Constraints on cosmic strings from gravitational waves, diffuse gamma-ray background and dark matter

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Outline

Introduction to cosmic strings

Cosmic strings[3]

1D topological defects

- Cosmic strings are 1D topological defects that may appear after a symmetry breaking phase transition
- After the phase transition the field falls into the new vacuum manifold $\mathcal{M}$
- Strings arise if $\mathcal{M}$ is not simply connected: there exist some closed paths on the vacuum manifold cannot be shrinked to a point
- Strings expected to form in most models of spontaneous symmetry breaking [1](some having currents)

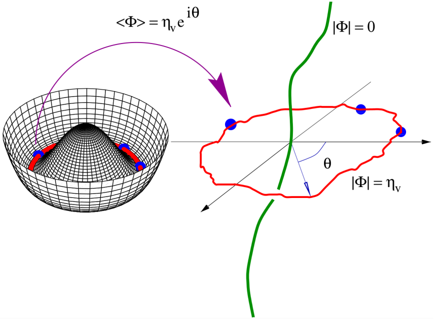


Figure: String formation in the "Mexican hat" potential $V(|\phi|)^2$

[1] Jeannerot, Rocher, and Sakellariadou 2003.
[2] Ringeval 2010.
[3] Kibble 1976.

Nambu-Goto strings: the one-dimensional limit

- Width of the string very small compared to other length scales in the problem. The thin string limit is commonly adopted.
- String simply modeled as a line with mass per unit length $\mu \propto T^2$ using the Nambu-Goto action which minimizes the area swept by the string

$$\mathcal{S} = -\mu \int \sqrt{-\det(\gamma)}\mathrm{d}^2\zeta$$

$\zeta^{\mathrm{a}} = (t, \zeta)$ and γ_{ab} the induced metric on the string

Energy scale	Width	Linear density
GUT : 10^{16} GeV	2×10^{-32} m	$G\mu \approx 10^{-6}$
3×10^{10} GeV	5×10^{-27} m	$G\mu \approx 10^{-17}$
10^8 GeV	2×10^{-24} m	$G\mu \approx 10^{-22}$
EW : 100 GeV	2×10^{-18} m	$G\mu \approx 10^{-34}$

Closed loops of cosmic strings

Oscillation and gravitational wave emissions

The general solution for a Nambu-Goto string in a Minkowski background is

$$\vec{X}(t,\zeta) = \frac{1}{2}\left[\vec{a}(\zeta - t) + \vec{b}(t+\zeta)\right]$$
$$\vec{a}'^2 = \vec{b}'^2 = 1$$

For a closed loop $X^\mu(t,\zeta+\ell) = X^\mu(t,\zeta)$. One can show that the loop oscillates with a period $T = \dfrac{\ell}{2}$.
These oscillations lead to a gravitational radiation. The *quadrupole formula* can give a **rough** estimate of the power emitted[4]

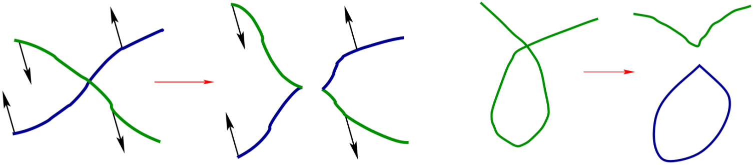
$$\dot{E} \approx G\left(\frac{\mathrm{d}^3 D}{\mathrm{d}t^3}\right)^2 \approx GM^2L^4\omega^6 \approx \Gamma G\mu^2$$

in which $D \approx ML^2$ is the quadrupole moment, $M = \mu L$ is the mass and $\omega \approx L^{-1}$ the characteristic frequency.
NOTE : it does not depend on the loop length !

[4] A. Vilenkin and Shellard 2001.

Typical properties of cosmic strings

Loop formation and scaling



- When strings intersect, they change partner
- Analytical arguments and numerical simulations suggest the existence of an attractor solution called scaling
- During scaling, all length-scales are proportional to t cosmic time.

$$\rho_\infty \propto t^{-2} \propto \begin{cases} a^{-4} & \text{during radiation era} \\ a^{-3} & \text{during matter era} \end{cases}$$

- It means loop can survive until today

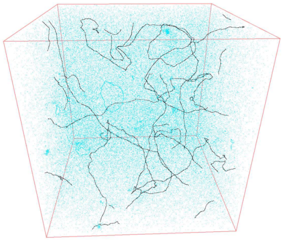


Figure: [5]

[5] Ringeval, Sakellariadou, and Bouchet 2007.

