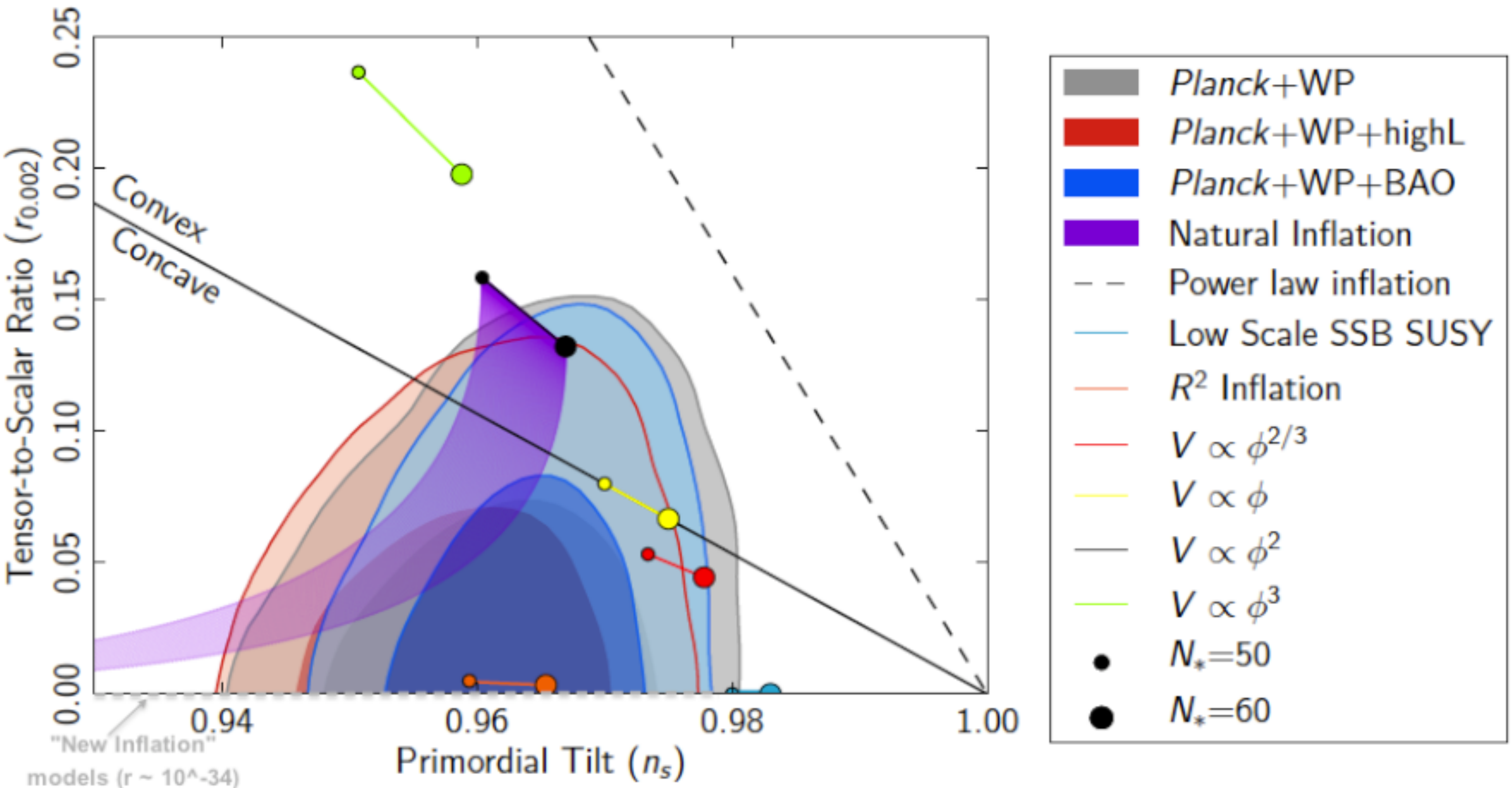
5. Data can be compresses with little loss