Observational signatures of cosmic strings

Selection of observational signatures

- CMB : line discontinuities in the temperature or polarization patterns, and statistical methods based on calculations of various correlation functions. $G\mu <$ few $\times 10^{-7}$
- 21-cm : brightness fluctuations or spatial correlations between the 21 cm and CMB anisotropies. Future experiments can in principle constrain $G\mu \approx 10^{-10} - 10^{-12}$
- The metric around a cosmic string can result in characteristic lensing patterns of distant light sources.

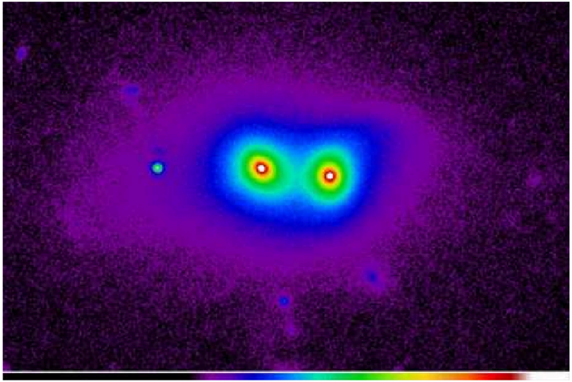


Figure: CLS-1, discovered in 2003, raised a lot of interest from the cosmic strings community but turned out to be two similar galaxies close to each other

Gravitational wave emission from cosmic strings, and constraints from LIGO/Virgo + forecast for LISA

P. Auclair et al. “Probing the gravitational wave background from cosmic strings with LISA”
P. G. Auclair “Impact of the small-scale structure on the Stochastic Background of Gravitational Waves from cosmic strings”

Bursts of gravitational waves

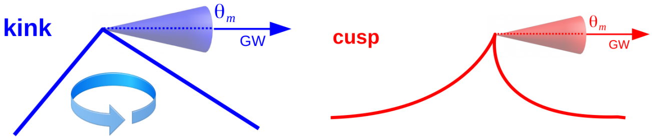


Figure: F. Robinet

A typical loop will have a number of kinks and cusps, and the spectrum of high frequency gravitational radiation emitted from a string depends on these features

- kinks are discontinuities in the tangent vector of the string. Kinks are formed when strings intercommute and travel along the string at the speed of light, $q = 5/3$.
- cusps travel instantly at the speed of light, $q = 4/3$.

The waveform of the gravitational wave arriving at the detector is known[6]

$$h_q(\ell, z, f) = A_q(\ell, z, f) f^{-q} \quad , \quad A_q = g_{1,q} \frac{G\mu\ell^{2-q}}{(1+z)^{q-1} r(z)}$$

[6] Damour and Alexander Vilenkin 2001.

Rate of bursts

For a given loop distribution, you can estimate the *GW burst rate*[7]

$$\frac{\mathrm{d}^2\mathcal{R}_q}{\mathrm{d}V\mathrm{d}\ell} = \frac{1}{1+z} \times \frac{\mathrm{d}^3\nu_q}{\mathrm{d}t\mathrm{d}\ell\mathrm{d}V} \times \Delta_q$$

as a function of

- Δ_q geometrical factor for the fraction of GWs you can access (linked to a beaming angle)
- $\frac{\mathrm{d}^3\nu_q}{\mathrm{d}t\mathrm{d}\ell\mathrm{d}V} = \frac{2}{\ell} N_q \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}$ number of events per space time volume per unit length
- N_q mean number of events per oscillation, which is supposed to be a fixed number.
- z redshift at emission

The effective burst rate in the detector depends on its sensitivity.

$$\mathcal{R}_q = \int \mathrm{d}A_q \, \mathrm{e}_q(A_q) \frac{\mathrm{d}\mathcal{R}_q}{\mathrm{d}A_q}(G\mu, N_q)$$

[7] Siemens et al. 2006.

LIGO/Virgo burst search during O1

- No cosmic string burst detected during O1 and O2 runs
- Parameter $G\mu$ is excluded at a 95% level when $\mathcal{R}_q$ exceeds $2.996/T_{\text{obs}}$ over an observation time T_{obs}.
- Allows to put upper bounds on the string tension which are not very competitive with respect to the Stochastic Background of GW
- I am currently involved in the LIGO/Virgo collaboration to produce constraints for the O3 run

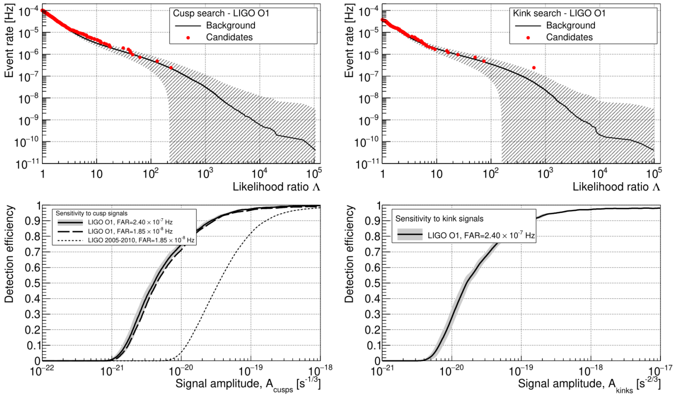


Figure: Results for O1[8]

[8] Abbott et al. 2018.

The stochastic background of gravitational waves

- The uncorrelated sum of all the GW signals produced by cosmic string loops constitutes a Stochastic Background of GW.
- We can estimate this background using energetic arguments

$$\Omega_{\rm GW}(\ln f) = \frac{8\pi G}{3H_0^2} f \rho_{\rm GW}$$

$$\rho_{\rm GW}(f) = \int_0^{t_0} \frac{\mathrm{d}t}{[1+z(t)]^4} \mathrm{P}_{\rm gw}(t, f') \frac{\partial f'}{\partial f}$$

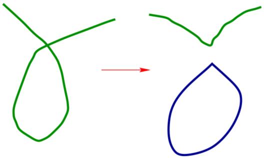
$$\mathrm{P}_{\rm gw}[t, f'] = G\mu^2 \sum_m \frac{2m}{f'^2} \mathrm{P}_m \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V}\left(\frac{2m}{f'}, t\right)$$

The loop distribution $\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V}$ remains to be specified

Number of cosmic string loops

The loop production function

- Infinite strings are stretched by the expansion of the Universe : $a(t)$
- They intersect each other and produce loops : $\mathcal{P}(\ell, t)$
- These loops decay by emitting gravitational waves : $\frac{\mathrm{d}\ell}{\mathrm{d}t} = -\Gamma G\mu$



$$\frac{\partial}{\partial t}\left(a^3 \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V}\right) + \frac{\partial}{\partial \ell}\left(\frac{\mathrm{d}\ell}{\mathrm{d}t} a^3 \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V}\right) = a^3(t)\mathcal{P}(\ell, t)$$

- The production of non-self intersecting loops is studied through numerical simulations and remains a matter of debate.
- We model the loop production function as[9]

$$t^5\mathcal{P}(\ell, t) = C\left(\frac{\ell}{t}\right)^{2\chi-3}$$

- LRS[10]: $\chi_{\mathrm{rad}} = 0.2$, $\chi_{\mathrm{mat}} = 0.295$
- BOS[11]: $\chi_{\mathrm{rad}} = 0.5$, $\chi_{\mathrm{mat}} = 0.655$

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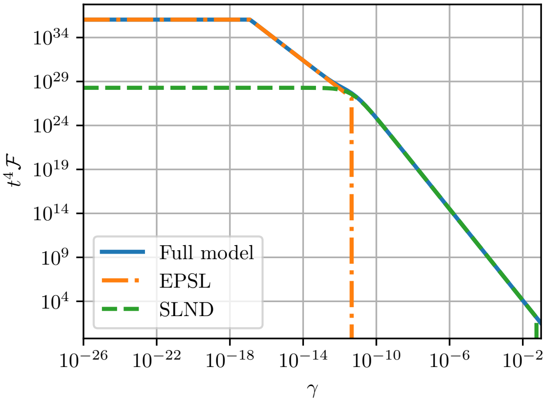
[9] Polchinski 2007.
[10] Lorenz, Ringeval, and Sakellariadou 2010.
[11] Blanco-Pillado, Olum, and Shlaer 2014.