6. Further data releases will allow GW searches to be performed.

Further work



Further work



Further work

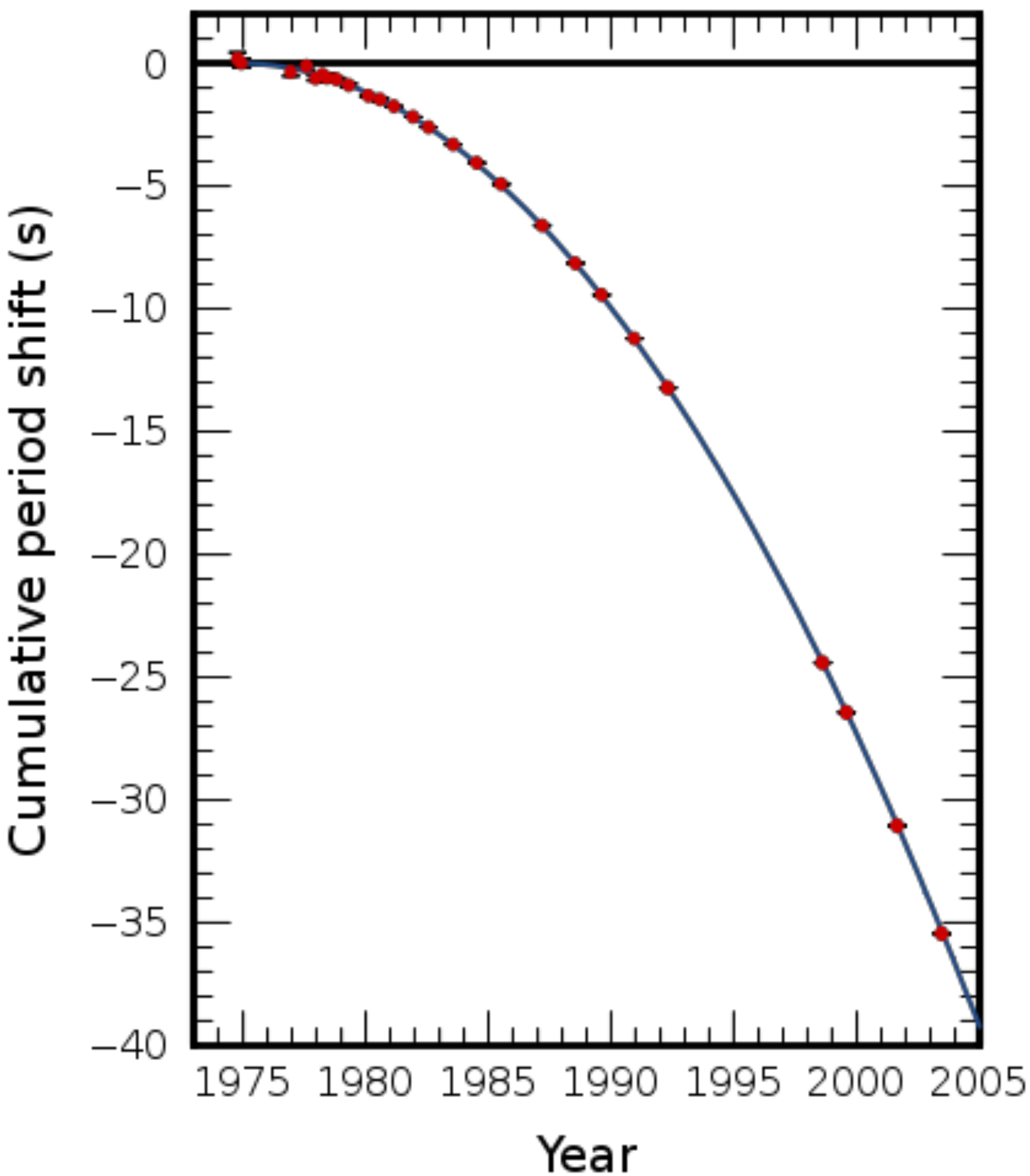
1. Systematics in Gaia DR4?
2. Pairwise velocities estimation
3. Anisotropy tests
4. Your ideas?

Acknowledgements

Max-Planck-Institut
für Gravitationsphysik
Albert-Einstein-Institut

gaia

Hulse-Taylor Binary Pulsar

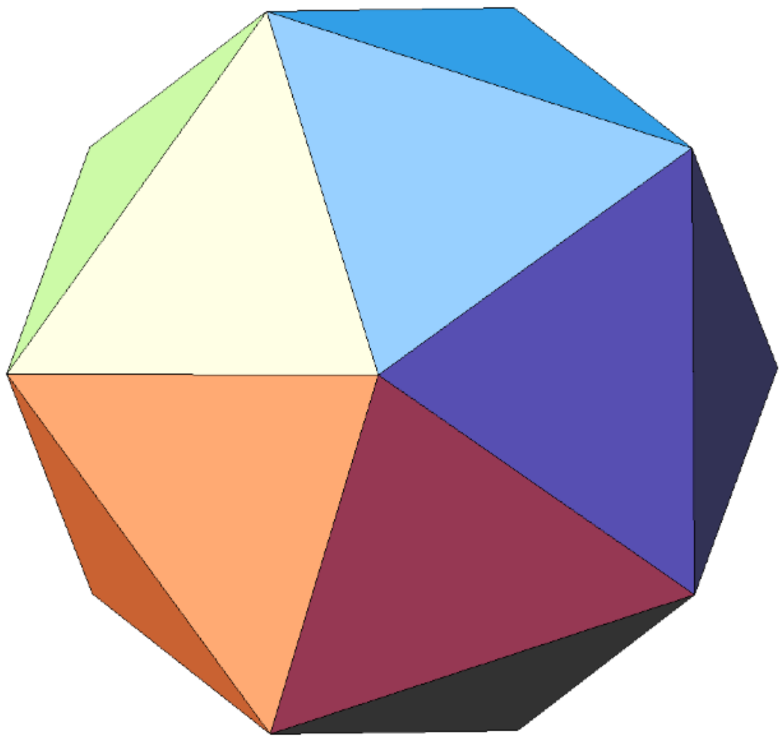
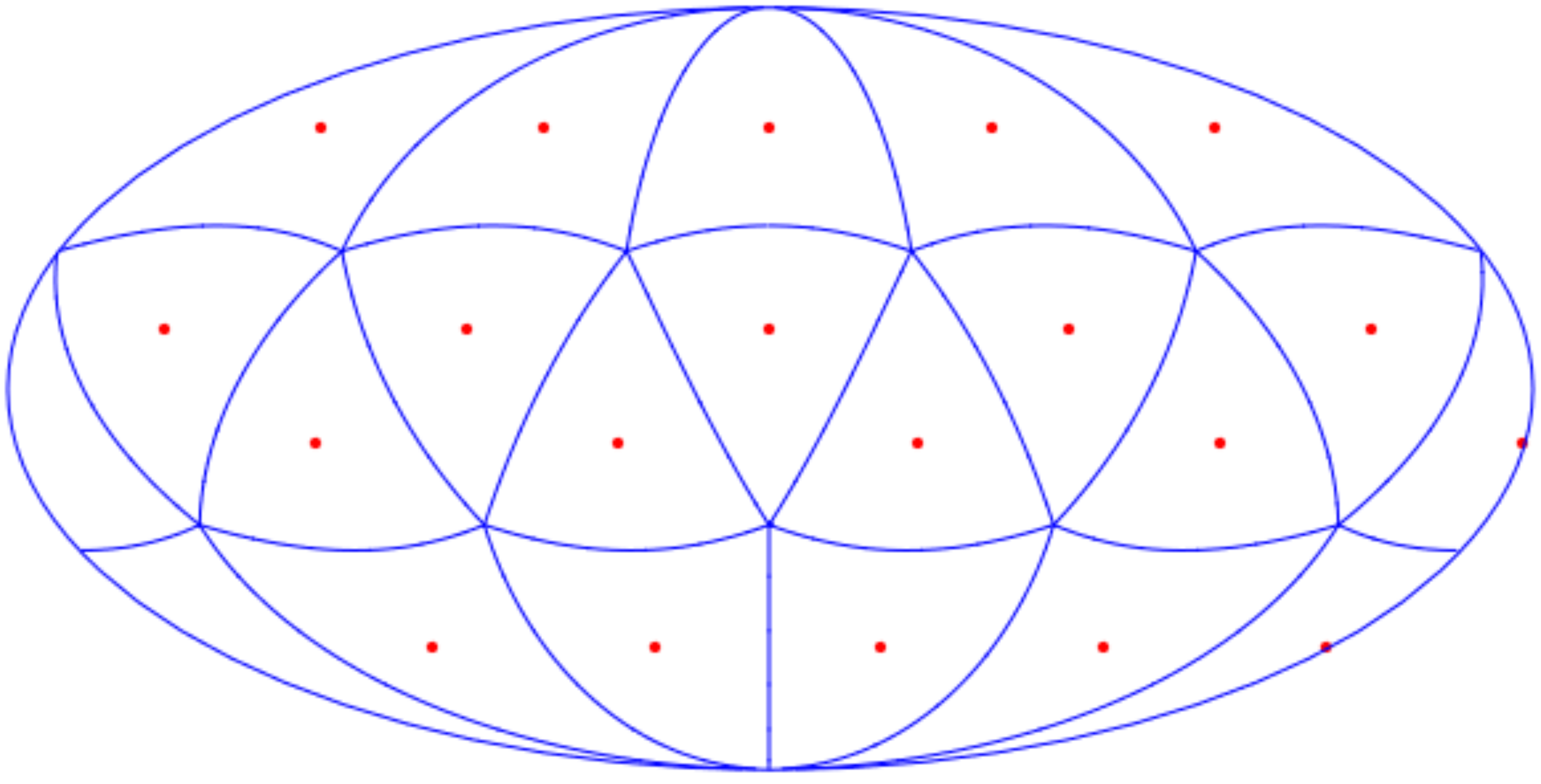