The extra population of small loops

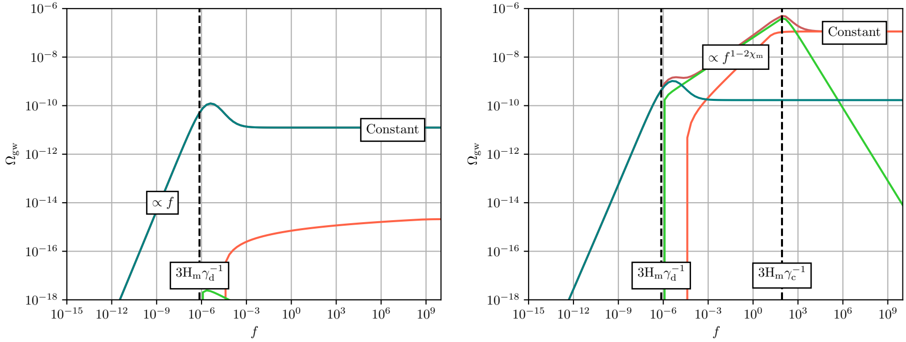
Solving the continuity equation, one finds two different populations of loops

- A **Standard Loop Number Density** qualitatively similar to the one predicted by the One-Scale Model $t^5\mathcal{P}(\ell,t)=\delta(\ell/t-\alpha)$
- An **Extra Population of Small Loops** which depends on additional theoretical inputs.



Production of gravitational waves

The stochastic background

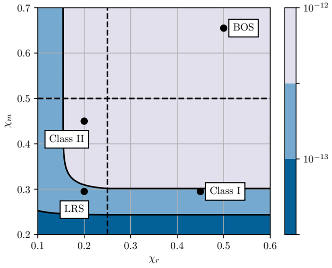


(a) BOS: $\chi_{\rm rad} = 0.5$, $\chi_{\rm mat} = 0.655$

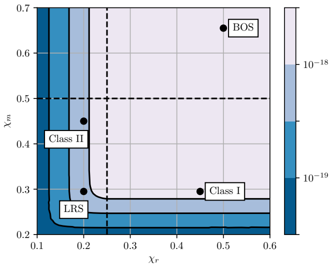
(b) LRS: $\chi_{\rm rad} = 0.2$, $\chi_{\rm mat} = 0.295$

A multi-frequency analysis[12]

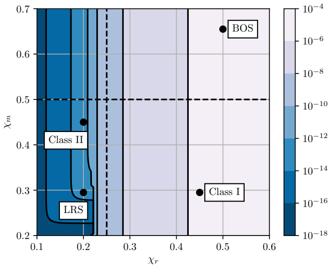
Probing the parameter space



(a) PTA, sensitivity of $\Omega_{\rm GW} = 10^{-12}$ taken at $f = 2 \times 10^{-9}$ Hz



(b) LISA, sensitivity of $\Omega_{\rm GW} = 10^{-13}$ taken at $f = 10^{-2}$ Hz



(c) LIGO/Virgo, sensitivity of $\Omega_{\rm GW} = 10^{-7}$ taken at $f = 20$ Hz

[12] P. G. Auclair 2020.

Nambu-Goto approximation and beyond: emission of massive particles and constraints from Fermi-LAT

P. Auclair et al. "Particle emission and gravitational radiation from cosmic strings: observational constraints"

Field-Theory (FT) simulations of individual loops

Formation, evolution and decay

- So far we have studied Nambu-Goto strings, ie. infinitely thin strings
- Large-scale field-theory simulations find that cosmic strings decay rapidly into particles [13]
- High resolution field theory simulation of single loops tend to show that their lifetime is actually longer that previously expected
- The rate at which strings emit particles has been measured in high-resolution numerical simulations
- We propose a first step to bridge the gap between Nambu-Goto strings and field-theory strings

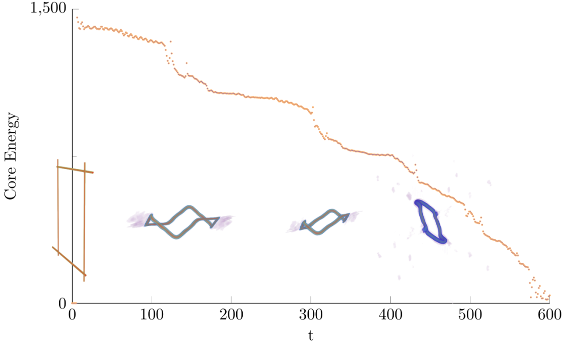


Figure: Energy of a loop with the initial size of 390 lattice spacings plotted vs time. [14]

[13] Hindmarsh, Stuckey, and Bevis 2009.
[14] Matsunami et al. 2019.

Energy budget for a cosmic string loop

We parametrize the energy lost by an average loop with $\mathcal{J}$, remember that for cosmic string loops, $E = \mu\ell$

$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = -\Gamma G\mu\mathcal{J}(\ell)$$

Where

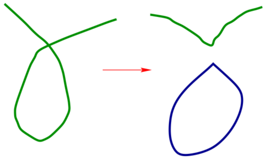
- $\mathcal{J}(\ell) = 1$ if GW emission is the only channel for losing energy
- $\mathcal{J}(\ell) = 1 + \ell_{\mathrm{k}}/\ell$ if kinks are present on the loop
- $\mathcal{J}(\ell) = 1 + \sqrt{\ell_{\mathrm{c}}/\ell}$ if cusps are present on the string

$$\ell_{\mathrm{k}} \sim \frac{w}{\Gamma G\mu} \propto (G\mu)^{-3/2}, \quad \ell_{\mathrm{c}} \sim \frac{w}{(\Gamma G\mu)^2} \propto (G\mu)^{-5/2}$$

Modeling the loop distribution with a continuity equation

Non self-intersecting loops are produced from the network of infinite strings and then lose energy

$$\frac{\partial}{\partial t}\left(a^3\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}\right)+\frac{\partial}{\partial \ell}\left(a^3\Gamma G\mu\mathcal{J}(\ell)\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}\right)=a^3\mathcal{P}(\ell,t)$$



Introducing new variables

$$\tau\equiv\Gamma G\mu t\ ,\qquad \xi\equiv\int\frac{\mathrm{d}\ell}{\mathcal{J}(\ell)}.$$

the continuity equation is rewritten

$$\left(\frac{\partial}{\partial\tau}\bigg|_{\xi}-\frac{\partial}{\partial\xi}\bigg|_{\tau}\right)\left(\Gamma G\mu\mathcal{J}a^3\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell\mathrm{d}V}\right)=a^3\mathcal{J}\mathcal{P},$$

Note that for a given loop, the following quantity is conserved along the flow

$$\Gamma G\mu t+\xi(\ell)$$

Modeling the loop distribution

Solution for a δ-function loop production function

- The shape of the loop production function is still a matter of debate[15]
- Simplest choice coming is to assume all loops are produced with the same size[16]

$$\mathcal{P}(\ell, t) = C t^{-5} \delta\left(\frac{\ell}{t} - \alpha\right)$$

- The solution for the loop number density is

$$t^4 \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V} = C \frac{1}{\mathcal{J}(\ell)} \frac{\mathcal{J}(\alpha t_\star)}{\alpha + \Gamma G\mu \mathcal{J}(\alpha t_\star)} \left(\frac{t_\star}{t}\right)^{-4} \left(\frac{a(t_\star)}{a(t)}\right)^3 .$$

  in which the loop formation time $t_\star$ satisfies the following equation : $\Gamma G\mu t_\star + \xi(\alpha t_\star) = \Gamma G\mu t + \xi(\ell)$,

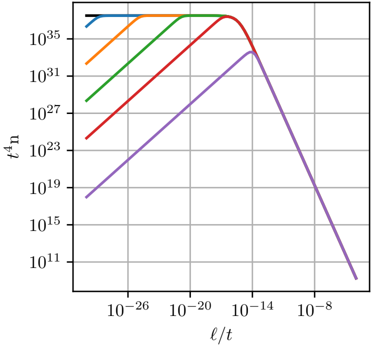
- For Nambu-Goto strings, $\mathcal{J}(\ell) = 1$ then $\xi(\ell) = \ell$ and $t^4 \dfrac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V} = C \dfrac{(\alpha + \Gamma G\mu)^{3-3\nu}}{(\ell/t + \Gamma G\mu)^{4-3\nu}}$

[15] P. Auclair, Ringeval, et al. 2019.

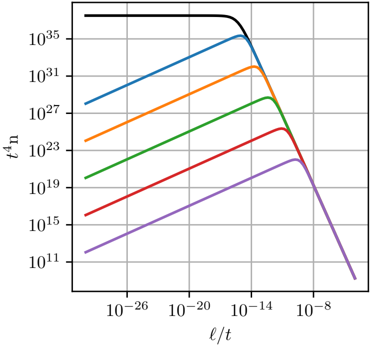
[16] Blanco-Pillado, Olum, and Shlaer 2011.