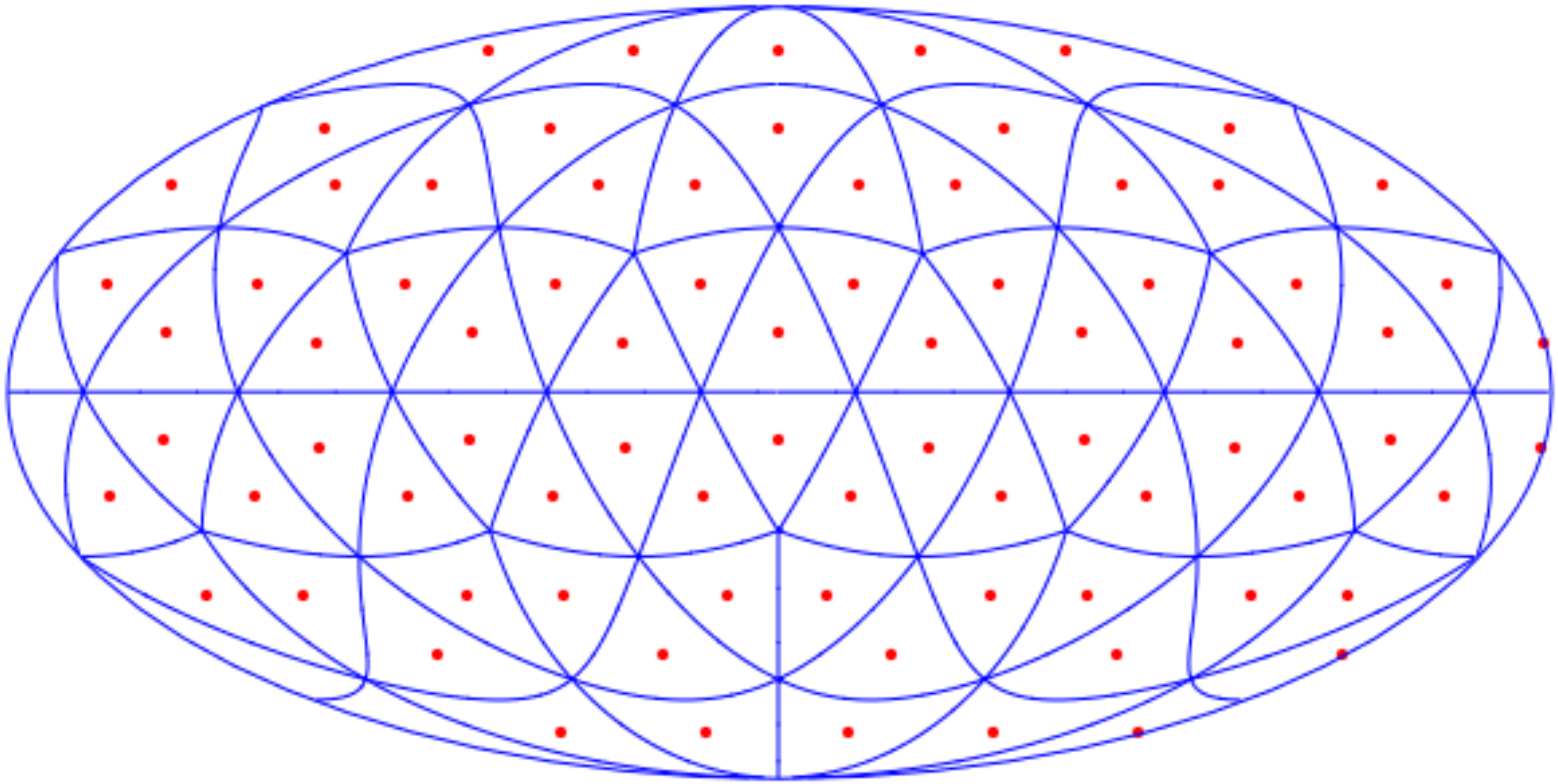
- First binary pulsar discovered in 1974
- Period decay consistent with GR
- Nobel prize in 1994