Consequences on the number of loops

Modeling the loop number density with both GW and particle emission



(a) Influence of kinks, $G\mu = 10^{-17}$



(b) Influence of cusps, $G\mu = 10^{-17}$

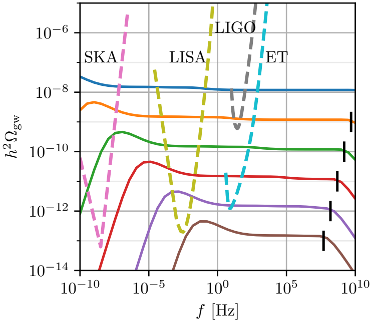
Figure: From bottom to top, the curves show snapshots of the loop distribution at redshifts $z = 10^{13}, 10^{11}, 10^{9}, 10^{7}, 10^{5}$, and the black curve is the scaling NG loop distribution

Impact on the SBGW

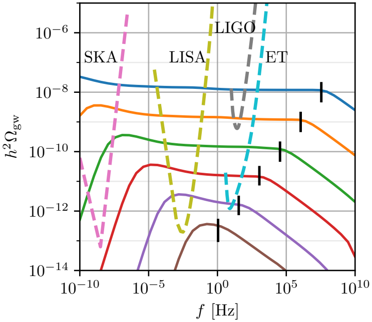
Breaking of the high frequency plateau

A consequence of the introduction of $\ell_{\mathrm{k}}, \ell_{\mathrm{c}}$ is that the high frequency plateau is cutoff at

$$f = \sqrt{\frac{2\mathrm{H}_0\sqrt{\Omega_{\mathrm{rad}}}c}{\ell_{\mathrm{c,k}}\Gamma G\mu}}$$



(a) SBGW : kinks



(b) SBGW : cusps

Particle emission bounds

Injected energy by cosmic strings[17]

- The emitted particles are heavy and in the dark particle physics sector corresponding to the fields that make up the string
- We assume that there is some interaction of the dark sector with the standard model sector

The energy density injected by cosmic strings per unit of time

$$\Phi_{\rm H}(t) = \int_0^{\alpha t} \mathrm{P}_{\rm c,k} \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}V} \mathrm{d}\ell'$$

in which

$$\mathrm{P}_{\rm k} = \Gamma G\mu \frac{\ell_{\rm k}}{\ell} \qquad \mathrm{P}_{\rm c} = \Gamma G\mu \sqrt{\frac{\ell_{\rm c}}{\ell}}$$

Then the emitted particle radiation will eventually decay, and a significant fraction of the energy $f_{\rm eff} \sim 1$ will cascade down into γ-rays.

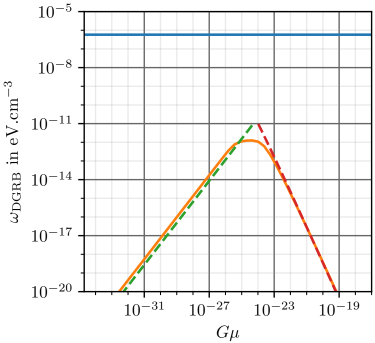
$$\omega_{\rm DGRB} = f_{\rm eff} \int_{t_c}^{t_0} \frac{\Phi_{\rm H}(t)}{(1+z)^4} \mathrm{d}t$$

[17] Mota and Hindmarsh 2015; Vachaspati 2010.

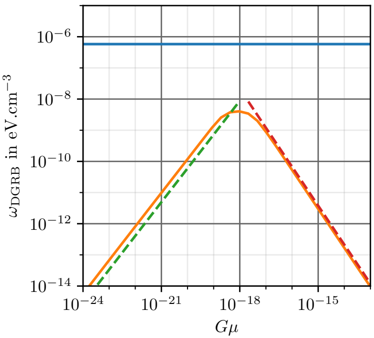
Contribution of cosmic strings to the Diffuse Gamma-Ray Background

Constraints from Fermi-LAT

$$\omega_{\mathrm{DGRB}}^{\mathrm{obs}} \leq 5.8 \times 10^{-7}\ \mathrm{eV.cm}^{-3}$$



(a) Contribution from kinks, for small $G\mu$, $\omega_{\mathrm{DGRB}} \propto \mu^{9/8}$ and $\mu^{-2}\log\mu$ for large $G\mu$



(b) Contribution from cusps, for small $G\mu$, $\omega_{\mathrm{DGRB}} \propto \mu^{13/12}$ and $\mu^{-5/3}$ for large $G\mu$

Beyond the Nambu-Goto approximation: abundance of vortons and Dark Matter

P. Auclair et al. "Irreducible cosmic production of relic vortons"

Current carrying strings[19]