Compressing the Gaia datasets

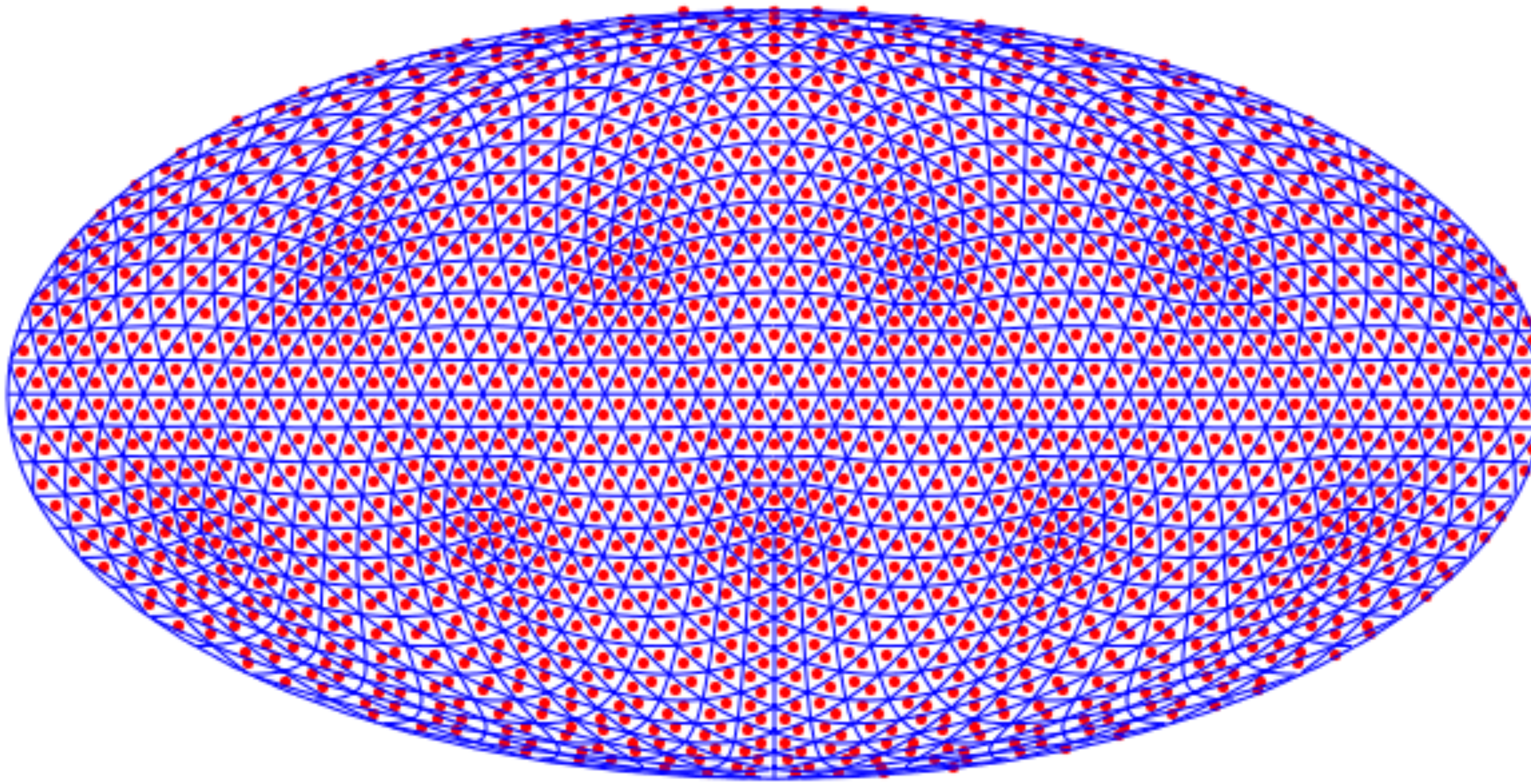
2 x 10^9 stars times 10^2 measurements = a lot of data