Existence of vortons

- In some theories, a current can condense on cosmic strings
- This current carries angular momentum and can, in some cases, prevent the loops from collapsing
- These stable loops of current carrying cosmic string are called **vortons**.
- We consider strings formed at one energy scale and subsequently carry a current in a secondary phase transition

$$\mathcal{R} = \frac{\mathrm{T}_\phi}{\mathrm{T}_\sigma} = \lambda\sqrt{\mu}$$

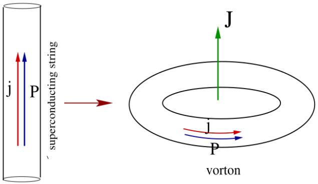


Figure: [18]

[18] Radu and Volkov 2008.
[19] Witten 1985.

Current carrying strings

Modeling the physics of vortons

- A loop is characterized by its proper length ℓ and a quantum number N
- At a length $\ell_0 = \dfrac{N}{\sqrt{\mu}}$, the loop stops shrinking and becomes a vorton
- The loop number density is the solution of a continuity equation

$$\frac{\partial}{\partial t}\left[a^3 \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}N}(\ell, t, N)\right] - \Gamma G\mu \frac{\partial}{\partial \ell}\left[a^3 \mathcal{J}(\ell, N) \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}N}(\ell, t, N)\right] = a^3 \mathcal{P}(\ell, t, N)$$

- The dynamics of the vortons can be parametrized by either

$$\mathcal{J}_1(\ell, N) = \Theta(\ell - \ell_0)$$
$$\mathcal{J}_2(\ell, N) = \frac{1}{2}\left[1 + \tanh\left(\frac{\ell - \ell_0}{\sigma}\right)\right] = \Sigma\left(\frac{\ell - \ell_0}{2\sigma}\right)$$

in which σ serves as a regulator.

3 types of loops

1. *Doomed loops*: they have an initial size too small to support a current, decay through gravitational radiation never becoming vortons

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\ell}_{\mathrm{doomed}}(\ell, t) = \int \mathrm{d}N \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}N}(\ell, t, N)\Theta(\mathcal{R} - N)$$

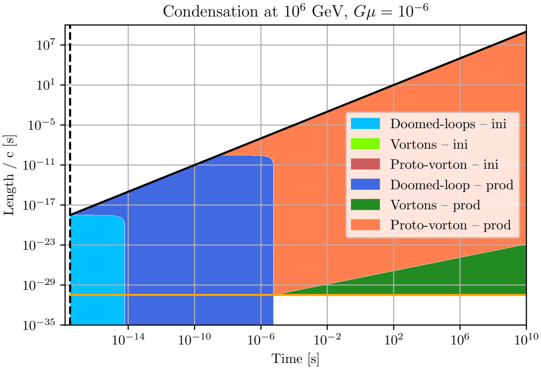
2. *Proto-vortons*: they are initially large enough to be stabilised by a current. They have not yet reached the vorton size ℓ_0.

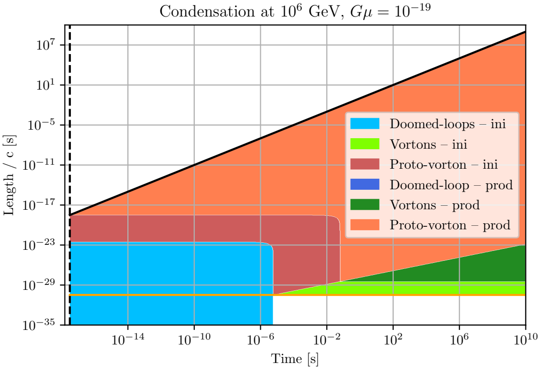
$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\ell}_{\mathrm{proto}}(t, N) = \int \mathrm{d}N\Theta(N - \mathcal{R})\frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}N}(\ell, t, N)\Theta\left[\ell - \ell_0(N)\right]$$

3. *Vortons*: all those proto-vortons which have decayed by gravitational radiation to become vortons.