Geodesic Dome

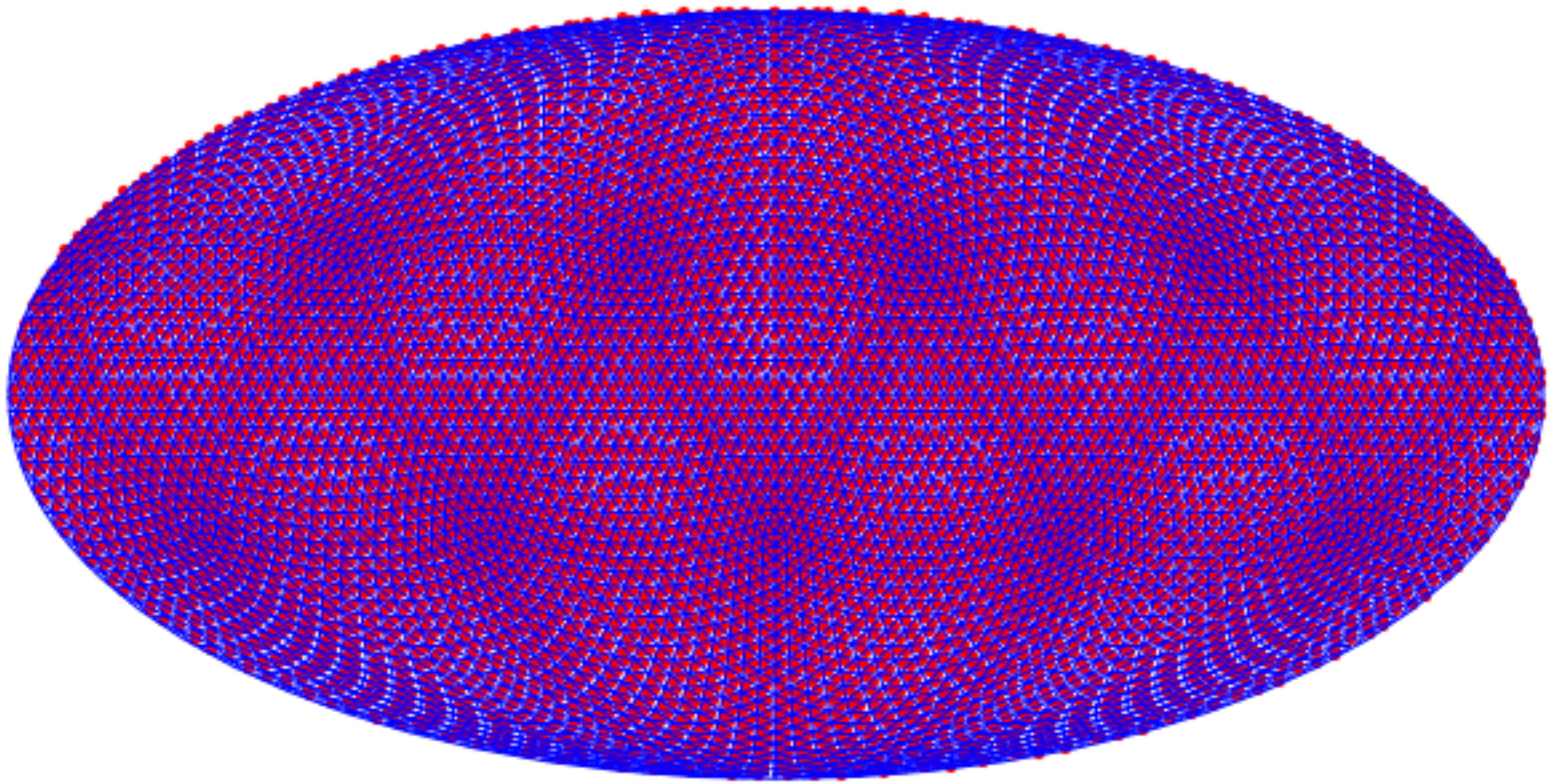





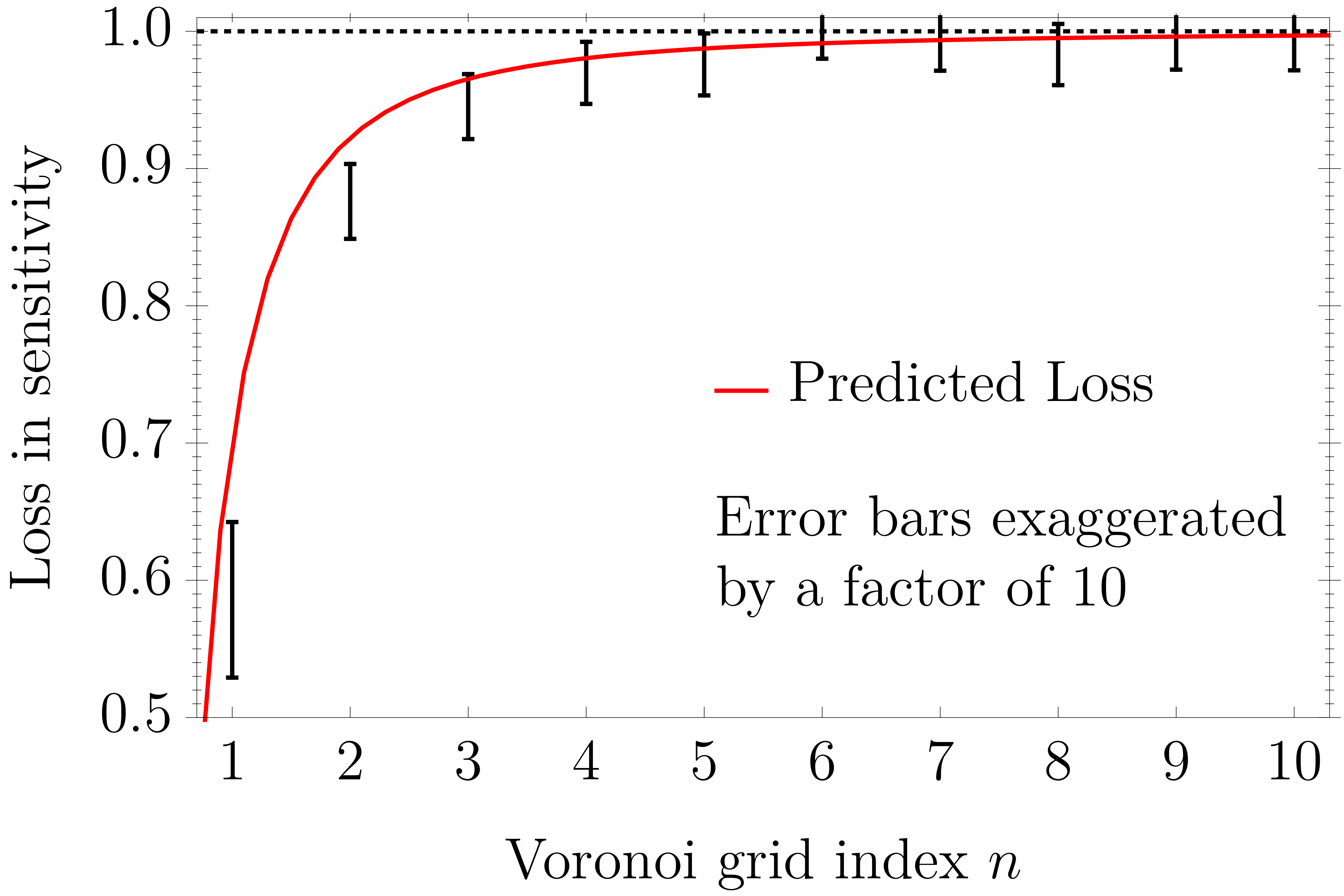
N = 2



N = 10



N = 20


1.0
0.9
0.8
0.7
0.6
0.5
Loss in sensitivity
1
2
3
4
5
6
7
8
9
10
Voronoi grid index n
Predicted Loss
Error bars exaggerated
by a factor of 10