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}N}_{\mathrm{vortons}}(t, N) = \Theta(N - \mathcal{R}) \int \mathrm{d}\ell \frac{\mathrm{d}^2\mathcal{N}}{\mathrm{d}\ell \mathrm{d}N}(\ell, t, N)\Theta\left[\ell_0(N) - \ell\right]$$

Population of loops

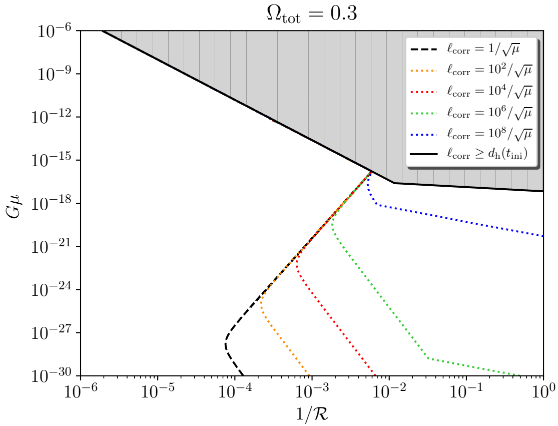




Results: Abundance of vortons

A dark matter candidate ?

$$\Omega_{\text{vortons}} \equiv \frac{8\pi G}{3H_0^2} \int \mu \ell \frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\ell}_{\text{vortons}} (\ell, t_0) \mathrm{d}\ell$$



Conclusion

Summary

- Cosmic strings are a general prediction of most symmetry-breaking models and reach a **scaling** solution meaning that the network survives until today.
- **Gravitational wave astronomy** is one of the most promising technique to probe for cosmic strings. LISA which will probe cosmic strings with tension $G\mu \geq 10^{-17}$
- Taking into account the emission of massive particles, the stochastic GW baground is **cutoff at high frequency** without conflicting with current and planned GW experiments
- Cosmic string contribute to the **Diffuse Gamma-Ray Background** without violating Fermi-LAT bounds
- Stable configuration of current carrying loops: **vortons**, can contribute to the **Dark Matter** content of the Universe

Conclusion

Future developments

- We need better estimates for the number of kinks and cusps on the loops to make stable predictions for cosmic string observables
- At present, we have assumed that all loops form with the same size. But evidence[20] suggests that non-self intersecting loops can be produced in a cascade of sizes.
- Assuming power-law loop production functions, more small loops will be produced and one expect to have stronger bounds from Diffuse Gammma Ray Background and the abundance of vortons[21]

[20] Polchinski and Rocha 2007; Ringeval, Sakellariadou, and Bouchet 2007.

[21] Work in progress

Conclusion

First detection of a Stochastic GW Background ?

NanoGrav latest results[22] have raised a lot of enthusiasm in the past weeks:

- Blasi et al. "Has NANOGrav found first evidence for cosmic strings?"
- Ellis et al. "Cosmic String Interpretation of NANOGrav Pulsar Timing Data"
- Vaskonen et al. "Did NANOGrav see a signal from primordial black hole formation?"
- De Luca et al. "NANOGrav Hints to Primordial Black Holes as Dark Matter"
- Nakai et al. "Gravitational Waves and Dark Radiation from Dark Phase Transition: Connecting NANOGrav Pulsar Timing Data and Hubble Tension"
- Addazi et al. "NANOGrav results and Dark First Order Phase Transitions"
- Buchmuller et al. "From NANOGrav to LIGO with metastable cosmic strings"
- Kohri et al. "Possible Solar-Mass Primordial Black Holes for NANOGrav Hint of Gravitational Waves"
- ...

[22] Arzoumanian et al. 2020.

Other works

First order phase transitions and turbulence

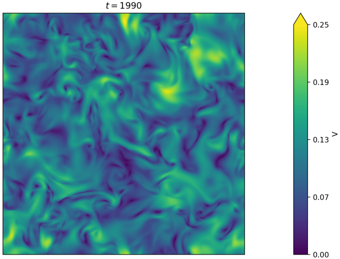


Figure: I run large scale simulations of turbulence to study their decay, decorrelation and the rate at which they emit GW. With M. Hindmarsh, D. Cutting and D. Weir.

Primordial Black Holes from preheating instability

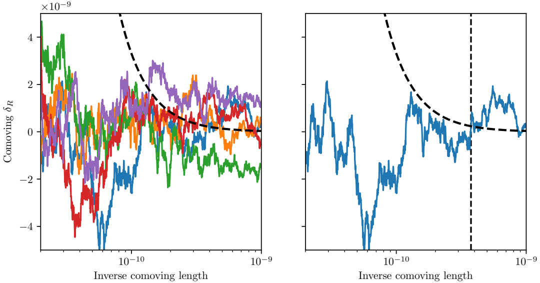


Figure: With V. Vennin, we determine the initial mass function of Primordial Black Holes during the preheating instability using the Excursion Set formalism.
Article in preparation